Nuclear Physics Approach

Nonlinear Barrier Penetration and Cold Fusion

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Abstract
The great difficulty of cold fusion as the nuclear reaction is the barrier penetration. The quantilativite calculation of the multistage chain reaction theory can explain some experimental facts of cold fusion. Further, we propose a new mechanism, the nonlinear barrier penetration. Its quantilativite results show some new characters even in a simplied model, for example, the penetration factor has a periodicity with the barrier thinckness. This is a new method which may be developed and applied further.

1. Introduction

At the present it shows by more and more experiments that cold fusion should be the nuclear reaction, including nuclear fusion and fission, because some new elements which do not exist in the beginning of experiments appear. For instance, a large number of triton appears [1], helium produces in Yamaguchi's and Takahashi's experiments, rhodium appears in deuterium of Karabut's glow discharge [2], the abundance ratio of Sr-88 to Sr-86 shifts [3], etc. The most puzzling problem on the theory of cold fusion is the barrier penetration, which is too small in comparison with the experimental facts. In order to explain this contradiction, based on the standart quantum theory, we have derived the multistage chain reaction theory [4]. In this paper we propose a new penetration mechanism based on the nonlinear quantum mechanics.

2. Multistage chain reaction theory of cold fusion

From various internal conversion mechanism on cold fusion, or from collisions and interactions among various particles and nuclei [4], we have derived the multistage chain reaction theory of cold fusion : the first chain reactions are

$$e^- + D \rightarrow 2n + \nu_e, \quad (1)$$

$$2n + D \rightarrow {}^4H^* \rightarrow {}^4He + e^- + \bar{\nu}_e, \quad (2)$$

$$e^- + D \rightarrow 2n + \nu_e \cdots \quad (1)$$

The second reactions are

$$e^- + p \rightarrow n + \nu_e, \quad (3)$$

$$n + p \rightarrow D, \quad (4)$$

$$e^- + D \rightarrow 2n + \nu_e \cdots \quad (1)$$

According to the barrier penetration of quantum mechanics, the penetration factor is

$$D_p = \exp\left\{ -\frac{2}{\hbar}\int_a^b [2m(U-E)]^{1/2} dr \right\}. \quad (5)$$

If U is the Coulomb potential ke^2/r, $a = R$ is the distance of interaction between incident particle and target, $b = r_0 = ke^2/E$,

$$D_p = \exp\left\{ -\frac{2}{\hbar}\sqrt{2m}\left[-\frac{ke^2}{\sqrt{E}} \operatorname{arctg}\left(\frac{ke^2}{RE} - 1\right)^{1/2} - \left(\frac{ke^2}{R} - E\right)^{1/2} R \right] \right\}. \quad (6)$$

For $D + Pd^{106}$, because $m = 1876.0289\text{MeV}/c^2$, $R = 1.25A^{1/3} = 5.9158\text{fm}$, $k = Z = 46$, even if $E = 240\text{KeV}$, $D_p = 9.64827 \times 10^{-52}$. It is too small. According to the multistage chain reaction theory, for the reaction (1), $m_e = 0.511\text{MeV}$, $E = 220\text{eV}$ is the energy of electron, $k = Z = 1$, $R = 1.5749\text{fm}$, so the penetration factor of electron-deuteron is $D_p = 1.7767 \times 10^{-13}$. Since the section ratio between deuteron and D-atom is about 3.814×10^{-11}, the penetration probability of nuclear reaction between an electron, whose energy is $E = 220\text{eV}$, and a D-atom is 6.777×10^{-24}, and the number of D-atom is 1.6×10^{15} per cm^2. It is consistent with Jones' experiment [5] and with Manduchi's result [6]. The current density is $\rho = 400\text{mA}/\text{cm}^2 = 2.5 \times 10^{18}$ electron / scm^2, the size of a rod cathode is $0.4 \times 1.25 = 0.5\text{cm}^2$, and $t = 6\text{day} = 5.184 \times 10^5\text{s}$ in Fig.1 of Ref. [7], so the total rate is $2.712 \times 10^{10}/\text{scm}^2$. From the total reaction

$$D + D \rightarrow {}^4He + \nu_e + \bar{\nu}_e({}^4He + 23.9\text{MeV}), \quad (7)$$

the total released energy is $2.69 \times 10^4\text{J}$. It agrees completely with this result 26KJ in the Fig.1. In a word, the nuclear reactions and the nuclear phenomena are complex process, in particular, when time accumulate continuously.

