NUCLEAR ENERGY IN AN ATOMIC LATTICE

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ABSTRACT

The distinct nature of the cold fusion regime is emphasized: electromagnetic selection rules suppress radiation, permitting excess energy transference to the lattice; the coherent nature of the wave-function is at variance with the standard separation between barrier penetration and nuclear reactivity. The discussion is restricted to tritium production, based on the dd reaction that populates the first excited state of ^{4}He, which decays into t+p, whereas the formation of ^{3}He+n is energetically forbidden. Production rates compatible with the broad range of experimental results are realized within a narrow parametric interval. The great sensitivity to the physical circumstances is reminiscent of the reproducibility problems that have plagued this field.

I believe it was Nils Bohr who defined an expert in some subject as one who has already made all possible mistakes. I stand before you as one who, in the field of cold fusion, is rapidly attaining expert status.

It began the night of March 23, 1989, when I caught the tail end of the MacNeil/Lehrer news report, which was somewhat amplified the next day by the New York Times article. As an old nuclear physicist I was, of course, quite excited by the remark — and here, for precision, I quote from the paper of Pons and Fleischmann as submitted a few days earlier — that "the bulk of the energy release is due to a hitherto unknown nuclear process or processes (presumably . . . due to clusters of deuterons)."

Surely, if something new is taking place, it ought to be associated with what — from a nuclear point of view — is new in the electrochemical arrangement. And that is the atomic lattice environment within which the putative nuclear reactions occur. Apart from a brief period of apostasy, when I echoed the conventional wisdom that atomic and nuclear energy scales are much too disparate, I have retained my belief in the importance of the lattice, as witnessed by the title of this talk.

But there must be more to the story. The early experimental situation suggested the hypothesis that two different nuclear reactions are at work. One is the apparently familiar DD reaction that, for example, produces tritium and hydrogen. The other stems from the inevitable contamination of D_2O by H_2O. An HD reaction produces ^{3}He — no tritium, no neutrons. Of course, a well trained hot fusioneer will instantly object that there must also be a 5.5 Mev γ-ray. He will not fail to point out that no such radiation has been observed. Indeed.

But consider the circumstance of cold fusion. At very low energies of relative motion, the proton and deuteron of the HD reaction are in an s-state, one of zero orbital angular momentum, and therefore of positive orbital parity. The intrinsic parities of proton, deuteron, and ^{3}He are also positive. Then, the usually dominant electric dipole radiation — which requires a parity change — is forbidden. To be sure, as in the capture of a slow neutron by a proton, magnetic dipole radiation can occur, but at a significantly slowed rate. It is not unreasonable, then, that a good fraction, or, indeed, all of the available energy will be transferred to the phonon excitations of the heavily deuterided lattice. No 5.5 Mev γ-rays.

Incidentally, I did not advance the HD hypothesis as something to be proved theoretically – that is not the nature of a hypothesis – but as the basis for obvious critical experiments in which the H_2O/D_2O ratio is altered in small steps and heat production is monitored. To my knowledge, no systematic tests along these lines have yet been completed.

From what has been said, it is clear that cold fusion and hot fusion are quite different physical domains. The following qualitative remarks present that fact in an extreme form.

To my knowledge, all treatments of nuclear fusion between positively charged particles represent the reaction rate as the product of two factors. The first factor is a barrier penetration probability. It refers entirely to the electric forces of repulsion. The second factor is an intrinsic nuclear reaction rate. It refers entirely to the nuclear forces.

This representation of the overall probability, per unit time, as the product of two independent factors, may be true enough under the circumstances of hot fusion. But in very low energy cold fusion one deals essentially with a single state, or wave function, all parts of which are coherent. It is not possible to totally isolate the effect of the electric forces from that of the nuclear forces. The correct treatment of cold fusion will be free of the collision-dominated mentality of the hot fusioneers.

This audience needs no reminder of the extreme reactions that cold fusion has engendered. Happily, the psychological situation has been stabilized by a Christmas present from Los Alamos, re-enforcing an earlier announcement by Oak Ridge, as supported by findings at Texas A & M. It is no longer possible to lightly dismiss the reality of cold fusion.

