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# THE CROSS SECTION FACTOR FOR THE REACTIONS $^{2}H(d,p)^{3}H + {}^{2}H(d,n)^{3}He$ AT VERY LOW TEMPERATURE

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## 1. The shadow model.

A determined value of the fusion cross section $\bar{\sigma}_f$ can be obtained by using the Rutherford differential cross section $\sigma_R(\vartheta)$, in fact for a fixed value of $\bar{\sigma}_f$ there exist a value $\vartheta_f$ of $\vartheta$ such that:

$$(1) \qquad \sigma_f = 2\pi \int_{\vartheta_f}^{\pi} \sigma_R(\vartheta)\ \sin(\vartheta)\ d\vartheta$$

Eq.(1) could be considered a merely numerical result, however this is not true if we assume that the subbarrier fusion process is the shadow of the elastic scattering[(1)], so that the particles which fuse are those which in the Rutherford scattering are detected in the shadow region. In fact, by assuming the "shadow" point of view there exist a value $\vartheta_f$ of the scattering angle $\vartheta$ such that the particles which fuse are those that in Rutherford scattering are detected in the angular range

$$(2) \qquad \vartheta_f \le \vartheta \le \pi$$

i.e. the "shadow" region, see fig.1, so that eq.(1) follows from the "shadow" assumption. Now we remind that in Rutherford scattering it is

$$(3) \qquad R_f = \eta/k \left( 1 + 1/\sin(\vartheta_f/2) \right)$$

where $R_f$ is the distance of closest approach of the Rutherford trajectory with $\vartheta=\vartheta_f$, $\eta$ is the Coulomb parameter and k is the wave number. By using eq.(3), eq.(2) becomes:

$$(4) \qquad 2\eta/k \le R \le R_f$$

so that in the "shadow" approach the particles which fuse are those having a distance of closest approach R that satisfies inequality (4).

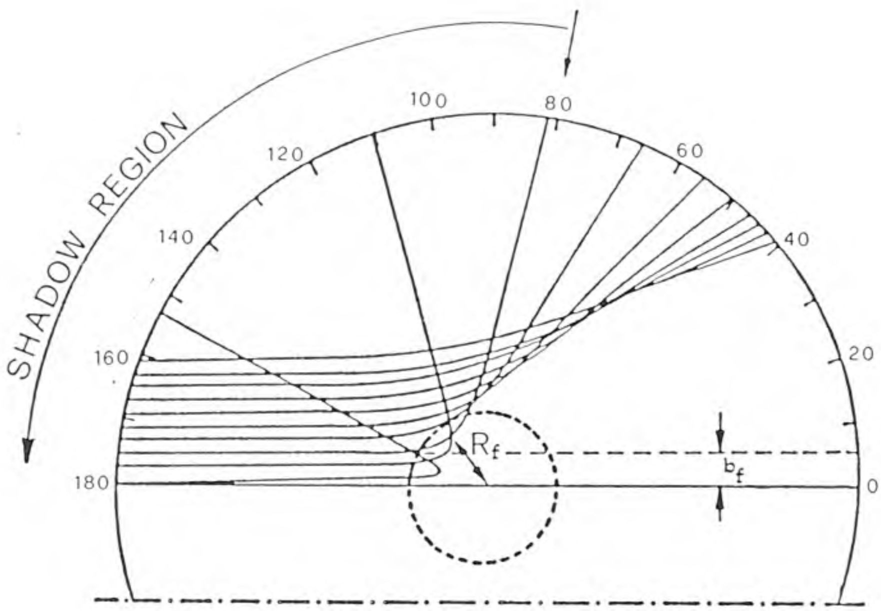


Fig.1 Rutherford trajectories for different values of the impact parameters. The dashed line shows the range of strong interaction.

By using eq.(3), eq.(1) can be rewritten as:

$$(5) \qquad \bar{\sigma}_f = \pi R_f^2 \left( 1 - 2\eta/kR_f \right)$$

where $R_f$ must be determined. From eq.(5) it follows that

$$(6) \qquad \frac{R_f}{2\eta/k} - 1 = -1/2 + 1/2\left( 1 + \frac{\bar{\sigma}_f \, k^2}{\pi \, \eta^2} \right)^{1/2}$$

and by putting in eq.(6) $\bar{\sigma}_f = \sigma_f^{ex}$, $\sigma^{ex}$ being the experimental fusion cross section, we have:

$$(7) \qquad \frac{R_f^{ex}}{2\eta/k} - 1 = -1/2 + 1/2\left( 1 + \frac{\sigma_f^{ex} k^2}{\pi\eta^2} \right)$$

