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RELATIVISTIC HYPERFINE INTERACTION AND THE SPENCE-VARY RESONANCE

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Introduction

Spence and Vary[1,2] have reported a resonance in calculations of positronium and hydrogen in the "axion" (0^-) channel. The energy and lifetime of the positronium resonances have led these authors to suggest this new state as an explanation for the anomalous e^+e^- peaks seen at GSI[3]. They and others[4] speculate that similar states in hydrogen may explain anomalous nuclear reactions reported in metal lattices at low temperatures ("cold fusion").

Spence and Vary calculate the Bethe-Salpeter four-leg amplitude using a Blankenbechler-Sugar reduction. They use single photon exchange in Coulomb gauge for the kernel of their equations. Their results seem to depend critically on the use of this gauge. Attempts by others[5] to reproduce their result in Feynman gauge have not been successful. The starting point in either calculation is gauge invariant so the reduction formalism must introduce spurious gauge dependence. Whether the results of Spence and Vary are spurious is not known at this time.

In an attempt to address this question in a qualitative yet gauge-invariant way, we have studied the two fermion system using the Breit equation. The wavefunctions explicitly obey current conservation so the Coulomb gauge terms can have no effect on the results. For the purposes of obtaining qualitative features of the affect of the hyperfine interaction at short distances we approximate the relative coordinate Breit equation by the equivalent Schrödinger-form equation for hydrogen ($m_1 \gg m_2$, for applications to positronium we use the reduced mass). We examine the hyperfine interaction in the axion channel and solve the equation in the energy range of interest ($0 \rightarrow 2$ MeV). We find the hyperfine interaction introduces an effective attractive interaction at very short distances (~10 fm for positronium), but find no evidence for a resonance in the energy range of interest.

Description of Calculation

The Breit equation assumes an additive hamiltonian in the space of the two interacting Dirac particles. This cannot be exact, but provides a reasonable and tractable model for examining qualitative relativistic features and provides quantitatively accurate results for weak interactions. The resulting equation separates into relative and center-of-mass coordinates[6]. The relative coordinate Breit equation with single photon exchange interaction is

$$[(\vec{\alpha}_1 - \vec{\alpha}_2)\cdot\vec{p} - \beta_1 m_1 - \beta_2 m_2 - \frac{e^2}{r}(1 - \vec{\alpha}_1\cdot\vec{\alpha}_2)]\Psi = E\Psi \qquad (1)$$

We use Feynman gauge for the interaction but note that since the wavefunction explicitly obeys current conservation the calculation is independent of gauge. This equation can be solved in the

16x16 matrix product space using the Fermi-Yang[7] representation. For the purpose of qualitatively studying a possible resonance, it is sufficient to consider the hydrogen atom limit ($m_1 \gg m_2$). This allows one to reduce the Breit equation to an effective Schrödinger equation[8]:

$$\left(\frac{-\nabla^2}{2m} + V(r)\right)\psi(\vec{r}) = E\psi(\vec{r}). \tag{2}$$

where

$$\begin{aligned} V(\vec{r}) =& \frac{-E}{m}\frac{e^2}{r} + \frac{l(l+1)-e^4}{2mr^2} - \frac{1}{2m(E+m+\frac{e^2}{r})}\frac{e^2}{r^3}(r\frac{d}{dr} - 2\vec{s}\cdot\vec{L}) \\ &+ \frac{e}{2m}\frac{2}{(E+m+\frac{e^2}{r})}\frac{e^2}{r^3}\mu\vec{I}\cdot(\frac{\vec{s}}{r} - \frac{\vec{r}\vec{s}\cdot\vec{r}}{r^3}) \\ &+ \frac{e}{2m}2\mu\vec{I}\cdot(\frac{\vec{L}-\vec{s}}{r^3} + \frac{3\vec{r}\vec{s}\cdot\vec{r}}{r^5} - 4\pi\delta^3(\vec{r})) \\ &+ \frac{e^2}{2m}\frac{\mu^2}{r^4}(I^2 - \vec{I}\cdot\hat{r}\vec{I}\cdot\hat{r}) \end{aligned} \tag{3}$$

and where $\vec{I}$ is the spin of the 'heavy' particle and μ is its magnetic moment. Equation (3) differs from the corresponding expression in Ref. (8) because no assumptions regarding the relative magnitudes of E and e^2/r have been made. This expression is appropriate for hydrogen and should provide qualitative results applied to positronium using a reduced mass.