Within the general context of "Nuclear Energy in an Atomic Lattice", my focus here is on the nuclear energy generation revealed by tritium production, as it occurs within the heavily deuterided palladium lattice. It is accepted that the DD reaction is the relevant mechanism. Two deuterons in close proximity can be thought of as an excited state of ^4He. It is advisable, then, to review what is known about such states.

Under cold fusion circumstances, the two deuterons have zero relative angular momentum, implying even parity, and a total spin angular moment of 0 or 2, as restricted by Bose-Einstein statistics. The energy of this state – twice the deuteron energy of binding – is -4.4_5 Mev.

The ground state of ^4He is a 0^+ state with a binding energy of 28.3 Mev. The first excited state, which is also 0^+, has a binding energy of 8.2 Mev. It is unstable, with a width ~ 0.3 Mev, decaying into a triton (^3H) and a proton (^1H), which continuum has its threshold at -8.5 Mev, 0.3 Mev below the 0^+ excited state. In contrast, the threshold for ^3He and a neutron is at -7.7 Mev, 0.5 Mev above the first excited state. Thus the formation of the first excited state produces a source of tritium, but not of neutrons.

Now, the two-deuteron state, with energy -4.4 Mev, lies 3.8 Mev above the first excited state of ^4He. How can this excess energy, of roughly 4 Mev, be carried away? Certainly not by electric dipole emission of a photon; that is forbidden by the lack of parity reversal. And there is no counterpart to the weak magnetic dipole radiation in the HD reaction, because angular momentum 2 and angular momentum 0 do not yield a vector. There are more exotic possibilities: two photon emission, the creation of an electron-positron pair. But, the essential point, as in the HD reaction, is the likelihood that the excess energy will be transferred to the lattice.

A good deal is known about the lattice structures of pure palladium and deteurided palladium, except for the situation relevant to cold fusion, that of heavy deuteron loading. At least, I am unaware of any x-ray (or possibly neutron?) studies that would help clarify matters. There is, however, a theoretical conjecture [1] that new D_2 sites come into being, with an equilibrium separation of 0.94 Å. Inasmuch as this is significantly greater than the equilibrium separation in D_2 gas, 0.74 Å, the authors concluded that fusion in the lattice is highly improbable. To the contrary, I propose to accept their hypothesis as a basis for attempting to validate the cold fusion concept.

I have given only two public lectures on cold fusion. The first one, in October, was delivered at Albuquerque; the second one, two months ago, at Dijon, whence comes the mustard and the cassis. Although not hostile, both audiences were certainly skeptical, thus requiring detailed mathematical proof for all assertions. In this, more receptive atmosphere, perhaps precise details are less important than a qualitative survey, although if one is to come to grips with quantum mechanical subtleties, the natural language of atom physics can hardly be absent.

I begin with two important atomic parameters — those of length and of frequency. Consider a deuteron, of mass M, in the ground state of some localized site, about which it has the oscillation angular frequency ω. Let the mean square displacement from the equilibrium position, along one direction, be called Λ^2. By equating the relevant average potential energy, $\frac{1}{2}M\omega^2\Lambda^2$ to half the corresponding zero-point energy, $\frac{1}{2}(\frac{1}{2}\hbar\omega)$, one learns that $\Lambda^2 = \hbar/(2M\omega)$.

The X-ray measurements on hydrided palladium, converted to the deuteron mass, indicate the Λ value: $\Lambda \approx 10^{-9}$ cm = 0.1 Å. I shall accept this, as well, for the hypothetical D_2 sites. Given Λ, one infers that $\hbar\omega = (\hbar/\Lambda)^2/2M \approx 0.1$ ev, which sets the scale of individual phonon energies. A convenient way to express the corresponding angular frequency scale is $2\pi\omega \approx 10^{15}\ s^{-1}$.

In the early days of radar, prior to, and at the start of the second World War, although the British had begun with radio waves that were called high frequency, HF, the need for better resolution led them to VHF, very high frequency, which inexorably brought about VHFI, very high frequency indeed. I mention this ancient history because, in our study of cold fusion, CF, it is useful, and implies no serious loss of relevance, to consider VCFI, very cold fusion indeed. That is, we examine the lattice state at absolute zero. Then there are no phonon excitations – it is the phonon vacuum state – at least initially.