Now we consider the function $y^{ex}(E)$ defined as follows:

$$(8) \qquad y^{ex}(E) = \ln\left\{ \ln\left[ -\ln\left( \frac{R_f^{ex}}{2\pi/k} - 1 \right)\right]\right\}$$

and we remember that the energy of the Coulomb Barrier $V_B$ can be obtained approximately by using the following expression:

$$(9) \qquad V_B = \frac{Z_1 Z_2 e^2}{1.07(A_1^{1/3} + A_2^{1/3})+2.72} \quad \text{MeV}$$

so that if we report $y^{ex}(E)$ versus $(V_B - E)$ we obtain that $y^{ex}(E)$ is a linear function of E[1,3]. In tables 1,2 some systems investigated in refs. 1 and 3 respectively, are reported. From the above arguments it follows that $R_f$ can be rewritten as

$$(10) \qquad R_f = 2\eta/k \left\{ 1 + \exp\left[ -\exp\left( \exp(y) \right) \right] \right\}$$

where

$$(11) \qquad y = (E_B - E)/E_S$$

and $E_B$ and $E_S$ are two parameters, expressed in MeV, to be determined. By using eq.(10), eq.(5) can be rewritten:

$$(12) \qquad \bar{\sigma}_f = \pi(2\eta/k)^2 \left[ 1 + G(y) \right] G(y)$$

where

$$(13) \qquad G(y) = \exp\left[ -\exp\left( \exp(y) \right) \right]$$

and the parameters $E_B$ and $E_S$ can be determined by comparing the experimental values of fusion cross section with those obtained by using eqs.(11-13).

Now we remind that eqs.(11-13) are not able to reproduce the experimental values of fusion cross section for light systems at very low energy[4,5] so that we suggested[4,5] to modify eq.(12) as follows:

$$(14) \quad \sigma_f = \bar{\sigma}_f \left[ 1 - g(y) \right] \left[ 1 - g_1(y) \right]$$

where

$$g(y) = \exp\left[ - \left( \frac{d - y}{d - y_m} \right)^{\gamma_1} 2.789 \right]$$

$$(15) \quad g_1(y) = \exp\left[ - \left( \frac{d - y}{d - y_1} \right)^{\gamma_2} 2.789 \right]$$

$$d = E_B / E_S \quad , \quad y_m = (E_B - E_m)/E_S \quad , \quad y_1 = (E_B - E_1)/E_S$$

the values of $E_B$, $E_S$, $E_m$, $E_1$, $\gamma_1$, $\gamma_2$, can be determined by fitting the experimental data. The values of $E_B$, $E_S$, $E_m$, $E_1$, $\gamma_1$, $\gamma_2$ for the reactions $^2H(d,p)^3H$ + $^2H(d,n)^3He$ are reported in table 1.

Table 1

| | | System $^2H + ^2H$ | | | |
|---|---|---|---|---|---|
| $E_B$(MeV) | $E_S$(MeV) | $E_m$(MeV) | $E_1$(MeV) | $\gamma_1$ | $\gamma_2$ |
| 0.18919 | 0.21250 | 0.00944 | 0.03660 | 4.48091 | 3.64950 |

A comparison between the experimental values of fusion cross section[7] and those obtained by using eqs.(11),(14),(15) for the above reactions is shown in fig.2.

## 2. The cross section factor[4-6].

The reaction rate $r_{12}$ in a thermal equilibrium distribution is given by weighting the cross section by the Maxwell-Boltzmann velocity distribution

$$(16) \quad r_{12} = n_1 n_2 \langle \sigma v \rangle / (1 + \delta_{12})$$

with

$$(17) \quad \langle \sigma v \rangle = \int_0^{\infty} \left[ 8/\pi \, M_{12} (KT)^3 \right]^{1/2} \sigma(E) \exp(-E/KT) \, E \, dE$$

where $M_{12}$ is the reduced mass of particles 1 and 2 $n_1$ and $n_2$ are the number densities of particles 1 and 2, v is the relative velocity, E is the center of mass energy, K is the Boltzmann constant, and T is the temperature.

