Abstract

We derive a criterion for coherent \((n,r)\) reaction between propagating neutrons and the nuclei at lattice sites in a crystal, and for coherent d-d reaction between itinerant deuterons and the lattice deuterons in a crystal of PdD\(_x\). We show that the reaction rate increases by \(N\), the number interacting nuclei (deuterons) in the coherence region. In the case of d-d fusion reactions, the effect of tunneling on the reaction rate is also considered. Some applications of coherent \((n,r)\) reactions are discussed.

1. Introduction

In-phase interactions between photons (or propagating quantum particles) and active centers in a medium can lead to coherence effects as a result of which total scattering amplitude (or interaction matrix) increases as \(N^{1/2}\), where \(N\) is the number of active centres in the medium. Dicke superradiance and laser phenomenon are the established examples of such coherence phenomena. The state-of-art neutron interferometry experiments have not only reaffirmed the wave nature of neutrons but also established persistence of coherence over long distances. The present paper considers coherent \((n,r)\) reactions between propagating monoenergetic neutrons and the nuclei in a crystalline solid. The coherent interactions between itinerant deuterons (deutrium ions) and lattice deuterons in crystalline PdD area also considered.

2. Coherent \((n,r)\) Reaction

The wave nature of propagating neutrons allows them to interact simultaneously with all lattice nuclei in the crystal. Consider a neutron beam having De Broglie wavelength \(\lambda = \hbar/(2\pi E)^{1/2}\) propagating through a single crystal of rhodium along [100] direction (Fig.1). The neutrons will form compound nucleus state \(\Psi_1(0)\) at all lattice sites \(R_i = R_i + u_i\) in the overlap region \(0<r<r_n\), where \(r_n\) is the size of the nucleus. As in normal or anomalous scattering of neutrons, we associate a phase factor \(\exp(ik \cdot u_i)\) with the state \(\Psi_1(0)\), where \(k\) is propagation vector and \(|k| = 2\pi/\lambda\). Hence, the total overlap wave function \(\Psi(0)\)

\[
\Psi(0) = \sum \Psi_1(0) = (1/\sqrt{\Sigma}) \cdot \Psi_1(0) \cdot [\sum \exp(ik \cdot u_i)]
\]

where \(\Sigma\) denotes sum over all the \(N\) sites and \(\Psi(0)\) is the wave function in the overlap region evaluated from the interaction.
potential using the WKB method. The rate of nuclear reaction

\[ {}^{104}\text{Rh} (n,r) {}^{104}\text{Rh}, \quad R = |\psi(0)|^2 \] 

can thus be expressed as

\[ R = \left( \frac{A}{V} \right) |\psi(0)|^2 \exp \left( -\frac{k^2 u_{ik}^2}{2} \right) \times S^2 \quad (2) \]

\[ S^2 = \Sigma \Sigma \exp \left( ik(E_{i0} - E_{j0}) \right); |k| = \frac{2\pi}{\lambda} \quad (3) \]

Here \( A \) is the nuclear rate constant and \( V \) is the volume of

coherence region. It is seen that the reaction rate \( R \) depends on

the Debye Waller factor and the structure term \( S^2 \). For neutrons

propagating along \([100]\) direction, phase coherence occurs when \( E \cdot (E_{i0} - E_{j0}) = 2\pi \) or

\[ \lambda = \left( \frac{a}{2m} \right) \quad (4) \]

where \( a \) is the lattice parameter and \( m \) is an integer (Fig.1)

From Eqs.(3) and (4), \( S^2 = N^2 \) at coherence. For \( \lambda \neq \left( \frac{a}{2m} \right) \), on

the other hand, \( S^2 = N \). Hence

\[ R_{\text{coh}} = N \cdot R_{\text{incoh}} \quad (5) \]

and the reaction rate increases by a factor \( N \) at coherence.

Coherence conditions for simple lattices are given in Table 1

The number of participating nuclei \( N \) can range from \( 10^3 \) to \( 10^5 \)

depending on the coherence of the incident neutron beam and the

mosaic size of the crystal. The experiments for studying

coherence enhancement using thermal neutrons from a research

reactor have been mentioned elsewhere.

2.1 Applications

Coherent \((n,r)\) reactions can be used for production of isotopes

at enhanced rates. Pure isotopes or their suitable compounds

must be used in form of single crystals. \(^{197}\text{Rh}, \; ^{181}\text{Ta}, \; ^{59}\text{Co}, \; ^{115}\text{In} \) and others which have nearly 100% natural abundance and

belong to crystal structures of high symmetry can be used in such experiments.