For the axion resonance (0^-) the spin-dependent operators are diagonal in the $|[[s_1,l]j_1,I]J>$ representation.

$$2\vec{s}\cdot\vec{L} = (j_1(j_1+1) - l(l+1) - 3/4)|[[s_1,l]j_1,I]J> \tag{4}$$

$$\vec{I}\cdot\hat{r}\vec{s}\cdot\hat{r}|[[s_1,l]j_1,I]J> = \frac{J(J+1)-j_1(j_1+1)-3/4}{8j_1(j_1+1)}|[[s_1,l]j_1,I]J> \tag{5}$$

$$\vec{I}\cdot\vec{s}|[[s_1,l]j_1,I]J> = \frac{j_1(j_1+1)-l(l+1)+3/4}{2j_1(j_1+1)}\frac{J(J+1)-J_1(j_1+1)-3/4}{2}|[[s_1,l]j_1,I]J> \tag{6}$$

The resulting effective potential for positronium is shown in Fig. 1. Note the four regions where the various terms in V dominate. For r greater than about one angstrom the familiar coulomb attraction dominates. Between about 10 fermis and 1 angstrom the centrifugal repulsion dominates. The hyperfine interaction dominates the effective interaction below about 10 fermis. In the axion channel for positronium the hyperfine interaction is attractive down to about 1 fermi below which the repulsive $1/r^4$ hyperfine term dominates. Note, especially, the large scale of the effective interaction at these short distances. For any hope of quantitative accuracy, one should include vacuum polarization effects. At short distances such corrections will be large on the scale of the electron rest mass, but still small on the scale of the main effective interaction term, and as such should not affect the qualitative features found by neglecting them.

The effective Schrödinger equation is solved numerically using Numerov with regular boundary conditions near the origin. To provide numerical stability over the enormous range of distance scales we use a dynamically rescaled (exponentially) grid spacing. These solutions may be matched onto asymptotic Coulomb wavefunctions and the hyperfine part of the phase shift extracted. We find that the relativistic hyperfine phase shift is small ($\sim .01$ radians) and slowly varying over the range of energies near the reported resonance. In this range the attractive hyperfine interaction puts a node in the wavefunction at around 10 fermis (see Fig. 2). The phase shift is given approximately by this distance times the asymptotic wave number. The behavior of the wavefunction in the short distance region is quite stable in this energy range. In conclusion we have presented an approximate, but gauge invariant, calculation of the relativistic hyperfine phase shift. The calculation, while not quantitatively accurate, should reveal qualitative features such as scattering resonances. We find no evidence for a resonance in the energy range of that reported by Spence and Vary.

Acknowledgements

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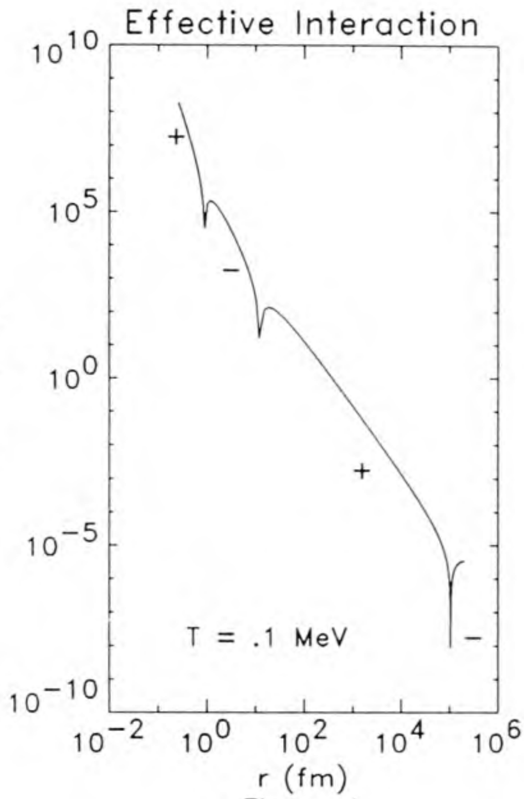

Figure 1

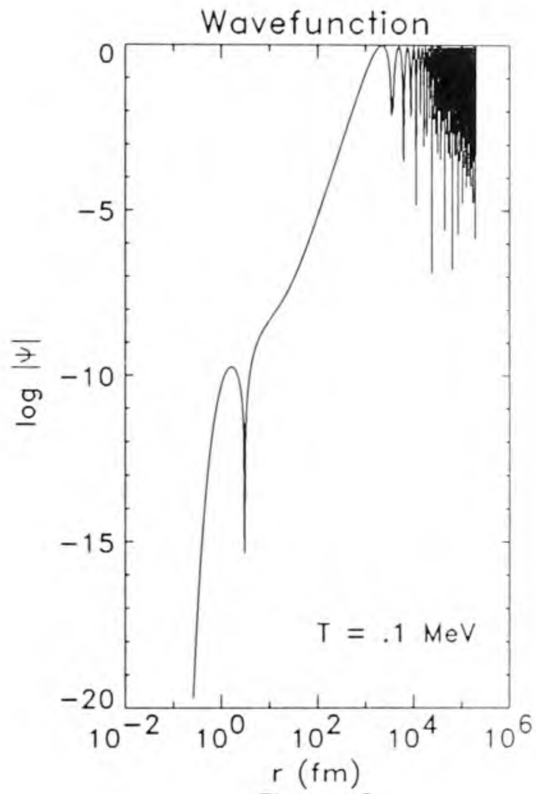

Figure 2

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