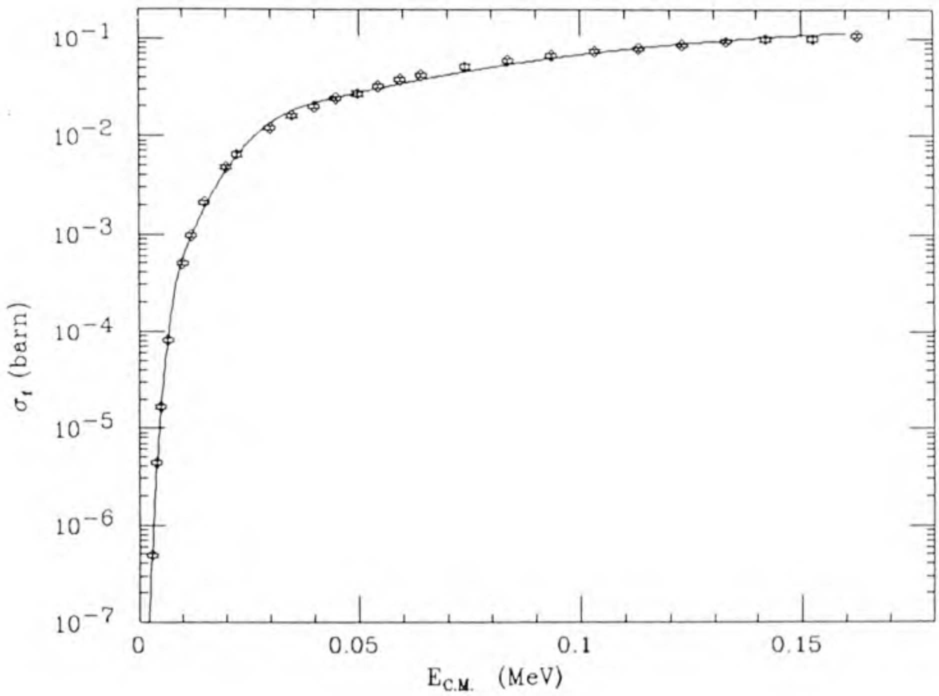


Fig.2 Comparison between the experimental values of fusion cross section[6] and those obtained by using eqs. (14) and (15)

For non resonant reactions the cross section $\sigma(E)$ is written usually[8]

$$\sigma(E) = S(E)/E \exp(-2\pi\eta)$$

with

$$2\pi\eta = 2\pi \frac{Z_1 Z_2 e^2}{\hbar v} = (E_G/E)^{1/2}$$

where $E_G$ is the Gamow energy, $\eta$ is the Coulomb parameter , $Z_1$ and $Z_2$ are the nuclear charges of the two particles, S(E) is the nuclear astrophysical factor. By assuming for the cross section $\sigma(E)$ the expression obtained in the framework of the "shadow" model, see eqs.(11),(14),(15), the cross section factor $\langle\sigma v\rangle$ can be rewritten

$$(18)\quad \langle\sigma v\rangle = \int_0^\infty \left[8/\pi\, M_{12}(KT)^3\right]^{1/2} \sigma_r(E)\, \exp(-E/KT)\, E\, dE = \int_{-\infty}^{d} M(T)\, I_3^T(y)\, dy$$

with

$$M(T) = \frac{(8\pi)^{1/2}\, (Z_1 Z_2 e^2)^2}{M_{12}^{1/2}\, (KT)^{3/2}}$$

and

$$I_3^T(y) = \left[1/(d-y)\right] G(y) \left[1 + G(y)\right] \exp\left[-\frac{E_B - yE_S}{KT}\right] \left[1-g(y)\right] \left[1-g_1(y)\right]$$

so that by using a numerical integration code we can obtain the values of $\langle\sigma v\rangle$ at different temperatures.

A comparison between the values of $N_A\langle\sigma v\rangle$ obtained by using our approach and those reported in the literature[9] is shown in table 2. $N_A$ is the Avogadro's number.

Table 2

| T(K) | System $^2H + ^2H$<br>$N_A\langle\sigma v\rangle^{(a)}$ ($cm^3 s^{-1} mol^{-1}$) | $N_A\langle\sigma v\rangle^{(b)}$ ($cm^3 s^{-1} mol^{-1}$) |
|---|---|---|
| $0.500\ 10^9$ | $1.061\ 10^7$ | $1.043\ 10^7$ |
| $0.300\ 10^9$ | $0.455\ 10^7$ | $0.473\ 10^7$ |
| $0.150\ 10^9$ | $0.118\ 10^7$ | $0.125\ 10^7$ |
| $0.100\ 10^9$ | $0.461\ 10^6$ | $0.485\ 10^6$ |
| $0.500\ 10^8$ | $0.611\ 10^5$ | $0.660\ 10^5$ |
| $0.250\ 10^8$ | $0.477\ 10^4$ | $0.496\ 10^4$ |
| $0.150\ 10^8$ | $0.473\ 10^3$ | $0.456\ 10^3$ |
| $0.500\ 10^7$ | $0.525\ 10^0$ | $0.443\ 10^0$ |
| $0.300\ 10^6$ | $0.283\ 10^{-8}$ | |
| $0.300\ 10^5$ | $0.646\ 10^{-15}$ | |
| $0.300\ 10^4$ | $0.151\ 10^{-21}$ | |
| $0.300\ 10^3$ | $0.353\ 10^{-28}$ | |

a)present work results b)ref.(9) data

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EDITORIAL NOTE TO THE PAPER "THE CROSS SECTION FACTOR ..."
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The authors assume that the fusion cross section is the shadow of the elastic scattering cross section. This assumption is plausible when energy is not small. At $E \simeq 0$ the assumption becomes implausible, for the Rutherford formula cannot disagree with the Gamow barrier factor, which emerges from the solution of the problem of the quantum electro-static interaction between charged particles.