COLD FUSION AS
BOSON CONDENSATION IN A FERMI SEA

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Abstract
Boson condensation and the important selection rule "Bosons in, Bosons out" were presented earlier as a means of predicting the primary products of Cold Fusion and understanding how two deuterons in a crystalline medium might fuse. An important connection with superfluidity and superconductivity was stated, since all three phenomena involve bosons. The Born-Oppenheimer separability of the nucleonic and electronic functions was an essential ingredient of the theory; the reformulation of the wave functions in the Wannier representation is equally important here. The problem is recast as a further modification of a BCS-Bose formulation of superconductivity for high Tc materials as proposed by Fujita and coworkers.

Treatment of Condensed Matter
There are two different approaches to treating condensed matter and molecular physics. In the first, the "Heitler-London" method of atomic eigenfunctions, one assumes widely separated atoms so that a specific electron belongs to a specific nucleus. The behavior of such an atom differs from that of an isolated atom only inasmuch as it is modified by the local dielectric constant. This is what Chubb and Chubb1 refer to as the "Free Space Paradigm". Aside from certain inner core energy level problems that can adequately be treated in this manner, a more proper quantitative approach is based on the "Hund-Mulliken"
approximation, namely, of closely spaced nuclei plus a collective treatment of the electrons involved. An electron then is not assumed to be assigned to a single nucleus, since it is acted on by the force field of all the surrounding nuclei and electrons. The mutual interaction between different electrons is treated as a small perturbation. This is represented by the "band model". The transition to the many-electron band model lies not in using an inter-electron interaction but simply in filling the energy levels of the one-electron problem with all the electrons which must be accommodated in the crystal.

In order for the energy of a system of atoms assembled in condensed matter to minimize itself, the corpuscular aspect of the electrons is replaced by their wave aspect with the latter spread throughout the solid. Consequently, (1) the locations of a "particle" cannot be defined, (2) only a small fraction of any one particle can be found within a unit cell of the material, and (3) each nuclear reactions which may occur will do so at all of the periodically equivalent crystalline sites.

The prerequisite boundary condition of the solid state model is that the system is periodic and the energy in the low temperature limit minimizes itself with respect to changes in the electronic structure. Each atom eigenfunction is split into a quasi-continuous band and the splitting becomes greater as the atoms become closer together. From the Bloch theorem, one obtains a band spectrum of broadened states. Using Brillouin's treatment of an electron as a wave, it is understandable that for some special wavelengths and propagation directions, the wave will encounter Bragg reflections. Interference with the lattice leads to reflection of an electron wave of exactly the same energy. But the apparent degeneracy is lifted by the periodic lattice; Bloch demonstrated that the discrete atomic levels are split into allowed bands of energy which arise from the periodicity of the lattice potential. Brillouin's treatment shows that the continuous energy of free-electron waves is further split into groups of forbidden energy bands. The Bloch functions are time-independent.

When the energy of an electron approaches the upper band edge, the condition for Bragg reflection is more nearly fulfilled and the lattice potential reflects more of the incident wave in the form of a reflected wave. When they become equal in magnitude at the band edge, they form a standing wave and no propagation occurs; the velocity of the electron under an accelerating field falls to zero since the electron mass becomes negative (it comes from the Bragg reflection of the lattice.) This at the heart of what Chubb and Chubb\textsuperscript{2} call the "Solid State Paradigm."

In the Free Space Paradigm, particles collide with similar
particles (or collision partners) — i.e., hydrogens with other hydrogen molecules. In contrast, because of their wave nature, electrons move unhindered and uniformly throughout a perfect lattice. In practice, a lattice may not be perfect even at 0° K——it may contain missing atoms, imperfections or impurities. The electrons are scattered off these disturbances. At non-zero temperatures, there is an additional type of "collision partner." Density changes occur in the lattice and these are treated as acoustic and optical vibrations, called phonons.

Local, inhomogeneous electrostatic fields bind the deuterium atoms into the palladium deuteride lattice and lattice-induced broadening occurs. So, in contrast to the free space situation, the nuclear charge is broadened. Further, many-body interactions give it an effective electrostatic volume which is comparable to the atomic volume since zero-point motion must be included.

