

The Combined Resonance Tunneling and Semi-Resonance Level in Low Energy D-D Reaction

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ABSTRACT

When nuclear potential wells are connected by an atomic potential well, a new kind of tunneling may happen even if there is no virtual energy level in nuclear potential wells. The necessary condition for this combined resonance tunneling is the resonance in the atomic potential well. Thus, the nuclear reaction may be affected by the action in atomic scale in terms of combined resonance tunneling. The nuclear spectrum data support this idea.

1. Introduction

While more evidences are confirming the anomalous nuclear phenomena in deuterium / solid system, there are still two puzzles remaining: i.e. the penetration of Coulomb barrier at low energy, and why the process in nuclear scale is affected by the parametric variation in atomic scale.

2. Combined Resonance Tunneling in a New Matrix Formalism

It is well known in quantum mechanics that the potential barriers in Fig.1 may be penetrated without any reflection when the energy of the incident nucleus is in resonance with the virtual energy level of the potential well between the barriers. The penetration may be expressed in a matrix formalism with the new base functions at each interface between the barrier and the well. Under the WKB approximation ⁽¹⁾, the solutions of Schrodinger equation in regions I, II, III, IV and V may be written as

$$\psi_I = \frac{1}{\sqrt{k}} \left\{ A_s \cos\left[\int_a^x k dx - \frac{\pi}{4}\right] + B_s \sin\left[\int_a^x k dx - \frac{\pi}{4}\right] \right\}$$

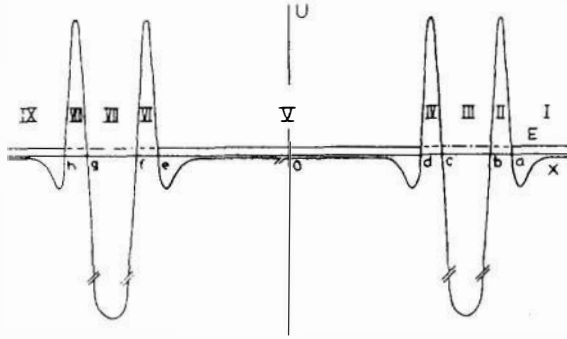


Fig.1 The Combined Resonance Tunneling of Two Barrier-Well-Barriers .

$$\begin{aligned} \psi_{\text{VI}} &= \frac{1}{\sqrt{\beta}} \left\{ C_a \exp\left[-\int_x^a \beta dx\right] + D_a \exp\left[\int_x^a \beta dx\right] \right\} \\ &= \frac{1}{\sqrt{\beta}} \left\{ C_b \exp\left[-\int_b^x \beta dx\right] + D_b \exp\left[\int_b^x \beta dx\right] \right\} \\ \psi_{\text{VII}} &= \frac{1}{\sqrt{k}} \left\{ A_b \cos\left[\int_x^b k dx - \frac{\pi}{4}\right] + B_b \sin\left[\int_x^b k dx - \frac{\pi}{4}\right] \right\} \\ &= \frac{1}{\sqrt{k}} \left\{ A_c \cos\left[\int_c^x k dx - \frac{\pi}{4}\right] + B_c \sin\left[\int_c^x k dx - \frac{\pi}{4}\right] \right\} \\ \psi_{\text{IV}} &= \frac{1}{\sqrt{\beta}} \left\{ C_c \exp\left[-\int_x^c \beta dx\right] + D_c \exp\left[\int_x^c \beta dx\right] \right\} \\ &= \frac{1}{\sqrt{\beta}} \left\{ C_d \exp\left[-\int_d^x \beta dx\right] + D_d \exp\left[\int_d^x \beta dx\right] \right\} \\ \psi_{\text{V}} &= \frac{1}{\sqrt{k}} \left\{ A_d \cos\left[\int_x^d k dx - \frac{\pi}{4}\right] + B_d \sin\left[\int_x^d k dx - \frac{\pi}{4}\right] \right\} \end{aligned}$$

Here $k^2 \equiv (2\mu/h^2) (E - U(x))$, $\beta^2 \equiv (2\mu/h^2) (U(x) - E)$. μ and E are the mass and the total energy of the particle, respectively. h is the Planck constant divided by 2π . $U(x)$ is the potential energy. Using the WKB connection formula, the relation between coefficients may be expressed in matrix form as

$$\begin{aligned} \begin{bmatrix} C_a \\ D_a \end{bmatrix} &= C_{\text{BW}} \begin{bmatrix} A_a \\ B_a \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} A_a \\ B_a \end{bmatrix} \\ \begin{bmatrix} C_b \\ D_b \end{bmatrix} &= B_{\text{ba}} \begin{bmatrix} C_a \\ D_a \end{bmatrix} = \begin{bmatrix} 0 & \theta_2 \\ \theta_2^{-1} & 0 \end{bmatrix} \begin{bmatrix} C_a \\ D_a \end{bmatrix} \\ \begin{bmatrix} A_b \\ B_b \end{bmatrix} &= C_{\text{WB}} \begin{bmatrix} C_b \\ D_b \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} C_b \\ D_b \end{bmatrix} \\ \begin{bmatrix} A_c \\ B_c \end{bmatrix} &= W_{\text{cb}} \begin{bmatrix} A_b \\ B_b \end{bmatrix} = \begin{bmatrix} \sin\gamma_{\text{cb}} & -\cos\gamma_{\text{cb}} \\ -\cos\gamma_{\text{cb}} & -\sin\gamma_{\text{cb}} \end{bmatrix} \begin{bmatrix} A_b \\ B_b \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \begin{bmatrix} C_c \\ D_c \end{bmatrix} &= C_{BW} \begin{bmatrix} A_c \\ B_c \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} A_c \\ B_c \end{bmatrix} \\ \begin{bmatrix} C_d \\ D_d \end{bmatrix} &= B_{dc} \begin{bmatrix} C_c \\ D_c \end{bmatrix} = \begin{bmatrix} 0 & \theta_4 \\ \theta_4^{-1} & 0 \end{bmatrix} \begin{bmatrix} C_c \\ D_c \end{bmatrix} \\ \begin{bmatrix} A_d \\ B_d \end{bmatrix} &= C_{WB} \begin{bmatrix} C_d \\ D_d \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} C_d \\ D_d \end{bmatrix} \end{aligned}$$

