Nuclear Physics Approach

The Theory of Bose-Einstein Condensation in Finite System for Explanation of Cold Fusion

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Abstract
The effect of contraction of potential well and Bose-Einstein condensation has been discussed. Due to the two processes of concentrating energy, cold fusion is possible at special conditions.

1. The effect of contraction of potential well

When deuterium ions (D^+) enter from electrolyte of heavy water into the gaps of palladium (Pd) lattice, the potential well containing D^+ changes from a big space to a small one. The process can be regarded appoximately as a contracted process of a potential well. In a cuboid well a × b × c, under Dirichlet boundary conditions, the energy level is[1]

$$E_{l_1,l_2,l_3} = \frac{h^2}{8m}\left[\frac{l_1^2}{a^2}+\frac{l_2^2}{b^2}+\frac{l_3^2}{c^2}\right], l_1,l_2,l_3 = 1,2,\cdots. \quad (1)$$

Under periodic boundary conditions, the energy level is

$$E_{l_1,l_2,l_3} = \frac{h^2}{2m}\left[\frac{l_1^2}{a^2}+\frac{l_2^2}{b^2}+\frac{l_3^2}{c^2}\right], l_1,l_2,l_3 = 0,\pm1,\pm2,\cdots. \quad (2)$$

If the well has contracted α^{-1} of the original well, from (1), we have

$$E_{n_1,n_2,n_3} = \frac{h^2}{8m\alpha^2}\left[\frac{n_1^2}{a^2}+\frac{n_2^2}{b^2}+\frac{n_3^2}{c^2}\right], n_1,n_2,n_3 = 1,2,3,\cdots. \quad (3)$$

The wave functions corresponding (1) and (3) are recpcetively

$$\psi_{l_1,l_2,l_3}(\vec{r}) = \sqrt{\frac{8}{abc}}\sin\left(\frac{l_1\pi}{a}x\right)\sin\left(\frac{l_2\pi}{b}y\right)\sin\left(\frac{l_3\pi}{c}z\right). \quad (1a)$$

$$\psi_{n_1,n_2,n_3}(\vec{r}) = \sqrt{\frac{8}{\alpha^3 abc}}\sin\left(\frac{n_1\pi}{\alpha a}x\right)\sin\left(\frac{n_2\pi}{\alpha b}y\right)\sin\left(\frac{n_3\pi}{\alpha c}z\right). \quad (3a)$$

If a particle is in an eigenstate (l_1,l_2,l_3) before contraction, the particle will no longer be in a definite energy eigenstate after contraction[2]. Let wave function after contraction be

$$\Phi(\vec{r},t) = \sum_{n_1,n_2,n_3} c_{n_1,n_2,n_3}\,\psi_{n_1,n_2,n_3}(\vec{r})\,e^{-iE_{n_1,n_2,n_3}t/\hbar}. \quad (4)$$

From (1), (3a) and (4), we have computed and obtained

$$C_{n_1,n_2,n_3} = \frac{8 l_1 l_2 l_3 \alpha^{3/2}}{\pi^3}(-1)^{l_1+l_2+l_3}\sin\left(\frac{n_1\pi}{\alpha}\right)\frac{1}{n_1^2-(l_1\alpha)^2}\sin\left(\frac{n_2\pi}{\alpha}\right)\cdot$$

$$\frac{1}{n_2^2-(l_2\alpha)^2}\sin\left(\frac{n_3\pi}{\alpha}\right)\frac{1}{n_3^2-(l_3\alpha)^2}, n_1 \neq \alpha l_1, n_2 \neq \alpha l_2, n_3 \neq \alpha l_3, \tag{5}$$

$$C_{\alpha l_1,\alpha l_2,\alpha l_3} = \frac{1}{\alpha^{3/2}}\delta_{n_1,\alpha l_1}\delta_{n_2,\alpha l_2}\delta_{n_3,\alpha l_3}.$$

From (5), we have proved the mean energy of particle at new states

$$\overline{E} = \sum_{n_1,n_2,n_3}\left|C_{n_1,n_2,n_3}\right|^2 E_{n_1,n_2,n_3} = E_{l_1,l_2,l_3}. \tag{6}$$

(6) shows that the conservation law of energy is obeyed before and after contraction, but energy can be redistributed. We have also obtain that the mean square error of particle energy at new states

$$\overline{(E-\overline{E})^2} = \sum_{n_1,n_2,n_3}\left|C_{n_1,n_2,n_3}\right|^2 E^2_{n_1,n_2,n_3} = \infty. \tag{7}$$