The Coulomb interactions between electrons and the ion cores are equal and opposite in sign. To a large extent, they cancel each other out. If we take a specific electron, then it interacts with the system of N ions and N-1 electrons; the system as a whole has a net charge of +e. It is desirable to distinguish between two types of condensed matter (a) amorphous or disordered and (b) crystalline or ordered.

A rephrasing of the statement of Chubb and Chubb is in order: "Just as electrons cease to be point particles when they are injected into a solid; so do particles such as deuterons." Chubb and Chubb also state that "...Just because a deuteron looks like [and acts like] a particle in one situation ..." it need not do so in other situations and ..." just because the energy from nuclear fusion is released entirely in one place ..." in one situation, the release need not be constrained or to be so highly localized in another situation. It can occur, for example, throughout the lattice as in the Mössbauer phenomenon.

Electrons, or similar particles, when treated by the Bloch theorem, are extended over the entire lattice and are in equilibrium. The important point is that the interaction strength of any specific electron or particle is greatly reduced by the screening of the N-1 electrons in the system.

Details of the Band Structure Model

In a crystal, there are energy levels E_n(k) and associated Bloch eigenstates. They are identified by a band index n and by a wave vector k having three Cartesian components. Each such band state has both ↑ and ↓ spins associated with it, so that each state is at least two-fold degenerate. The Bloch electrons (or particles) obey Fermi-Dirac statistics independent of any interaction between
A denumerable set of Fourier components $<G_i>$ can be determined from the lattice potential and each is labelled by a reciprocal lattice vector $G_i$. A further condition is that dot-product $G_i \cdot R_n$ is equal to $2\pi x$ integer where $R_n$ is a distance to the $n^{th}$ atom in the unit cell, i.e., a basis vector of the structure. An arbitrary amount of momentum $\hbar G_i$ can be given to any particle in the solid (where $\hbar$ is Plank's constant) but the momentum of a specific particle is not conserved over a long time-scale. That is, the solid as a whole can coherently absorb an arbitrary amount of momentum without altering the Fourier transforms of $V_{\text{elct}}$. The band energies of $V_{\text{elct}}$ are defined by a continuous set of eigenvalues by the condition

$$E_n(k) = E_n(k+G_n)\quad (1)$$

The energy band is infinitely degenerate at each value of $k$. Bloch's theorem eliminates this huge degeneracy - that is, a single eigenfunction $\Phi^*$ for the $n^{th}$ energy band has the property that

$$\Phi^*_{\text{bloch}}(k, r+R_n) = \Phi^*_{\text{bloch}}(k, r) \exp(ik \cdot R_n).\quad (2)$$

The particle density can then formally be written as

$$n(r) = \sum_{n, k} |\Phi^*_{\text{bloch}}(k, r)|^2 F(E_n(k))\quad (3)$$

where $F$ is a statistical weight factor, namely

$$F[E_n(k)] = \left[ \frac{\exp(E_n(k) - \mu(T))}{k_B T} + A \right]^{-1} \quad (4)$$

Here $A= +1$ for $A$ corresponds to Fermi-Dirac statistics, (namely, to the occupation of $N^f$ Fermions but only two per state - differing by their spin. When $A=-1$, $F$ corresponds to Bose-Einstein statistics, (occupation of $N^b$ Bosons, i.e., to many bosons being in the same state). $A=0$ corresponds to Boltzmann statistics. The chemical potential is $\mu$ and $k_B$ is the Boltzmann constant. The Bloch functions are time-independent.

**Born-Oppenheimer Separability**

An important ingredient is the Born-Oppenheimer separability of the electronic and nuclear functions. For any deuterium ion located in the lattice at or near an s site, the nucleon wavefunction $\psi$ can be written as
\[ \psi_{\text{nucleon}}(\mathbf{r} - \mathbf{r}_{\text{cm}}) = \psi_{\text{nucleon}}(\mathbf{r} - \mathbf{r}_{\text{cm},s}) \theta_{\text{elect}}(\mathbf{r}_{\text{cm},s}) \]  
\( (5) \)

where \( \mathbf{r}_{\text{cm},s} = (\mathbf{r}_p + \mathbf{r}_n)/2 - \mathbf{R}_s \) is the center-of-mass of a deuteron located to within a small radius of the center of the zero-point motion in cell \( s \).

