ON BOSE-EINSTEIN CONDENSATION OF DEUTERONS IN PdD

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Abstract

In this paper we consider the aspects connected with the screening of coulomb interaction between deuterons in PdDx. We propose an experiment for increasing deuteron number density, their fluctuations and screening in a single crystal of PdDx, at low temperatures, to approach conditions for the Bose condensation of deuterons for achieving high d-d fusion rates.

1. Introduction

Following the work of Fleischmann and Pons, several careful electrolytic and gas effusion experiments performed on palladium deuteride in the last four years have reported observation of excess heat, neutrons and charged particles. Measurements of excess heat which exceed several $\sigma$, observation of neutron emissions in PdDx and DsWO3 systems, and of $^4$He from in-situ mass-spectrometer experiments have steadily accumulated evidence in support of cold fusion. Several theoretical models have been suggested to account for the enhancement of deuteron-deuteron (d-d) fusion rate from a low value of about $10^{-74} \text{s}^{-1} (\text{dd})^{-1}$ in an isolated D$_2$ molecule to about $10^{-24} \text{s}^{-1} (\text{dd})^{-1}$ deduced from some of the experiments. In this paper we shall consider the electron-deuteron screening, tunneling and coherent interaction mechanisms which have been proposed for enhancement of d-d fusion rate. We propose an experiment for elucidating the role of deuteron screening and for increasing the fusion rate.

2. Electron-Deuteron Screening

Fusion cross-sections of $^2\text{H}(d,r)^4\text{He}$, $^3\text{He}(d,p)^4\text{He}$ and $^3\text{He}(d,n)^4\text{He}$ nuclear reactions show an increase for centre-of-mass energies $E_{CM} < 15$ Kev. This increase has been attributed to the screening of nuclear interaction by the electrons in target atoms. The screening by conduction electrons in metals is well known, and it forms the basis of the free electron gas model. The d-d interactions in metal deuterides at $E_{CM} < 1$ ev are likewise screened by the conduction electrons and, as a result, the d-d fusion rate $R$ increases to about $10^{-40} \text{s}^{-1} (\text{dd})^{-1}$. The formation of electron-deuteron (deuteron ions) plasma against the background of palladium lattice, under the experimental conditions, can increase the screening and $R$. At low energies, the electrons and ions in the plasma instantaneously rearrange themselves so that the polarization charge follows the
interacting ions. Carraro et al.\textsuperscript{12} have shown that both the electrons and ions make full contribution of screening at low $E_{\text{CM}}$ but the screening falls off as the $E_{\text{CM}}$ increases.

The effective interaction potential in an ideal plasma of electrons and deuterons\textsuperscript{4,5} is $(e^2/r)\exp(-Kr)$ with screening parameter

$$k^2 = k_e^2 + k_d^2 = \left[6\pi e^2n_e/E_F\right] + \left[4\pi e^2n_d/k_BT\right] \left[\frac{g_{3/2}(z)}{g_{3/2}(z)}\right]$$

(1)

Here $n_e$ and $n_d$ are the electron and deuteron number densities, $E_F$ is the Fermi energy and $g_{3/2}(z) = \zeta_{3/2}(n_{-3/2}^z)$. The electron-deuteron screening model for PdD gives $R \sim 10^{-16}$ s\textsuperscript{-1}(dd)\textsuperscript{-1} with $n_e = 1.36 \times 10^{23}$ cm\textsuperscript{-3} and $n_d = 6.8 \times 10^{23}$ cm\textsuperscript{-3}.

The deuteron screening parameter $k_d$ decreases rapidly if the deuteron number density and mobility are small. This also follows from the expression\textsuperscript{13}

$$k_d = \frac{4\pi e^2\langle y^2\rangle}{k_BT} ; \langle y^2\rangle = \langle n^2\rangle - \langle n\rangle^2$$

(2)

where $\langle y^2\rangle$ is the number density fluctuation and $n$ is the average site occupancy number.

Bose condensation temperature of an ideal deuteron gas is

$$T_B = \left(\frac{2\hbar^2}{M_kk_B}\right)\left(\frac{n_d/2.612}{2}\right)^{2/3}.$$  

(3)

For PdD, Eq. (3) gives $T_B = 6.65K$. As the temperature $T \rightarrow T_B$ and Bose condensation is approached, the screening parameter $k_d$ in Eq. (1) $\rightarrow \infty$ and the d-d fusion rate becomes very large. The superconducting transition temperature of PdD\textsubscript{x} ($T_C = 11K$) is higher than that of PdH\textsubscript{x} ($T_C = 8K$) for $x = 0.6 - 0.7$. The inverse isotope effect of $T_C$ suggests that at low temperatures the screening of the interactions between electrons (which form Cooper pairs) is higher in PdD\textsubscript{x} than in PdH\textsubscript{x}.

Whaley's cluster model calculations\textsuperscript{13} show that d-d fusion rate $\sim 10^{-20}$ s\textsuperscript{-1}(dd)\textsuperscript{-1} requires large deuteron density fluctuation $\langle y^2\rangle \sim 0.1$. Migration of deuterons in PdD\textsubscript{x}, as in other transition metal hydrides and deuterides is at low temperatures governed by tunneling, and at high temperatures by over-the-jump mechanisms which are activated processes. The tunneling process, as opposed to other processes, preserves phase correlation among the migrating deuterons. Hence the increase in $\langle y^2\rangle$ by correlated tunneling of deuterons at low temperatures will increase the d-d fusion rate due to coherence as well.

