Abstract

Based on the similarity between cold fusion (CF) materials and such systems as cellular automata and a set of recursion relations that occur in phenomena characterized by complexity, we show that typical experimental data sets obtained in the cold fusion phenomenon (CFP) are qualitatively explained using concepts of complexity. Using the adjustable parameter \( n_n \) introduced in the Trapped-Neutron Catalyzed Fusion model for the CFP, as a parameter specifying a set of recursion relations, classic experimental data sets such as those by Fleischmann et al. (1989), De Ninno et al. (1989), and McKubre et al. (1993) have been reexamined as typical examples showing some phases of the CFP related with complexity. Thus, we have to expect only qualitative or statistical reproducibility in the CFP and the controversial questions such as quantitative reproducibility and controllability of events in the CFP should be considered meaningless.

1. Introduction

It is well known now that complex dynamical systems consisting of many interacting components (sub-units or agents) exhibit behavior called complexity [1, 2] that is interesting but at the same time is not an obvious consequence of the known interaction among the agents. The complexity includes such phenomena as (1) emergence, which refers to the appearance of laws, patterns or order through the cooperative effects of the agents, (2) fractals, complex geometric shapes with fine structure at arbitrarily small scales [3] and (3) power-laws, the average frequency of a given size of event is inversely proportional to some power (close to 1) of its size [4] in out-of-equilibrium states, and (4) chaos, which appears in a property of some non-linear dynamical systems showing extreme sensitivity to initial conditions and long-term aperiodic behavior that seems unpredictable [5].

The cold fusion phenomenon (CFP) in CMNS (condensed matter nuclear science) or SSNP (solid-state nuclear physics) including various events occurs automatically or induced by external effects in various complex systems consisting of many interacting components (agents) in various manners [6, 7]. The external effects include thermal neutrons, electric field, temperature change, irradiation of laser or charged particle beam, etc.

The relation between the complexity and the CFP has been explored in our works [6, 8]. There are several features of the CFP corresponding to characteristics of the complexity such as the inverse-power relation, the bifurcation and chaos; thus, we can understand that the CFP is a phenomenon characterized by nonlinear interactions between multi-component agents in the
system composed of host nuclei and hydrogen isotopes both in periodic arrays and in ambient thermal neutrons.

In this paper, we investigate qualitatively further relations between mathematical models of complexity and the CFP. It is shown that the cellular automaton and a set of recursion equations have common characteristics with CF materials where the CFP occurs. The mathematical tools used in these mathematical models could be used in investigation of events in corresponding events of the CFP.

2. Complexity in Systems with Nonlinear Interactions between Components

2.1 The Cellular Automata and the CFP

A very interesting situation arises in systems consisting of a lattice of identical boolean components interacting locally in space. Such dynamic systems are referred to as cellular automata [1, §3.12]. A factor in the CFP closely related to the cellular automata is the occupation number of hydrogen isotopes in a host lattice if we assume the latter is perfect. The occupation number \( X(r_i; t) \) of D or H (D/H) at a site \( r_i \equiv (x_i, y_i, z_i) \) and a time \( t \) satisfies the following relation:

\[
X(r_i; t + 1) \equiv X(x_i, y_i, z_i; t + 1) = F_i \left\{ X(r_j; t) \right\}
\]

(2.1.1)

where \( \{ X(r_j; t) \} \) is a set of values \( X(r_j; t) \) for \( r_j \) adjacent to \( r_i \) and \( F_i(\{ X(r_j; t) \}) \equiv F_i \) is a function characterizing the change of the occupation number of D/H at \( r_i \) by processes in the system such as (1) diffusion of D/H and (2) consumption of D/H by nuclear reactions \( n + d = t + \phi, n + p = d + \phi \) and so forth (\( \phi \) means phonons in these equations).

The function \( F_i \) is therefore a function of the parameter \( n_n \) (number density of thermal neutrons in the CF material) of the Trapped-Neutron Catalyzed Fusion model if we use the model to describe the CFP.

2.2 Recursion relations and the CFP

Recursion equations \( x_{n+1} = \lambda f(x_n) \) have been investigated in nonlinear dynamics for its usefulness to describe a variety of problems [9]. From our point of view, it is interesting to notice that a few experimental data obtained in the CFP seems to have characteristics of the type seen in recursion equations, which suggests complexity in the CFP.

2.2.1 Recursion Relations

The recursion relations

\[
x_{n+1} = \lambda f(x_n),
\]

(2.2.1)

which have been well investigated in nonlinear dynamics, have interesting characteristics, as shown by Feigenbaum [9]. A large class of recursion relations (2.2.1) exhibiting infinite bifurcation possesses a rich quantitative structure – essentially independent of the recursion functions \( f(x) \) – when they have a unique differentiable maximum \( x \). With \( f(x) - f(x) \sim |x - x|^z \) (for \( |x - x| \) sufficiently small) and \( z > 1 \), the universal details depend only upon \( z \). In particular,
the local structure of high-order stability sets is shown to approach universality rescaling in successive bifurcations, asymptotically by the ratio $\alpha (\alpha = 2.5029078750957\ldots$ for $z = 2$) [9].