The nuclear behavior is governed by the highly localized, i.e., short ranged nuclear function \( \psi_{\text{nucleon}} \), and vanishes when the range exceeds a few Fermis. Inasmuch as the internucleon force is charge independent, one could have included the isospin function \( \zeta(t) \) as a factor in \( \psi \), which distinguishes between the two charge states of the nucleons.

On the other hand, \( \theta_{\text{elect}} \) is a slowly varying function over a length-scale associated with the electrostatic force between deuterons in nearby cells. The electrostatic potential is periodic, hence each \( \psi \) can be approximated either by the single-particle Bloch state \( \Phi_{\text{Bloch}}(k, \mathbf{r}) \) or by its Wannier representation \( \psi_s(\mathbf{r}_m, T_s) \). These two forms of wave-function appear to be rigorously valid in the limit in which the energy-scale of the nuclear interaction is very different from the scale associated with the single-particle electrostatic potential or its first order perturbations. Moreover, the nuclear forces associated with the possible interaction between two deuterons (in a boson condensate) within a common cell, are invariant under a rigid translation of all the deuterons.

Without belaboring the point, Chubb and Chubb\(^3,4,5\) have shown that non-separable nuclear-electrostatic interactions are prohibited. Since \( \psi_{\text{nucleon}} \) is a Fermion field while \( \theta_{\text{elect}} \) is a Boson field, it becomes impossible to construct non-vanishing forms, when these fields are quantized, in which both sets of functions (or their complex conjugates) evolve independently of each other. Consequently the short range nuclear wave-functions are constrained to be independent of the electrostatic wave-function only during an intermediate stage when the nuclear portions are constructed of paired neutron-proton bosonic wave-function \( \psi_{np} \). Recently Chubb and Chubb\(^6\) have investigated this question further using Poisson Brackets and the condition for separability is even stronger than initially supposed.

The calculation of Switendick\(^7\) inter alia\(^8\) of the band structure and of the charge on the hydrogen atom show that the protonic model of PdD with \( (D^+) \) is not more correct than the anionic mode \( 1 \) with \( (D^-) \). That is, when the content of deuterium atoms in the compound approaches \( x = 1 \), principally anti-bonding states are occupied and the \( \text{H} \) atoms to the first approximation are neutral. The optical phonons bring about an ionization of atoms into deuterons and excited electrons.
Wannier Representation

It is illuminating to cast the wave-function into the Wannier representation \( \Psi_n(r, R_z) \) where each state is specific to a given site and is normalized to the volume of the solid. Note that each \( \Psi \) is orthogonal to all the other Wannier functions centered on different sites. The applicable formula is

\[
\Phi_{\text{Bloch}}(k, r) \exp(-E(k) t/\hbar) = \left(1/N_L\right)^{1/2} \sum_n \Gamma_n(r, T_R) \exp(ik \cdot R_n) \tag{6}
\]

The summation runs over all the \( N_L \) lattice sites. Because each deuteron which occupies a band state is indistinguishable from all others in like states, we can define a new cooperative state which Chubb and Chubb\(^9\) calls a Bose Bloch Condensate or BBC. The periodic order in PdD\(_x\) leads to the BBC as a means of reducing the localized lattice strain by allowing a small number of deuterium atoms (injected into the solid by electrolytic charging) to occupy empty band states. The Coulomb repulsion between deuterons is uncorrelated (and overlap could occur at a specific site) on a time-scale compatible with a nuclear reaction but too short to be compatible with electrostatic processes. In terms of this particle-wave *duality, a deuteron could appear particle-like on a short time-scale but wave-like with respect to electrostatic processes.

The present treatment is in terms of both the time-independent Bloch functions which spread throughout the solid and their time-dependent Wannier representations which are localized on individual lattice cells. The selection rule, "boson in, bosons out!", is developed and leads to the conclusion that \(^3\)He will be the primary reaction product, rather than tritium or \(^4\)He which are fermions.