The latter phrase is the recognition that the phonon vacuum state is unstable. Through nuclear fusion, the deuteron constituents of the lattice are transformed, in pairs, at a certain rate, into tritium, hydrogen, and phonon energy. The anticipated number of phonons, per reaction, is measured by the ratio 3.8 Mev/0.1 eV ~ 4×10^7.

Recall that the simple independent phonon description of lattice excitations is based on the approximation of linear restoring forces or quadratic potential energies. The Hamiltonian for that phonon system will be called H_L.

Now consider a particular D_2 pair in the heavily loaded lattice. When the two deuterons are so close that fusion can occur, one is far outside the phonon domain of linear restoring forces. An additional potential energy – call it $V(\vec{R})$, with $\vec{R}$ the spatial displacement between the deuterons – comes into play. The Hamiltonian of this system is $H = H_L + V(\vec{R})$. Contact is made between $V(\vec{R})$ and the phonon description by introducing $\vec{R}_0$, the equilibrium separation of the deuterons, and writing $\vec{R} = \vec{R}_0 + \vec{r}$. Of course, an expansion in powers of $\vec{r}$, for $V(\vec{R}) = V(\vec{R}_0 + \vec{r})$, begins with cubic terms; any lesser power is already incorporated in H_L.

Time dependent perturbation theory gives the rate of transition out of the phonon vacuum state, which is the reciprocal of the mean lifetime T, as the vacuum expectation value

$$\frac{1}{T} = \frac{2\pi}{\hbar}\; {}_0\langle V\, \delta(H-E)\, V\rangle_0 \quad ,$$

where ${}_0\langle\ ,\ \rangle_0$ symbolize the phonon vacuum state. Although, for simplicity, V is written, the symbol stands for the part of V that generates one or more phonons. As already noted, it must, in fact, be a rather large number of phonons. Also, whereas one would ordinarily assign zero energy to the lattice excitations, in the phonon vacuum state, E = -4.4 Mev appears here as the nuclear measure of energy appropriate to the two deuterons.

The next steps require some mathematical details. First, one introduces the Fourier integral representation for the delta function that enforces energy conservation:

$$\frac{1}{T} = \frac{1}{\hbar}\int_{-\infty}^{\infty} d\left(\frac{\tau}{\hbar}\right)\; {}_0\langle V\, e^{-\frac{i}{\hbar}(H-E)\tau}\, V\rangle_0 \;.$$

Then one adopts a Fourier integral representation for $V(\vec{R}) = V(\vec{R}_0 + \vec{r})$:

$$V = \int \underbrace{\frac{(d\vec{p})}{(2\pi\hbar)^3}\, V(\vec{p})}\; e^{\frac{i}{\hbar}\vec{p}\cdot(\vec{R}_0+\vec{r})}$$

$$\equiv \int dV(\vec{p})\; e^{\frac{i}{\hbar}\vec{p}\cdot\vec{R}_0}\; e^{\frac{i}{\hbar}\vec{p}\cdot\vec{r}} \quad ,$$

along with the complex conjugate version, which is used for the left-hand V-factor:

$$\frac{1}{T} = \frac{1}{\hbar}\int_{-\infty}^{\infty} d\left(\frac{\tau}{\hbar}\right)\, dV(\vec{p})^*\, dV(\vec{p}\,')\; e^{-\frac{i}{\hbar}(\vec{p}-\vec{p}\,')\cdot R_0}$$

$$\times\; {}_0\langle e^{-\frac{i}{\hbar}\vec{p}\cdot\vec{r}}\; e^{-\frac{i}{\hbar}(H-E)\tau}\; e^{\frac{i}{\hbar}\vec{p}\,'\cdot\vec{r}}\rangle_0 \;.$$