We can investigate the CFP from the point of view described by the recursion relations (2.2.1). The cold fusion phenomenon (CFP) contains events related to nuclear reactions accompanying excess energy production. In these events, we have experimentally determined parameters governing the occurrence of the CFP such as the loading ratio $\eta$ (=D/Pd, H/Ni, etc.), temperature $T$, current density $i$ to the cathode in electrolytic and discharge systems, etc. The events also show a characteristic of the recursion relations (2.2.1).

2.2.2 Structure of the Population (Density) Equation

The recursion equations (2.2.1) provide a description for a variety of problems. For example, a discrete population (density) satisfies the formula

$$p_{n+1} = f(p_n),$$

which determines the population (density) at one time in terms of its previous value [9].

The events we observe in the CFP seem to belong to this type of quantities obeying the relations (2.2.1) as we have partly shown in recent works [8].

If the population (or density) referred to is such that of a dilute group of organisms (or agents e.g. the density of trapped neutrons $n_n$ in the CFP), then the population (density) equations,

$$p_{n+1} = b_p p_n,$$

accurately describe the population (density) growth with a growth rate $b$ so long as it remains dilute, with the solution of the equation

$$p_n = p_0 b^n.$$ (2.2.4)

For a given species of organism (agent) in a fixed milieu, $b$ is a constant – the static birth rate for the configuration.

As the population grows, the dilute approximation will ultimately fail: sufficient organisms (agents) are present and mutually interfere. At this point, the next value of the population (density) will be determined by a dynamic or effective birth rate $b_{eff}$:

$$p_{n+1} = b_{eff} p_n$$ (2.2.5)

with $b_{eff} < b$. Clearly, $b_{eff}$ is a function of $p$, with

$$\lim_{p \to 0} b_{eff} (p) = b,$$ (2.2.6)

the only model-independent quantitative feature of $b_{eff}$. It is also clear that

$$\lim_{p \to \infty} b_{eff} (p) = 0.$$ (2.2.7)

Accordingly, the simplest form of $b_{eff}$ to reproduce the qualitative dynamics of such a population (density) should resemble Fig. 1, where $b_{eff} (0) = b$ is an adjustable parameter.
A simple specific form of $b_{\text{eff}}(p)$ is written with a constant $a$;

$$b_{\text{eff}}(p) = b - a \, p,$$

so that

$$p_{n+1} = b_{\text{eff}} \, p_n - a \, p_n^2.$$ 

By defining $p_n = (b/a)\, x_n$, we obtain the standard form of logistic difference equations (l.d.e.)

$$x_{n+1} = bx_n (1 - x_n).$$ (2.2.9)

In (2.2.9), the adjustable parameter $b$ is purely multiplicative. With a different choice of $b_{\text{eff}}$, $x_{n+1}$ would not in general depend upon $b$ in so simple a fashion [9]. Nevertheless, the internal $b$ dependence may be (and often is) sufficiently mild in comparison to the multiplicative dependence that at least for qualitative purposes the internal dependence can be ignored. Thus, with $f(p) = p \, b_{\text{eff}}(p)$, any function like Fig. 1,

$$p_{n+1} = b f(p_n)$$ (2.2.10)

is compatible and representative of the population (density) equation discussed in [9].

![Figure 1](image)

Figure 1. Dependence of $f(p) = p \, b_{\text{eff}}(p)$ on $p$ after Feigenbaum [9].

To investigate the structure of the population (density) equation (2.2.9), we can use such a study of the l.d.e. as given by J. Gleick [5].

3. Discussion – The Cold Fusion Phenomenon in Composite Systems with Hydrogen Isotopes

As discussed in the preceding two subsections, the CF systems where the CFP occurs have characteristics common with cellular automata and the set of recursion equations which are well investigated mathematically. We discuss further here only the relation of the CFP with cellular automata.
Cellular automata, explained in Section 2.1, have a corresponding structure to the CF systems. In the present situation of analysis of the CFP, it is difficult to determine the exact form of the function $F_i$ in Eq. (2.1.1) for an event in the CFP. A possible investigation of a cellular automaton in the CFP is facilitated on the quantal approach given by us [6 §3.7]. The number of neutrons $n_i(\tau)$ is a function of values of $n_i(\tau')$ and $n_i(\tau')$ where $\tau = \tau' + d\tau$;

$$n_i(\tau) = f(n_{up}(ii'j; \tau')).$$  \hspace{1cm} (3.1.1)

Unfortunately, this approach is not complete, and we are far from a position of being able to determine the functional form $F_i$ in Eq. (2.1.1) or $f$ in Eq. (3.1.1) which will be objects of future works.

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**References**

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