Condensed Matter Effects for Cold and Hot Fusion

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ABSTRACT

In dense plasmas, the ensemble of fusing particles has a significant exchange of kinetic and potential energies. Because of this condensed matter effect (CME), the higher Z nuclei thus have a larger reduction in fusion rates. Our proposed solution of the solar neutrino problem finds a larger reduction for $^7Be(p,\gamma)^8B$ than for $p(p,e^+\nu_e)D$. Our CME predictions are consistent with neutrino detection experiments. CME have broad ranging astrophysical implications; may account for the anomalous branching ratio in cold fusion; and may be testable in laboratory beam fusion experiments with solid targets.

1. Introduction

It has recently been shown [1,2] that the conventional nuclear physics description of fusion reaction rates needs to be modified in laboratory and astrophysical environments due to CME. The conventional fusion rate R_{conv} for nuclear fusion reactions between two nuclei is written as $R_{conv} \propto \langle \sigma v \rangle$ where v is the relative velocity representing the incident flux and σ is the fusion cross-section. However, in condensed matter environments, R_{conv} needs to be modified to a new expression $R_{new} \propto \langle \tilde{\sigma} \overline{v} \rangle$, where $\tilde{\sigma}$ is a modified fusion cross-section and $\overline{v}$ is an average velocity of the incident particle in condensed matter which can differ significantly from the incident velocity. As a specific application of our new formulation of R_{new} to hot fusion, condensed matter effects for the solar neutrino problem will be presented. Application of our new formulation to cold fusion rates explaining the anomalous branching ratio for deuterium-deuterium fusion is in progress.

FRONTIERS OF COLD FUSION

2. Effective Flux Velocity

For the purpose of obtaining quantitative estimates of CME, we use a "shifted" screened Coulomb potential with an interior square–well nuclear potential $V_s(r)$:

$$V_s(r) = \begin{cases} -V_0 \,, & r < r_N \\ Z_i Z_j e^2\left(\frac{1}{r} - \frac{1}{r_s}\right) \,, & r_s > r \geq r_N \\ 0 \,, & r_e > r \geq r_s \end{cases} \quad (1)$$

For a number density of condensed (target) matter, n, and elastic cross–section σ_e, the mean free path is $\lambda_e = (n\sigma_e)^{-1} = r_e - r_2$ and for the fusion cross–section σ_f, $\lambda_f = (1/n\sigma_f) + r_2 = (\lambda_e\sigma_e/\sigma_f) + r_2$, where r_2 is chosen as the classical distance of closest approach (turning point) for a given barrier. The average interatomic distance is $\sim 2r_e$ in condensed matter. Then the average velocity ($\propto$ effective flux) can be written as $\langle v\rangle = \lambda_f/[(\sigma_e/\sigma_f)(t_i+t_0)+t_B+t_N]$ $= [(r_e-r_2)(\sigma_e/\sigma_f)+r_2]/[(t_i+t_0)(\sigma_e/\sigma_f)+t_B+t_N]$ where t_i, t_0, t_B and t_N are the times for the incident particles to traverse distances (r_e-r_s), (r_s-r_2), (r_2-r_N), and r_N, respectively. For the case of $\sigma_e >> \sigma_f$, $\langle v\rangle$ reduces to $\langle v\rangle \approx \overline{v} = (r_e - r_2)(t_i + t_0) = [(r_e - r_s) + (r_s - r_2)]/(t_i + t_0)$.

The traversal time $t(j) = t_0$ of the incident particle i in the presence of the barrier due to the target species j can be estimated in the WKB approximation for the case of $V_s(r)$, eq. (1), with $E < B = V_s(r_N)$ as $t(j) = \int_{r_2(j)}^{r_s(j)} [(2/\mu_{ij})|E - V_s(r)|]^{-1/2} dr$ where $\mu_{ij} = m_i m_j/(m_i + m_j)$. Using $t(j)$, we calculate $\overline{v}(v)$ and $F(E) = \overline{v}/v$ as a function of v or $E = \mu v^2/2$. Our calculations show that $\overline{v} \leq v$ where v is given by $v = v_i = (r_e - r_s)/t_i$.

3. Fusion Rates

The conventional fusion rate between two species of nuclei, i and j, with number densities n_i and n_j is conventionally written as $R_{conv} = [n_i n_j/(1 + \delta_{ij})]\langle\sigma v\rangle_{conv}$ where

$$\langle\sigma v\rangle_{conv} = \int d^3v_i \int d^3v_j f(\vec{v}_i) f(\vec{v}_j) v\sigma(v) = \int \sigma(v) v f(v) d^3v \quad (2)$$

with kinetic energy $E = \mu v^2/2$, in the center of mass (CM) frame and the Maxwell–Boltzmann velocity distribution, $f(v) = (\mu/2\pi kT)^{3/2}\exp(-\mu v^2/2kT)$ where μ is the reduced mass.

Because of the continuous exchange of kinetic and potential energies in dense plasmas, the original velocity $\vec{v}$ whose distribution is described by $f(\vec{v})$ $= (\mu/2\pi kT)^{3/2}\exp(-\mu v^2/2kT)$ is reduced to $\overline{v} = u(v) < v$. The original pair total energy, $m_i v_i^2/2 + m_j v_j^2/2 = (m_i + m_j)V^2/2 + \mu v^2/2$, becomes $(m_i + m_j)V^2/2 + \mu u^2/2 + W(u)$ where $W(u)$ is the potential energy, $W(u)$ $= \mu v^2/2 - \mu u^2/2$. Therefore, the original velocity distributions $f(\vec{v}_i)f(\vec{v}_j) = f(\vec{V})f(\vec{v})$ are replaced by $f(\vec{V})f(u)$ where $f(u)$ is given by $f(u) = (\mu/2\pi kT)^{3/2}$

$\exp[-(\mu u^2/2 + W)/kT]$. Since $f(u)$ does not conserve particle number due to $\int f(u)d^3u \neq 1$, $f(u)$ is replaced by $\tilde{f}(u)$, $\tilde{f}(u) = f(u)\, d^3v/d^3u = (\mu/2\pi kT)^{3/2}$ $\exp(-\mu v^2/2kT)\, d^3v/d^3u$ which gives particle number conservation $\int \tilde{f}(u)d^3u = 1$.