The exponential factor $e^{\frac{i}{\hbar}\vec{p}\,'\cdot\vec{r}}$ acts on the phonon vacuum state to produce states with every possible number of phonons. The simplification adopted here ignores all details about the phonons except their number. The related neglect of the vector character of $\vec{p}'$ will be corrected shortly. Then, the probability amplitude for creating n phonons is

$$_n\langle e^{\frac{i}{\hbar}\vec{p}'\cdot\vec{r}}\rangle_o = \frac{1}{[n!]^{1/2}}\left[\frac{2^{1/2}\,p'\Lambda}{\hbar}\right]^n e^{-(p'\Lambda/\hbar)^2},$$

which satisfies the physical requirement of unit total probability,

$$\sum_{n=0}^{\infty}\left|_n\langle e^{\frac{i}{\hbar}\vec{p}'\cdot\vec{r}}\rangle_o\right|^2 = \sum_{n=0}^{\infty}\frac{[2(p'\Lambda/\hbar)^2]^n}{n!}\,e^{-2(p'\Lambda/\hbar)^2} = 1\ .$$

Also correctly incorporated is the fact that, as a description of the relative motion of the two deuterons, the appropriate mass is the reduced mass ½ M.

One function of the Hamiltonian H is to record the total energy of the n phonons, $n\hbar\omega$. Then the phonon part of the vacuum expectation value,

$$_o\langle e^{-\frac{i}{\hbar}\vec{p}'\cdot\vec{r}}\,e^{-\frac{i}{\hbar}(H-E)\tau}\,e^{\frac{i}{\hbar}\vec{p}'\cdot\vec{r}}\rangle_o\ ,$$

is

$$\sum_{n=0}^{\infty} e^{-(p\Lambda/\hbar)^2}\frac{1}{[n!]^{1/2}}\left[\frac{2^{1/2}p\Lambda}{\hbar}\right]^n e^{-\frac{i}{\hbar}n\hbar\omega\tau} \times\left[\frac{2^{1/2}p'\Lambda}{\hbar}\right]^n\frac{1}{[n!]^{1/2}}\,e^{-(p'\Lambda/\hbar)^2}$$

$$\longrightarrow e^{-(p\Lambda/\hbar)^2}\,e^{2\vec{p}\cdot\vec{p}'(\Lambda/\hbar)^2 e^{-i\omega\tau}}\,e^{-(p'\Lambda/\hbar)^2}$$

$$= e^{-(\vec{p}-\vec{p}')^2(\Lambda/\hbar)^2}\,e^{2\vec{p}\cdot\vec{p}'(\Lambda/\hbar)^2(e^{-i\omega\tau}-1)}$$

where the correct vectorial relation between p and p' appears. In particular, it satisfies the requirement that only $\vec{p}-\vec{p}'$ occurs for $\tau = 0$.

The other function of the Hamiltonian H is to describe the formation of the 0^+ excited bound state of ^4He:

$$e^{-\frac{i}{\hbar}(H-E)\tau} \longrightarrow\ \rangle_b\, e^{\frac{i}{\hbar}E_N\tau}\,{}_b\langle$$

$$E_N = 8.2-4.4 = 3.8\ \text{Mev}$$

where $\rangle_b$, $_b\langle$ symbolize the excited bound state. E_N, the nuclear energy released, sets the scale for τ, $\tau \sim \hbar/E_N \sim 10^{-22}$ s. As a result, $\omega\tau \sim 10^{14}\times10^{-22} \sim 10^{-8}$, and $e^{-i\omega\tau} - 1 \cong -i\omega\tau$.

The special treatment of the two deuterons requires a corresponding splitting off, from the phonon vacuum state, of the description given the relative motion of the two deuterons, each in the ground state about its equilibrium position. That is the meaning of the subscript E, for Einstein, in the following summary:

$$_o\langle e^{-\frac{i}{\hbar}\vec{p}'\cdot\vec{r}}\,e^{-\frac{i}{\hbar}(H-E)\tau}\,e^{\frac{i}{\hbar}\vec{p}'\cdot\vec{r}}\rangle_o$$

$$\longrightarrow e^{-(\vec{p}-\vec{p}')^2\left(\frac{\Lambda}{\hbar}\right)^2} e^{-\frac{i}{\hbar}\left[\frac{\vec{p}\cdot\vec{p}'}{M}-E_N\right]\tau}\left|_b\langle\ \rangle_E\right|^2.$$