Coherent \((n,r)\) reaction can produce simultaneously a large number of nuclei \( n^{+1}A^* \) in excited state. This may be used for creating

a large population of required nuclei \( n^{+1}A^* \) for fission action.

Coherent \((n,r)\) reactions can produce intense gamma rays since, in analogy with Eq.(5),

\[ I_{\text{coh}} = N \cdot I_{\text{incoh}} \quad (6) \]

The interference among the intense gamma rays produced by coherent interaction will give intensity maximums. The maximum for the gamma ray of wavelength \( \lambda \) will occur along the directions \( OP \) at an angle \( \phi \) to the direction of incidence \( OA \) when

\[ d \cdot \sin \phi = n \cdot \lambda \quad (7) \]
Table 1

Coherence condition for simple lattices.

<table>
<thead>
<tr>
<th>lattice; cell constant</th>
<th>example</th>
<th>direction of incidence</th>
<th>n\lambda at coherence</th>
</tr>
</thead>
<tbody>
<tr>
<td>face centered cubic; a</td>
<td>Au,NaCl</td>
<td>[100]</td>
<td>a/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[110]</td>
<td>a/2\sqrt{2}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[111]</td>
<td>a/\sqrt{3}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[112]</td>
<td>(a/2\sqrt{6})</td>
</tr>
<tr>
<td>body centered cubic; a</td>
<td>Ta</td>
<td>[100]</td>
<td>a/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[110]</td>
<td>a/\sqrt{2}</td>
</tr>
<tr>
<td>hexagonal close packed; a,c</td>
<td>Co/Tb</td>
<td>[0001]</td>
<td>c/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1000]</td>
<td>a/2</td>
</tr>
</tbody>
</table>

where n is an integer and d is the lattice periodicity along OA (Fig.2). Large scale application of coherent (n,r) reactions will require technology for production of high flux, tunable, monoenergetic neutron beams from nuclear reactor or other sources.

3. Coherent d-d Reaction in PdD_x

In some theoretical studies, electron-deutron screening, tunneling or coherent interactions have been suggested as possible mechanisms for enhancement of d-d reaction rate. According to the coherent interaction mechanism\(^1\), the of fusion reaction between the mobile deuterons and the lattice deuterons increases by N when the deuteron wavelength \( \lambda_d = \hbar/(2MDv)^{1/2} \) meets the coherence condition. For deuterons propagating along [100] direction, in analogy with Eq.(4), coherence occurs when

\[
\lambda_d = (a/2m) \quad ; \quad m \text{ integer .} \tag{8}
\]

In general, coherence criterion can be expressed as

\[
\lambda_d = x \cdot L \tag{9}
\]

where L is the periodicity and x is a number which depends on the direction of propagation (Table 1). A propagating deuteron can tunnel through an array of one dimensional potentials having periodicity L (Fig.3) when the length of classically accessible region in each well is an odd multiple of \( \lambda_d/4 \):

\[
(L-2r_c) = (2m+1) \cdot \lambda_d/4. \tag{10}
\]
Here $r_t$ is the turning distance which depends on the screening length. Following Turner and Bush we shall assume that the same condition is applicable for tunneling of deuterons through the crystal. Tunneling preserves the phase of propagating deuteron and hence, under ideal conditions, the coherence length $l_c$ will be very large - equal to the length of the length of the crystal. There is however considerable reduction in $l_c$ on account of non-monochromaticity of deuterons and finite mosaic size of the crystal.

The coherence criterion Eq.(9), and tunneling criterion Eq.(10) are met simultaneously along a certain direction of propagation of deuterons if

$$\frac{2r_t}{L} = 1-(2m+1)x/4; \quad \lambda_d = xL \quad (11)$$

Maximum d-d fusion rate will be attained when Eq.(11) is fulfilled.

We have here attempted to bring out the role of tunneling and coherence mechanisms which have hitherto been discussed separately. In case of (n,r) reactions, the tunneling criterion can be ignored since neutrons do not experience coulomb repulsions at lattice nuclei. Tunneling of deuterons can be increased by the application of longitudinal acoustic waves to the PdD$_x$ crystal. This will enhance the d-d reaction rate by increasing the screening parameter $k_d$. These aspects are discussed in the accompanying paper at this conference.

References


FIG. 1 PROJECTION OF FCC LATTICE ON C AXIS

FIG. 2

FIG. 3