Fermi Surface and Band Structure of Palladium and Its Deuteride

The theoretical basis for interpreting experimental measurements of the Fermi surface of a material is the collection of \( E_n(k) \) curves. In Figure I, such a collection is plotted versus the wave vector \( k \). The Fermi level is determined by summing all of these energy states, taking into account the degeneracy \( p' \) of each \( k \) states (which can hold \( 2p' \) electrons) until the number of conduction electrons per atom is reached. In the present case, the number is 11. In addition, one must keep track of the band index \( n \). Notice the shaded areas near points \( X \) and \( L \) in the Brillouin Zone. These are unoccupied states at 0°K. They correspond to pockets in the Fermi surface. The Fermi level corresponds to the chemical potential \( \mu \) of adding an electron to the solid.
Figure 1. A plot of the band structure of palladium hydride, where the individual $E_n(k)$ values are plotted versus the wave vector $k$. Note that two shaded regions lie above the Fermi Level. The labels on the horizontal axis of the graph refer to high symmetry points in the Brillouin Zone.
The Fermi surface determination of Bakker et al.\textsuperscript{10} is based on de Haas-van Alfen measurements for the hydride, the deuteride and the tritide of palladium and one of these is shown as Figure II.

**Figure II.** Plot of the Fermi Surface of Palladium Hydride determined by Bakker et al. The cross section is on a (100) plane passing through the zero momentum state $\Gamma_1$. Note the pockets in the $X$ and $L$ directions. The band index indicates which sheet of the Fermi surface is involved, i.e., on which sheet the unoccupied pocket lies.

The external cross-section (which is what is actually measured) is indicated in this figure by cross hatching.

The effect of adding either of the hydrogen isotopes to palladium is to lower the d-bands associated with the palladium as well as to introduce additional states below $\Gamma_1$ of the metallic element. The former effect lowers the position of both the $\Gamma_{25}$ and
$\Gamma_{12}$ states relative to the Fermi level. The net effect is to lower $N(E_F)$, the density of states at the Fermi level, as seen in Figure III based on the calculation discussed in Reference 8.

Figure III. Three Plots of the Densities of States of Palladium Hydride for Different Hydrogen Contents, i.e., different $x$ values. The lower portion of each graph is the Partial Density of States of only the Hydrogen Atoms. Note the shift of the Fermi level $E_F$ with $x$. 
Note that on the right hand scale, the number of electron is given as a function of energy. The sum $\sum N(k_n)$ is shown in each upper panel of the figure as a continuous line.

When more deuterium atoms are introduced into the PdD$_x$, where $x$ is approximately 0.67, they occupy octahedral sites - for higher $x$ values they may occupy tetrahedral sites as suggested by Johnson. Each of these deuterons must have a wave function solution of the Schrödinger equation of a bound particle imbedded in a periodic electrostatic potential.

A further consequence of the lower density of states, is that introducing more deuterium atoms can be "expensive." Because any electrostatic energy must be overcome, only an infinitesimal number of electrons can be added and these go in above the Fermi level in the model of Chubb and Chubb. The "single particle" electrostatic potential is a periodic function of the Bravais lattice vectors $R_n$ which define the solid. That is

$$V_{\text{elect}}(r) = V_{\text{elect}}(r + R_n)$$

(7)

The net charge of these pairs $2e(Q_e + Q_d)$ is distributed throughout the crystal and the electrostatic repulsion greatly reduced by rearrangement of the electrons and ions in the bulk crystal. The strain which occurs when two correlated deuterons try to occupy the same site, i.e., compete for a common location, is minimized when the small number of injected deuterons is distributed uniformly over all the unit cells of the lattice. Clearly these solid state energy and physical charge distributions differ radically from the comparable free-space distributions of widely spaced nuclei. It is important to point out here that this wave-function differs from other many-boson eigenfunctions in which the bosons interact weakly with each other, because these deuteron bosons, in contrast, interact weakly with the periodic lattice potential.