The first, Gaussian, factor limits the magnitude of $\vec{p}-\vec{p}'$ to momenta $\sim \hbar/\Lambda$, which, Λ being $\sim 10^{-9}$ cm, is small on the nuclear scale. Now a crucial step is taken by recalling the additional factor $e^{-\frac{i}{\hbar}(\vec{p}-\vec{p}')\cdot R_0}$, and by writing

$$e^{-(\vec{p}-\vec{p}\,')^2 (\Lambda/\hbar)^2} \; e^{-\frac{i}{\hbar}(\vec{p}-\vec{p}\,')\cdot R_0}$$

$$= e^{-\left[(\vec{p}-\vec{p}\,')\frac{\Lambda}{\hbar}+\frac{i}{2}\frac{\vec{R}_0}{\Lambda}\right]^2} \; e^{-\frac{1}{4}(R_0/\Lambda)^2} \quad .$$

The complex substitution

$$\vec{p} \to \vec{p} - \frac{i}{4}\frac{\hbar}{\Lambda}\frac{\vec{R}_0}{\Lambda}, \qquad \vec{p}\,' \to \vec{p}\,' + \frac{i}{4}\frac{\hbar}{\Lambda}\frac{\vec{R}_0}{\Lambda},$$

which has little effect elsewhere, converts this into

$$e^{-(\vec{p}-\vec{p}\,')^2 (\Lambda/\hbar)^2} \; e^{-\frac{1}{4}(R_0/\Lambda)^2} \quad .$$

That Gaussian factor can be approximated by a delta function:

$$e^{-(\vec{p}-\vec{p}\,')^2 (\Lambda/\hbar)^2} \to \pi^{3/2}\left(\frac{\hbar}{\Lambda}\right)^3 \delta(\vec{p}-\vec{p}\,') \quad ,$$

which permits a further reduction to a statement about magnitudes,

$$e^{-(\vec{p}-\vec{p}\,')^2 (\Lambda/\hbar)^2} \to \frac{\pi^{1/2}}{2}\left(\frac{\hbar}{\Lambda}\right)^3 \frac{1}{Mp}\,\delta\left(\frac{p^2}{M}-\frac{p'^2}{M}\right).$$

But, prior to the last step, one would have carried out the τ-integration that exhibits energy conservation. With the distinction between $\vec{p}$ and $\vec{p}\,'$ removed, this integral appears as, for example,

$$\int_{-\infty}^{\infty} d\left(\frac{\tau}{\hbar}\right) e^{-\frac{i}{\hbar}\left[\frac{p'^2}{M}-E_N\right]\tau} = 2\pi\delta\left[\frac{p'^2}{M}-E_N\right].$$

Then one can return to the Gaussian function and give it the equivalent form

$$e^{-(\vec{p}-\vec{p}\,')^2 (\Lambda/\hbar)^2} \to \frac{\pi^{1/2}}{2}\left(\frac{\hbar}{\Lambda}\right)^3 \frac{1}{Mp}\,\delta\left(\frac{p^2}{M}-E_N\right)$$

$$= \hbar\omega\left(\frac{2\pi\hbar\omega}{E_N}\right)^{1/2} \delta\left(\frac{p^2}{M}-E_N\right) \quad . \qquad \left[\frac{(\hbar/\Lambda)^2}{2M} = \hbar\omega\right]$$

Here is where we are now:

$$\frac{1}{T} = \frac{1}{\hbar}\int dV(\vec{p})^* \, dV(\vec{p}\,') \; 2\pi\,\hbar\omega\left(\frac{2\pi\hbar\omega}{E_N}\right)^{1/2}$$

$$\times\, \delta\left(\frac{p^2}{M}-E_N\right)\delta\left(\frac{p'^2}{M}-E_N\right)$$

$$\times\, e^{-\frac{1}{4}(R_0/\Lambda)^2} \left|{}_b\langle\;\;\rangle_E\right|^2 \quad .$$

Attention is directed to the squared presence of the factor

$$v = \int dV(\vec{p})\;\delta\left(\frac{p^2}{M}-E_N\right)$$

$$\equiv \int \frac{(d\vec{p})}{(2\pi\hbar)^3}\, V(\vec{p})\;\delta\left(\frac{p^2}{M}-E_N\right) \quad ,$$

which is dimensionless and predominantly nuclear in content. That gives us

$$\frac{1}{T} = 2\pi\omega\left(\frac{2\pi\hbar\omega}{E_N}\right)^{1/2} v^2\, e^{-\frac{1}{4}(R_0/\Lambda)^2}\left|{}_b\langle\;\;\rangle_E\right|^2 .$$