3. Nonlinear Barrier Penetration

So far, all theories of the barrier penetration are based on the linear quantum mechanics, for example, the Schrödinger equation. Combining the nonlinear quantum theory, which was developed at present, assume that a separate incident particle likes a solitary wave, and obeys a nonlinear Schrödinger equation

$$\psi_{xx} + i\psi_t - 2a^2\psi^3 = 0. \tag{8}$$

For the stable state and a square potential barrier,

$$U = \begin{cases} 0 & (x < 0,\ x > d) \\ V_0 & (0 < x < d) \end{cases}$$

the equation (8) bacomes,

$$\frac{d^2\psi}{dx^2} + k_1^2\psi - 2a^2\psi^3 = 0 \quad \text{for} \quad x < 0, \text{ or } x > d. \tag{9}$$

$$\frac{d^2\psi}{dx^2} + k_2^2\psi - 2a^2\psi^3 = 0 \quad \text{for} \quad 0 < x < d. \tag{10}$$

where

$$k_1^2 = \frac{2m}{\hbar^2}E, \qquad k_2^2 = \frac{2m}{\hbar^2}(E - V_0),$$

Let the integral constants $c_0 = 0$, $c_1 = k_1^4 / 4a^2$, when $|\psi| < k_1 / \sqrt{2}a$, a particular solution is

$$\psi_1 = \frac{k_1}{\sqrt{2}a} \cdot \frac{A^2 e^{\sqrt{2}k_1 x} - 1}{A^2 e^{\sqrt{2}k_1 x} + 1} \quad (\text{for } x < 0), \tag{11}$$

or

$$\psi_3 = \frac{k_1}{\sqrt{2}a} \cdot \frac{C^2 e^{\sqrt{2}k_1 x} - 1}{C^2 e^{\sqrt{2}k_1 x} + 1} \quad (\text{for } x > d). \tag{11'}$$

It shows that this particle corresponds a soliton. When $V_0 > E$, $k_2^2 < 0$, let $k_2'^2 = -k_2^2 = 2m(V_0 - E)/\hbar^2 > 0$, $c_0 = 0$, $c_1 = k_2'^4 / 4a^2$,

$$\psi_2 = k_2' \text{tg}(k_2' x / \sqrt{2} + B) / \sqrt{2}a \tag{12}$$

According to the continuity conditions at points $x = 0$ and $x = d$,

$$\text{tg}B = \frac{1}{k}\frac{A^2 - 1}{A^2 + 1}, \qquad \text{tg}\left(\frac{k_2'}{\sqrt{2}}d + B\right) = \frac{1}{k}\frac{C^2 e^{\sqrt{2}k_1 d} - 1}{C^2 e^{\sqrt{2}k_1 d} + 1}, \tag{13}$$

where $$k = \frac{k_2'}{k_1} = \sqrt{\frac{V_0}{E} - 1}\ . \tag{14}$$

From above we obtain

$$A^2 = \frac{\mathrm{tg}\left(\frac{k_2'}{\sqrt{2}} d\right)[C^2 e^{\sqrt{2}k_1 d}(1-k^2) - (1+k^2)] + 2kC^2 e^{\sqrt{2}k_1 d}}{\mathrm{tg}\left(\frac{k_2'}{\sqrt{2}} d\right)[C^2 e^{\sqrt{2}k_1 d}(1+k^2) - (1-k^2)] + 2k}. \tag{15}$$

One-order approximation is $C^2 / A^2 = e^{-\sqrt{2}k_1 d}$. In the linear theory, the penetration factor is $D_p = |C^2 / A^2|$.