The BBC many-body wave function for $N_s$ deuterons in band states can be written as

$$\Psi(E_p) = (1/N_s!^{1/2}) \sum_{(r_m)} \prod_n \Phi_{\text{Bloch}}(k_{r_m}, r_n)$$

(8)

where the summation over $(r_n)$ includes the interchange of each coordinate with the remaining $N_s-1$ coordinates and the product is symmetric as required for bosons. This, in turn, can be substituted back into the expression above for $\Psi_{\text{Bloch}}(k,r) \exp(-E(k)t/\hbar)$. The resulting sum of products in which the Wannier index $s$ of a cell
in $\Phi_s(r_n, T_s)$ is almost never repeated since $N_\Phi \ll N_L$. However, a
small fraction of the terms contain products in which the cell
designator $s$ appears twice. These terms contain a product of
cospatial eigenfunctions centered at the center of unit cell $s$. They also involve the zero-point motion of the excited $D'$ ions in
the 3-D potential well. Those cells labelled with a common index
have a double occupancy by deuteron bosons. The mere existence of
the overlap in such cells does not, in itself, lead to nuclear
interaction. The important point is that the BBC must evolve in a
quantum-mechanically controlled manner.

Chubb and Chubb estimate the fractional occupancy of a unit
cell by excited $D''$ to be $10^{-7}$. When the initial state is formed
exclusively from excited deuterons, all intermediate states prior
to energy release must involve an integer number of neutron-proton
pairs. This leads to the selection rule, "bosons in, bosons out!"
which prohibits the formation as primary products of tritons,
$^3$He, neutrons or protons --- which are all Fermions --- these
reaction products are known to occur in the Free Space Paradigm.
Reaction paths with $n$-p pairs have a large overlap with the initial
BBC. The normally large electrostatic repulsion between deuterons
in free space is substantially reduced when these excited species
occupy band states. The Chubb-Chubb theory applies to $0^\circ$ K.

The BCS Theory of Superconductivity

Bogoliubov, working with the Fröhlich electron-phonon
Hamiltonian, used his method developed for superfluidity, and
independently derived the principal results and formulae of the
BCS theory. He showed that the creation of a pair of "virtual"
particles from vacuum, without phonons, had some unpleasant or
"dangerous" effects. The transition energy of such paired electrons
is small in comparison with the phonon energy $\hbar\nu$. In the second
paper from Bogoliubov's group, there are small terms in the
perturbation operator (see eq. 17) of Tolmachev and Tiablikov
which describe the creation of four particles from the vacuum. They
discuss the dynamics of electrons near the Fermi surface induced
by the electron-phonon interaction. The third paper by Bogoliubov
deals with diagrammatic methods to obtain the BCS equations.

Waber discussed the connection between superconductivity,
superfluidity and cold fusion. He treated deuterons as bosons and

See Reference 6.
indicated the influence of phonons and rotons (which may be appropriate for higher temperatures).

The BCS-Bose Formulation of Superconductivity

The connection is that superconductivity involves a sea of bosons which do not interact with the crystalline lattice. If the particle is in a field of phonons, a pair of Bloch particles can interact by exchanging massless phonons and lower their energy. Yukawa pointed out that nucleons, either neutrons or protons, would experience attraction by exchanging pions which are massive bosons. It is a similar process here since band deuterons (spin 1 massive bosons) involve interactions with phonons (massless bosons).

It is postulated here that near the Fermi Level, there is an instability that is induced by the phonons which leads to the formation of two Cooper pairs (i) a Cooper pair of electrons and as well as (ii) the formation of Cooper-like pair of deuterons (in place of the usual electron holes). Each of the D⁺ ions has a spin-paired (neutron and proton) in their band states.

Figure IV. Feynman Diagram showing the Two Cooper and Cooper-like Pairs. With momenta \( k_e \) of the paired spins of the electrons and \( k_D \) of the pair of bosonic deuterons. The optical phonon \( q \) is indicated.

If the distance between the centers of mass of either type of Cooper pair is large, the effective Coulomb interaction is modified
by the Debye-Hückel screened potential. The small Coulomb interaction between the deuteron states (reduced by the electrons and ions) further stabilizes the pairs.

One can infer good equations of motion diagonalized in a net momentum $q$. The Cooper pairs, both above and below the condensation temperature $T_c$, move independently because the net momentum $q$ is a constant of motion. The wave functions $A_j(k,q)$ are coupled with respect to $q$—this implies, according to Fujita and Watanabe that the true wave function is a superposition of "electron pair" and "hole (or deuteron, in our case) pair" plane-wave functions. The conditions for the separability of the "superimposed" wave function still applies (vide supra).