The velocity $\overline{v}$ is the appropriate value to be used for the fusion rate rather than $v(= v_i)$ which has been used in the conventional eq. (2). For dense astrophysical or laboratory plasmas, when a projectile particle i moves through the plasma with velocity $u = \overline{v}$, the probability of a fusion reaction per unit path length of the projectile is given by $P_x = n_j\sigma$ where n_j is the number of target nuclei per unit volume of the plasma. Because of the elastic scattering processes described previously, the projectile traversal distance per unit time is u, and not v. Then the fusion reaction probability for the projectile path length per unit time is $P = uP_x = un_j\sigma$. The rate at which n_i projectiles per unit volume, each moving with a speed $u(v)$ but in random directions, will react is then $R(u) = n_iuP_x = (n_iu)(n_j\sigma)$. If a projectile velocity distribution $\tilde{f}(u)$ is given, then the new fusion rate is given by $R_{new} = [n_in_j/(1+\delta_{ij})]\langle\sigma v\rangle_{new}$ where

$$\begin{aligned}\langle\sigma v\rangle_{new} &= \int \sigma(u)u\tilde{f}(u)d^3u = \int \sigma(u)uf(v)d^3v \\ &= \left(\frac{8}{\pi\mu}\right)^{1/2}\frac{1}{(kT)^{3/2}}\int_0^\infty \sigma(\overline{E})F(E)E\exp\left(-\frac{E}{kT}\right)dE\,,\end{aligned} \tag{3}$$

and $\overline{E} = EF^2(E)$. We note that $\langle\sigma v\rangle_{new} \leq \langle\sigma v\rangle_{conv}$.

4. Results

The new fusion rate formula, eq. (3), is applied to the solar neutrino problem. Table 1 shows our results of applying CME reduction factors of $\langle\sigma v\rangle_{new}/\langle\sigma v\rangle_{conv} = 0.79$ and 0.48 for $p(p, e^+\nu_e)D$ and $^7Be(p,\gamma)^8B$, respectively, to the standard solar model (SSM) calculations of Refs. [3], [4], and [5]. Data collection periods (month.year – month.year) are indicated for Refs. [6] and [7].

The observed neutrino flux due to $^7Be(p,\gamma)^8B(e^+\nu_e)^8Be^*(\alpha)^4He$ by the Kamiokande-II detector [10,11] is $2.7\times10^6(1\pm0.11$ (stat.)±0.13 (syst.)) $cm^{-2}s^{-1}$ compared with the SSM result [3] of 5.7×10^6 $cm^{-2}s^{-1}$. Our result of applying the CME reduction factor of $\langle\sigma v\rangle_{new}/\langle\sigma v\rangle_{conv} = 0.48$ yields 2.74×10^6 $cm^{-2}s^{-1}$. which agrees with the experimental data [10,11].

5. Conclusions

We conclude that anomalies in both hot and cold fusion have a common origin in CME. It is noteworthy that overlooked CME may supply both the solution to the long standing solar neutrino problem, as well as an explanation for the anomalous branching ratio for $D-D$ fusion.

Table 1. Comparison of CME results with the experimental data [6–9].

	Rate $R^{Cl}_{\nu_e}$ with ^{37}Cl Detector (SNU)			Rate $R^{Ga}_{\nu_e}$ with ^{71}Ga Detector (SNU)		
Neutrino reaction	BU [3]	TCCD [4]	BPML [5]	BU [3]	TCCD [4]	BPML [5]
$p(p, e^+\nu_e)D$ (CME)	0	0	0	70.8 (56.0)	70.6 (55.8)	70.0 (55.3)
$p(p + e^-, \nu_e)D$	0.2	0.2	0.22	3.1	2.8	3.0
$^7Be(e^-, \nu_e)^7Li$	1.2	1.0	1.06	35.8	30.6	32.5
$^8B(e^+\nu_e)^8Be^*$ (CME)	6.2 (3.0)	4.1 (2.0)	5.73 (2.75)	13.8 (6.6)	9.3 (4.5)	13.1 (6.3)
$^{13}N(e^+\nu_e)^{13}C$	0.1	0.1	0.1	3.0	3.9	3.53
$^{15}O(e^+\nu_e)^{15}N$	0.3	0.4	0.318	4.9	6.5	5.58
SSM Total	8.0 ± 1.0 (1 σ)	5.8 ± 1 (1 σ)	7.43	131.5^{+7}_{-6} (1 σ)	124 ± 5 (1 σ)	127.7
SSM + CME Total	4.0	3.7	4.45	109.5	104.5	106.2
Experimental Results	(3.70 – 85) [6]: 2.1 ± 0.3 (3.70 – 3.88) [7]: 2.33 ± 0.25 (8.86 – 3.88) [7]: 4.2 ± 0.7			GALLEX [8]: $83 \pm 19 \pm 8$ SAGE [9]: $20^{+15}_{-20} \pm 32$		

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