In the nonlinear theory, a probability current density is still

$$J = \frac{i\hbar}{2m}(\psi \nabla \psi^* - \psi^* \nabla \psi), \tag{16}$$

In this case,

$$\psi^* = \frac{k_1}{\sqrt{2}a} \mathrm{tg}\left(\frac{k_1}{\sqrt{2}} x + A'\right), \tag{17}$$

Therefore, the penetration factor is

$$D_p = \frac{(A^2 e^{\sqrt{2}k_1 d} + 1)^2 \cos^2\left(\frac{k_1}{\sqrt{2}} d + A'\right)}{(C^2 e^{\sqrt{2}k_1 d} + 1)^2 \cos^2\left(\frac{k_1}{\sqrt{2}} d + C'\right)} \times \frac{[(C^2 e^{\sqrt{2}k_1 d})^2 - 1 - 2\sin(\sqrt{2}k_1 d + 2C')]}{[(A^2 e^{\sqrt{2}k_1 d})^2 - 1 - 2\sin(\sqrt{2}k_1 d + 2A')]}. \tag{18}$$

where $\mathrm{tg}A' = \frac{1 - A^2}{1 + A^2}$, $\quad \mathrm{tg}C' = \frac{1 - C^2}{1 + C^2}$.

The one-order approximation is $D_p \sim 1$. It is just a character of the soliton, which has an invariant shape for the collision. But, the calculating results show that D_p has a periodicity with d (barrier thickness), when those other quantities are difinited (Fig. 1). Although it is irrational that D_p tends toward infinite because of the tangert function, Fig. 1 possesses some similar rules with the shape of Fig. 4 in Ref.[7], where d becomes the four-dimensional time-space, and D_p is directly proportional to the number of reaction particle, the released energy and temperature. Therefore, it may be a new basis of the various resonance-penetration theories of cold fusion.

4. Discussion

Based on the known theory, some experimental results may be explained quantilativitely by the multistage chain reaction theory. But a fundamental outlet is possibly the barrier penetration of nonlinear quantum mechanics. It shows some new characters even in a simplied model. Further, the different nonlinear equations and their various solutions (kink, chaos, instanton, string, etc.) may be obtained, this new method can be discussed by the details, and may be applied to explain other experiments.

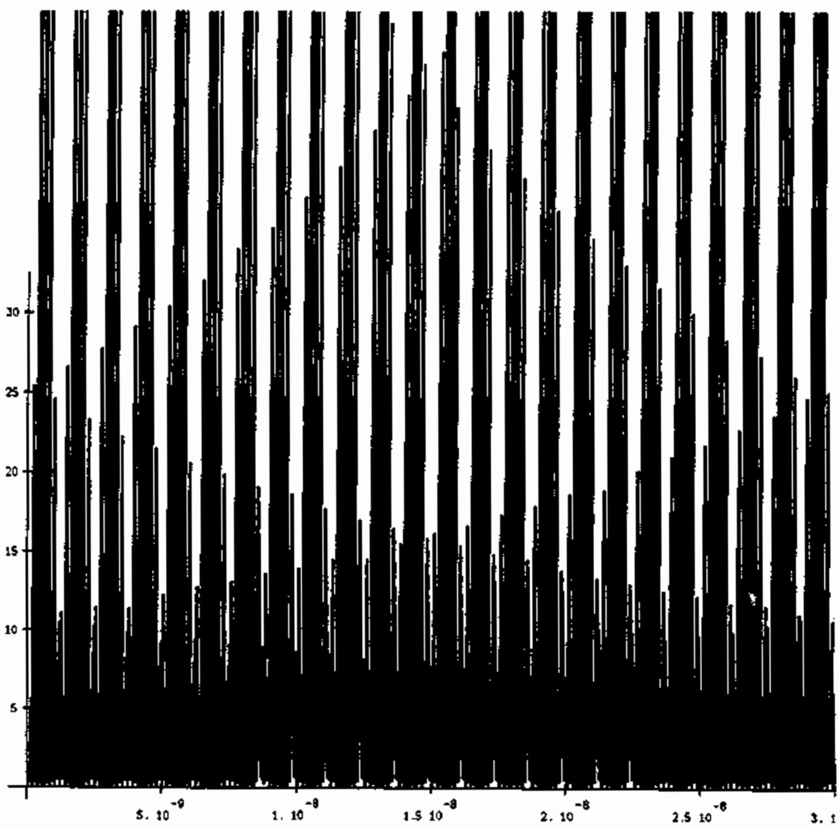

Fig. 1. Segment of D_p–d

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