A compound semiconductor provides a medium in which optical phonons can easily be virtually excited. Further, it is likely to have more than one Fermi sheet so that the effective masses and local curvatures can be different. The hypothesis of supercondensation brought about by the exchange of massive bosons may be checked by the sign of the isotope effect (which is known to be anomalous in PdD$_x$). The lowest frequency is $\pi/a$ with a mass dependence of $M^{-1/2}$.

The Bose-Einstein Condensation temperature $T_{\text{BE}}$ can be estimated in the following manner, given that the experimental value of $T_{\text{BE}}(\text{4He})$ for helium is about 2.2°K

$$
\frac{T_{\text{BE}}(\text{PdD}_x)}{T_{\text{BE}}(\text{4He})} = \left( \frac{\rho_{\text{PdD}}}{\rho_{\text{4He}}} \right)^{2/3} \left( \frac{M_{\text{4He}}}{M_{\text{PdD}}} \right)
$$

$$
= \left( \frac{10^3 \text{ g/L}}{0.2 \text{ g/L}} \right)^{2/3} \times 2,700 = 2,700
$$

so $T_c$ is approx. 6,000 K. Here $\rho$ is the density.

Fujita's theory accounts for the coherence distance of the electronic Cooper pair in high $T_c$ materials being about 10 Å instead of the familiar 10,000 Å for conventional materials. However, the coherence length $\lambda$ is mass dependent; thus $\lambda$ is further reduced because of the heavy deuteron bosons. Since a deuteron weighs more than 3,000 electron masses, the value of $\lambda$ becomes roughly 30 Fermis.

**Requirements for Applicability of the FW Model**

An important condition for "cold fusion" using this model is that the Fermi Level of occupied states must be in the immediate
vicinity of the Brillouin Zone boundary. That is, one possibility is that the lattice is fully loaded and essentially stoichiometric, i.e., \( x = 1 \pm \delta \). Recent experimental evidence by the IMRA group suggests that \( x > 0.8 \) is adequate for Cold Fusion. It is interesting that the Density of States at the Fermi level, namely \( N(E_F) \) or chemical potential \( \mu \) falls to a minimum as a function of \( x \) near \( x = 0.85 \) as shown in Figure V.

![Figure V. Variation of the Chemical Potential \( \mu \) as a Function of the Hydrogen Content. (After Reference 8)](image)

The chemical potential is the partial molar free energy of an electron at the highest occupied electronic level. The implication of this graph is that it costs progressively less energy to add an occupied electronic state at the Fermi level as \( x \), the number of hydrogen atoms, is incorporated into the lattice until \( x \) becomes approximately 0.85.

Several *ab-initio* band calculations have shown that in such a case bonding occurs via anti-bonding states and these are associated with the hybridization between the s-electrons, provided by the additional deuteriums, and the 4d plus 5s states, provided by the metal.
Fujita\textsuperscript{21} and Fujita and Watanabe\textsuperscript{22} postulate that the Coulomb interaction generates a correlation among (but not between) the pairs of negative and positive charges. Because of the low density of states of the anti-bonding orbitals, the D atoms are only weakly bound and undergo large zero-point fluctuations.

Fujita and Watanabe\textsuperscript{23,24} point out that a requirement of their theory is that there be one or more pockets in the Fermi Surface where the density of states is high, and secondly that the phonons involved in creating the two pairs be optical, rather than acoustical.

The energies of the excited Cooper pairs form a continuous energy band. Chubb and Chubb call these ionic band states (IBS) and a Boson-Bloch-Condensate BBC can form in excited states above the Fermi Level. In contrast to their assumption, the deuteronic Cooper pair of the present theory has energies below the Fermi Level, but they are still band states.

Fujita and Watanabe (who assume electron holes as the positive charge carrier) postulate that the Coulomb interaction generates a correlation among (but not between) the pairs of negative and positive charge. They modified the BCS theory as stated elsewhere. We have further modified their theory while retaining most of their changes, by replacing the electron holes by deuterons. We will label the electronic pairs with a subscript \(e\) and the deuterons by \(d\). The corresponding individual correlation strengths are called \(V_{dd}\), \(V_{ee}\), \(V_{ed}\) and \(V_{de}\) and in the same vein, we have assumed that

\[
V_{dd} = V_{ee} < V_{de} = V_{ed} \quad (10).
\]

It should be noted that these interactions which they assumed were between fermions whereas the inequalities just cited mix fermions and bosons. The energy gap constants \((\Delta_1, \Delta_2)\) for electrons (and deuterons) may be on different Fermi sheets and have different densities of states \(N_g(0)\) and \(N_e(0)\).