For periodic boundary conditions, the same proof can be completed. Equations (3), (5) and (7) show that after contraction, Maxwell-Bo ltzamann distribution (MBD) is not satisfied. From (7), we know that the possibility of distinct deviation of particle energy from mean energy increases greatly. If $\alpha >>1$, from (3), we know that the energies of near ground state decrease clearly. Before contraction, the distribution of particle energy satisfies MBD. Mean energy of a particle is $\overline{E} = \frac{3}{2}k_BT$. Let T=273K, one obtains $\overline{E}=3.5\times10^{-2}$ ev. Let a=b=c, a' =b'=c'= α a=1 nm, from (3), we have obtained $E_{111}=3.1\times10^{-4}$ ev, $E_{121}=6.2\times10^{-4}$ ev. Under periodic boundary conditions, $E_{000}=0$, $E_{100}=4.1\times10^{-4}$ ev. Clearly, $\overline{E}>>E_{111}$, E_{121}, E_{100}. After contraction, when a particle is at states near ground state, its energy was transfered to a particle at state with high energy. This is a process of concentrating energy.

2. The effect of Bose-Einstein condensation (BEC)

Deuterium nucleus $_1D^2$ is a boson. If the system of $_1D^2$ is regarded as a system of ideal bosons, its teperature of BEC is

$$T_c(\infty) = \frac{h^2}{2\pi m k_B}\left(\frac{n}{2.612}\right)^{2/3}. \tag{8}$$

Let n_D equal the density of Pd, $n_D=0.67\times10^{23}/cm^3$. Substituting this in (8), one obtains $T_c(\infty) \approx 13.15$K. Considering the effects of finiteness of system[1], under Dirichlet and Periodic boundary conditions, we have respectively

$$t_c = \frac{T_c - T_c(\infty)}{T_c(\infty)} = \frac{4\pi^2}{75\zeta\left(\frac{5}{2}\right)}\left(\frac{\pi}{15}\right)^{\frac{1}{4}} N^{\frac{1}{4}}, \tag{9}$$

$$t_c = \frac{8}{15}\frac{\zeta\left(\frac{3}{2}\right)}{\zeta\left(\frac{5}{2}\right)} N^{-1}. \tag{10}$$

From the two equations, finiteness can increase T_c, but T_c is still less that room temperature 273k.

Diameter of Pd is 0.275 nm and one of atom D is 0.092 nm. When a lattice point of Pd is displaced by atoms D, the number of atoms D, which can be contained, is

$$\left(\frac{0.275}{0.092}\right)^3 \approx 26.71.$$

If four neighboring lattice points of Pd are displaced by atoms D, the number of D is 106.84. This equivalent to a increase in N_D by 106.84 times. From (8) and (9), we obtain $T_{c'}(\infty)$=296.1K and $T_{c'}$=320.1K. Then, the system can produce BEC. The N_0 of particles at ground state is

$$N_0 = N\left[1 - \left(\frac{T}{T_c}\right)^{3/2}\right]. \tag{11}$$

We obtain N_0=0.212N. These particles transfers their energy to the particles at excited states. The contribution of energy which is supplied by one particle of ground state equals about 10^{-2} ev. But the process of concentraing energy has only a less probability to supply at energy several ev to particles at excited states. Thus, cold fusion connot take place.

If the space of size 27.5 nm in Pd lattice is occupied by D atoms, n_D increases to 26.7×10^6 times over the original value. We obtain $T_c = 1.17 \times 10^6$K. From (11), we obtain that the number of particles at exited states is $N' = 3.5 \times 10^{-6} N_0$. Each particle at ground state contributes energy 10^{-2} ev. A particle at excited states can obtain energy $10^{-2} \times (N_0/N') = 2.8 \times 10^3$ ev averagely. Deuterium nucleus ${}_1D^2$ having 10 ev can lead cold fusion at considerable probability through quantum tunnel effect.

Therefore, when atoms D concentrate in large quantities or a large cluster of atoms D are poured into Pd lattice, BEC can lead cold fusion. This work was supported by Chinese National Science Foundation Grant and by Yunnan Science Foundation Grant.

Reference

1. Peng, Kuangding, Phys. Rev. B, 48(1993), 9636.
2. J.G. Cordes et al., Am. J. Phys., 52(1986), 155.