It would be pointless to ask a precise number for v from this crude nuclear model. But, as a pure number that involves only nuclear magnitudes, the natural provisional value is $v \sim 1$. The dimensional factor that appears is

$$2\pi\omega\left(\frac{2\pi\hbar\omega}{E_N}\right)^{1/2} \sim 10^{15}\left(\frac{10^{15}}{10^{22}}\right)^{1/2} \sim 10^{12}\ s^{-1} \quad .$$

So, the inverse mean lifetime, measured in inverse seconds, is

$$\frac{1}{T} \sim 10^{12}\, e^{-\frac{1}{4}(R_0/\Lambda)^2}\left|{}_b\langle\;\;\rangle_E\right|^2 \quad .$$

The wavefunction for the relative motion of the two deuterons, in their respective ground states, is

$$\psi_E(\vec{R}) = \text{const.}\,\exp\left[-\frac{1}{2}\frac{\frac{1}{2}M\omega}{\hbar}(\vec{R}-\vec{R}_0)^2\right]$$

$$= \frac{1}{(4\pi)^{3/4}}\frac{1}{\Lambda^{3/2}}\exp\left[-\frac{1}{2}\frac{1}{4\Lambda^2}(\vec{R}-\vec{R}_0)^2\right].$$

What counts in

$$_b\langle\ \rangle_E = \int (d\vec{R})\ \psi_b^*(\vec{R})\ \psi_E(\vec{R})$$

is the behavior at short distances, $R \ll \Lambda$:

$$\psi_E(R \ll \Lambda) \cong \psi_E(0) = \frac{1}{(4\pi)^{3/4}} \frac{1}{\Lambda^{3/2}} e^{-1/8 (R_0/\Lambda)^2} .$$

As for the wavefunction of the bound state, in this oversimplified model, one can say that it has a linear dimension $\ell \sim 10^{-13}$ cm, and a corresponding amplitude $|\psi_b| \sim \ell^{-3/2}$. That gives the estimate

$$\left|\int (d\vec{R})\ \psi_b^*\ \psi_E\right| \sim \ell^3\ \ell^{-3/2} \frac{1}{\Lambda^{3/2}} e^{-1/8 (R_0/\Lambda)^2}$$

$$= (\ell/\Lambda)^{3/2}\ e^{-1/8 (R_0/\Lambda)^2} ,$$

from which one gets

$$e^{-1/4 (R_0/\Lambda)^2} \left|_b\langle\ \rangle_E\right|^2 \sim (\ell/\Lambda)^3\ e^{-1/2 (R_0/\Lambda)^2},$$

where $(\ell/\Lambda)^3 \sim (10^{-13}/10^{-9})^3 \sim 10^{-12}$. The final outcome is

$$\frac{1}{T} \sim e^{-1/2 (R_0/\Lambda)^2} .$$

With the two parameters chosen as R_0 = 0.94 Å and Λ = 0.10 Å, one gets

$$\frac{1}{T} \sim e^{-44} \sim 10^{-19}\ s^{-1} ,$$

per D_2 pair. If, for example, there are 10^{22} pairs, the rate of tritium production is 10^3/s. Changing Λ to 0.125 Å, a 25% increase, yields

$$\frac{1}{T} \sim e^{-28} \sim 10^{-12}\ s^{-1} ,$$

which, for 10^{22} pairs, is a production rate of 10^{10}/s. To my knowledge, these two examples more than span the observed range of tritium production.

But what is particularly striking is that a change in a parameter by 25% alters the production rate by a factor of ten million, a degree of sensitivity that verges on chaos. Inasmuch as the single parameter R_0/Λ combines, albeit crudely, the effects of all the forces at work within the lattice, the difficulties encountered in reproducing the cold fusion phenomena become more understandable.

REFERENCE

[1] Z. Sum and D. Tomanek, Phys. Rev. Lett. 63 p. 59:1989.