The presently modified BCS theory has been constructed with pairs of electrons (and deuterons) with opposite spins \((k\uparrow, -k\downarrow)\), and the two different momenta have been restricted to energies in the narrow range of 0 to \(\hbar \omega_b\), relative to \(E_F\), or \(\xi\) (the chemical potential) where \(\omega_b\) is the Debye frequency. Fujita and Watanabe lifted the BCS restriction of equal momenta. The two pair are restricted to lie in a thin shell either side of the Fermi level surface. While the energy difference is small, the effect of these pairs is quite large. Fujita and Watanabe point out that permitting unequal interactions and unequal momenta has some important consequences. It is interesting, for example, that Type I, Type II, organic superconductors as well as high \(T_c\) cuprates can be handled by the same theory.

One can infer good equations of motion diagonalized in a net momentum \(\tilde{q}\) involving \(\tilde{B}\) which will be defined in terms of creation and annihilation operators below. The Poisson brackets are
\[ [H, \tilde{B}_{kq}^{(d)}] = (E^{(e)}_{k+q} + E^{(e)}_{-k+q}) \tilde{B}_{kq}^{(e)} - v_{ee} \sum B_{k'q}^{(e)} - v_{ed} \sum B_{k'q}^{(d)} \]

and the bracket \([H, \tilde{B}_{kq}^{(d)}]\) terms are equivalent for the excited deuteron \(D''\). The creation operator for the excited, non-zero momentum Cooper pair is defined by

\[ B_{kq} = B_{k+q}^{(e)} C_{k+q} + C_{k+q}^{(e)} B_{k+q} \]

Equation (11) shows the Cooper pairs, both above and below the condensation temperature \(T_c\), move independently because the net momentum \(q\) is a constant of motion. The conditions for the separability of the "superimposed" electron pair and deuteron (bosonic) pair wave functions still applies. It should be noted in passing that Chubb and Chubb are slightly vague about the electrons which are introduced by the ionization of the deuterium atoms and which accompany the formation of deuterons in their BBC. They say they go into the next unoccupied electronic state but do not discuss pairing.

Phonons

Cho and Leisure\(^{25}\) observed, in addition to the usual Zener-type of relaxation peak found in both compounds, a strong optical phonon in the "alpha-prime" deuteride \((x > 0.64)\) which was not present in the hydride. These frequency dependent peaks are shown in Figure VI.

They interpreted the larger peak, i.e., the lower frequency one as arising from a longitudinal optical phonon propagating in the \([110]\) direction polarized along \([001]\). Also it was found to be very persistent. The relevant Einstein temperature was 590°K. Thus all the requirements of the FW theory are met.

Effect of Pions and Other Bosons

The exchange forces between electrons, i.e., the spin information is conveyed by virtual photons. The exchange of acoustical phonons between particles is long range since they are massless bosons and the interaction is appropriate for Type I superconductors where the coherence distance lies in the range of 1000 to 10,000 Å. While acoustical phonons are probably extant for the high \(T_c\), they cannot account for the smaller value of lambda, namely two unit cell dimensions. Optical phonons on the other hand have the lowest energy and their wavelengths are of the order of lattice spacings. The attraction generated by the exchange of massive bosons is short-range like the nuclear forces between two nucleons discussed by Yukawa\(^{26}\). More generally speaking all the possible causes for pair attraction should be examined. The presence and exchange of pions has been overlooked so far. Kenny\(^{27}\) points out that the attraction caused by the exchange of these spin 0 bosons is seven times that of the nucleonic constituents. He has presented a table of several reaction paths.
The effect of the pion exchange is to stabilize the deuteronic bosons and despite any strong repulsive effect of the deuterons being 50 Fermis apart, the exchange will bring them close enough that they will fuse.