Here $\theta_2 \equiv \exp\left[\int_b^a \beta dx\right]$, $\theta_4 \equiv \exp\left[\int_d^c \beta dx\right]$, and $\theta_2 = \theta_4 = \theta$ is assumed hereinafter due to the symmetry. Usually, θ is a very large number.

$\gamma_{cb} \equiv \int_c^b k dx$. We may define a matrix, T_{da} , to transform the coefficients

$$\begin{bmatrix} A_a \\ B_a \end{bmatrix} \text{ in region I to the coefficients } \begin{bmatrix} A_d \\ B_d \end{bmatrix} \text{ in region V.}$$

$$\begin{aligned} T_{da} &\equiv (C_{WB} B_{dc} C_{BW}) W_{cb} (C_{WB} B_{ba} C_{BW}) \\ &= \begin{bmatrix} -\sin\gamma_{cb} & -(2\theta)^2 \cos\gamma_{cb} \\ -\left(\frac{1}{2\theta}\right)^2 \cos\gamma_{cb} & \sin\gamma_{cb} \end{bmatrix} \end{aligned}$$

An outgoing wave in region I, $\frac{1}{\sqrt{k}} \exp\left[i\left(\int_a^x k dx - \frac{\pi}{4}\right)\right]$, corresponds to

$$\begin{bmatrix} A_a \\ B_a \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix}. \text{ It may be connected to an incident wave in region V,}$$

$$\frac{1}{\sqrt{k}} \exp\left[-i\left(\int_x^d k dx - \frac{\pi}{4}\right)\right], \text{ which corresponds to } \begin{bmatrix} A_d \\ B_d \end{bmatrix} = \begin{bmatrix} 1 \\ -i \end{bmatrix}. \text{ When the}$$

energy of incident particle, E , is in resonance with the meta-stable energy level in potential well III, then $\gamma_{cb} = (n + \frac{1}{2})\pi$, and this connection is possible, which means a perfect penetration of barrier-well-barrier combination. However, experiments do not show any evidence for this resonance energy level. Instead we propose another possible connection, i.e. the barrier-well-barrier (abcd) is combined to another barrier-well-barrier (efgh) by an atomic well region V. Now, if $\gamma_{cb} = \gamma_{gf} = n\pi$ ($n = 0, 1, 2, \dots$) and $\gamma_{ed} = (m + \frac{1}{2})\pi$, ($m = 0, 1, 2, \dots$), the connection matrix for this combination is

$$T_{ha} \equiv T_{he} W_{ed} T_{da} = (-1)^{n+1} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

This is again the right connection matrix for a perfect penetration through (hgfe) and (dcba) in Fig. 1. We may call this the combined resonance tunneling, because two semi-resonances in nuclear wells ($\gamma_{cb} = \gamma_{gf} = n\pi$) are combined by an atomic resonance well ($\gamma_{ed} = (m + \frac{1}{2})\pi$).

3. Semi-Resonance Energy Level in D-D System

We must first answer the question whether there is any semi-resonance level near $E=0$ in Fig.1. Fig.2 shows the experimental data ⁽²⁾ for states of the mass number 4 with charge number 2. It is interesting to notice that there are two energy levels just equally above and below 23.8 MeV, (In Fig.2 $E=0$ has been set for ⁴He ground state; therefore, $E=23.8$ MeV in Fig.2 corresponds to $E=0$ in Fig.1). The energy of these two levels are 25.5MeV and 22.1MeV, respectively. These two resonance levels may correspond to condition

$$\left[\int_c^b k dx \right]_{E=22.1\text{MeV}} = (n + \frac{1}{2})\pi \quad (n = 0, 1, 2, \dots)$$

and $\left[\int_c^b k dx \right]_{E=25.5\text{MeV}} = (n + 1 + \frac{1}{2})\pi$

The linear interpolation of these two levels would give

$$\left[\int_c^b k dx \right]_{E=23.8\text{MeV}} = (n + 1)\pi$$

This is just the condition for semi-resonance energy level in a nuclear potential well. In order to obtain the combined resonance tunneling, a resonance energy level for atomic potential well (region V in Fig.1) is the key point.

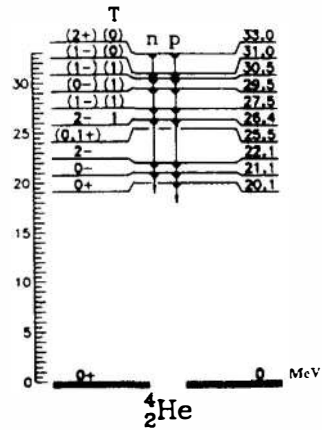


Fig.2 Data Showing Semi-Resonance Level.

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