Figure VI. Plot of the Ultrasonic Attenuation as a Function of Frequency Observed in Palladium Deuteride but not in the Hydride. (After Reference 25)

Oppenheimer-Phillips\textsuperscript{28} paper indicates that the range of the neutron portion of the deuteron wave-function extends to about 5 Fermis. Rarita and Schwinger\textsuperscript{29} discuss the deuteronic wave-functions and the potential well they deduced from scattering
DISCUSSION

We propose that the closely spaced pair of D' in ionic band states (with the modification of Chubb and Chubb that they lie below the Fermi Level) become the precursor of 4He ionic band states with the energy release spread over the crystal. So coalescence of the two D' bosons becomes quite feasible.

Fujita and Watanabe together with the Chubb and Chubb provide a means of resolving three or more issues raised as objections by many others, namely that (1) the deuterons must behave as particles, (2) they must penetrate a strong Coulomb barrier of the Gamow type, and (3) the length scale associated with nuclear reactions must be of nuclear dimension and (4) the by-products of reaction must be released from the solid since there are no known electrostatic processes capable of "trapping" product particles with energies in the vicinity of tens of MeV, without violating momentum or energy conservation.

SUMMARY

The solid state nature of cold fusion and the important fact electrons and similar particles are not localized has been largely overlooked in the literature. The connection with superfluidity and superconductivity is also not apparent as long as the deuterium atoms in palladium deuteride are treated as isolated particles in free space. Chubb and Chubb have repeatedly drawn attention to the extended wave nature of the electrons involved in the Bloch state and the Brillouin zone treatment of solids. This explains how the electron move in virtually a zero potential field, since they are screened by the other N ion cores and N-1 electrons. Further, the correct prediction of 4He as the principal product stems from the selection rule they derived, on the basis of solid state theory, namely boson in, bosons out.

The role of bosons as Cooper pairs links cold fusion with superconductivity. Extension of the BCS theory by Fujita and Watanabe have indicated certain requirements for superconductivity. Their theory is further modified herein for cold fusion by allowing the optical phonons to create two kinds of Cooper pairs, namely (a) excited electrons and instead of electron holes, (b) Cooper-like pair of deuteron band states which lie below the Fermi level. Both deuterons in the pair are bosons and their spins (±1) are also paired. The coherence length is shown to be of the order of a few Fermis due to the mass dependence and the larger mass of the deuterons. The Bose-Einstein condensation temperature of the bosonic deuterons is estimated to be as high as 6,000°K, so that one is well within the estimated condensate regime. The FW requirement of strong optical phonons in the deuteride is also met as is the requirement of pockets in the Fermi surface. Given these facts, the exchange separation of the two D' ions brings them
within the range of attractive nuclear forces, Exchange of virtual pions further increases the likelihood of two deuterons fusing to form "He as the "ash."

Note added in Proof

Johnson raised the question of whether the Meissner Effect could be observed at temperatures well above room temperature? If it were observed, the present proposal would have powerful support. However, there might not be sufficient of these Cooper-like bosonic pairs to exclude the magnetic field. Chubb and Chubb estimate the concentration of excited deuterons to be only $10^{-7}$, whereas there are roughly $10^{-4}$ conventional Cooper pairs per electron in metallic superconductors. Because of the low concentration of the deuteronic pairs, the skin depth will be large and thus scattering of the electrons will be significant. Thus little evidence of superconductive current or Meissner effect would be anticipated. This does not argue against the pairing of the deuterons.

Two papers at this conference discuss aspects of deuterons being bosons, although they come to negative conclusions. Tsuchiya et al. treat the interaction between two particles as though it occurred in free space. Vaidya does consider screening of the deuterons by conduction electrons and proposes a method of producing additional optical phonons which he says would enhance the fusion rate. However, he bases his overall conclusion on penetration of a Gamow-like barrier. It is the present author's opinion that the enhanced mobility of deuterium can be attributed to a strong interaction with optical phonons not with barrier penetration.

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References


12. The possibility of tritium etc, as secondary products is not specifically covered at this point.

13. This restriction applies equally to References 1 through 6.


19. The distinction is made here that the electron and electron holes in the original Cooper pairs involve fermions, where the deuterons are bosons.


31. Ken-ichi Tsuchiya, Kazutoshi Ohashi and Mitsuru Fukuchi, "Mechanism of Cold Fusion II," This Conference Paper T 3.12