3. Proposal for an Experiment

We propose, taking into consideration the aspects discussed in Sec.2, the following experiment for realizing high values of $R$ in PdD\textsubscript{x} crystal. We take a single crystal of palladium grown along
a principal crystallographic axis such as [100] or [111] with flat ends normal to the cylinder axis (Fig.1). The cylindrical surface is coated with gold as has been done by Yamaguchi and Nishiota. The crystal is charged with deuterium by electrolysis or by gas loading to high D/Pd ratio. Flat ends of the crystal are capped by two similar transducers for propagating longitudinal acoustic waves of high frequency $\omega > 10^9$ Hz. The transducers are aligned to direct the ultrasonic waves along the cylinder axis. The gold coating on the cylindrical surface helps ensure preferential migration of deuterons along the cylinder axis.

At low temperatures, deuterons occupy lowest energy levels in their potential wells in the face-centered-cubic lattice, have low mobility and deuteron transport is mainly due to tunneling between adjacent sites. The tunneling probability can be enhanced to yield high values of $n_3$ and $<y^2>$ by application of external perturbation in form of ultrasonic waves. The experiment consists in driving deuterons to the centre of the crystal by means of longitudinal acoustic (LA) waves from the two transducers which are operated at same amplitude and frequency. The phase difference between the transducers and the amplitude are varied to achieve maximum tunneling rate.

4. Effect of Longitudinal Acoustic Waves on Tunneling

In order to understand the effect of LA waves, we shall review the results of typical analysis of the effect of periodic perturbation on tunneling in a double-well potential. The
Hamiltonian of a particle in a quartic double-well can be expressed in dimensionless variables, following Grossmann et al., as

$$\begin{align*}
H(x) &= -\frac{1}{2} \frac{d^2}{dx^2} x^2 + \frac{x^4}{4} + x S \sin \omega t, \\
&= \frac{1}{64D} \left( \frac{d^2}{dx^2} x^2 \right) + x S \sin \omega t.
\end{align*}$$

Here $\omega_0$ is the frequency of small oscillation at the bottom of the well, $E_B$ is the barrier height, $S$ is the amplitude of perturbation in units of $(\hbar/M_0\omega_0^3)^{1/2}$ frequency $\omega$ and $D=(E_B/\hbar\omega_0)$ (Fig.2). Using Floquet formalism it has been shown that the wave function

$$\Psi_k(x,t) = \exp(i\epsilon_k t) \phi_{k,l}(x,t)$$

has periodicity $T = (2\pi/\omega)$ and quasi-energies corresponding to the unperturbed state $\epsilon_k$ are

$$\epsilon_{k,l} \equiv \epsilon_k + lw; \quad w = 0, \pm 1, \pm 2, \ldots$$

The values of $E_B$ and $\omega_0$ for $D$ in Pd will be of same order of magnitude as those for $H$ on W(100) surface, namely, $E_B = 240$ mev and $\omega_0 = 2.4\times10^{13}$ Hz. Compared to this, the frequency of LA wave $w = 10^9$ Hz is quite small. Tunneling probability for $w > \omega_0$, under adiabatic approximation, shows periodic variation with $n$, the number of cycles of $w$. The results of numerical calculations for $(E_B/\hbar\omega_0) = 1$, $(w/\omega_0) = 0.06$ and $s = 0.028$ reproduced in Fig.3 show variation of $P$ between nearly 0 and 1 as a function of $n$. Under repeated application of perturbation $\Psi_w$, the particle is progressively raised to states having higher $l$, and the $P$ increases. As the tunneling progresses, the system returns to the original state and the cycle starts all over again.

The results of this analysis can be extended to an array of periodic potentials in the same way as Bohm's tunneling criterion is extended to a periodic array.

The application of ultrasonic waves, by changing the tunneling rate, will lead to periodic increases in $n_d$ and to large fluctuations $<y^2>$. As increased number of mobile deuterons are driven to form a Bose condensate, there will be a substantial increase in the rate of production of heat and neutrons. The warming and cooling of the crystal will produce periodic bursts of heat and neutrons in such an experiment.

5.Conclusion

The above semi-classical analysis suggests that the application of LA ultrasonic waves to a crystal of PdD$_x$ can increase the $d-d$ fusion rate $R$. There might be further a) increase in $R$ due to phase coherence among the tunneling deuterons.

The deuteron charge $Z$ in PdD$_x$ depends, among other things, on
PARTICLE IN DOUBLE WELL POTENTIAL UNDER PERIODIC PERTURBATION

$W_0 =$ NAT. VIB. FREQUENCY
$E_B =$ BARRIER HEIGHT
$D = E_B / \hbar W_0$
$S =$ AMPLITUDE OF PERTURBATION
$W =$ FREQ. OF PERTURBATION

FIG. 2

$P =$ TUNNELING PROBABILITY FOR $W < W_0$
(APPLICABLE TO ULTRASOUND)

$D = (E_B / \hbar W_0) = 1$
$W / W_0 = 0.06$
$S = 0.028$
$n =$ PERIODS OF $W$

$P$ SHOWS PERIODIC INCREASE WITH $n$

FIG. 3
their velocity. For Z<+1, the d-d fusion is hindered by the veil of electron around the deuteron as in a D₂ molecule or D₂⁺ ion. It may not also be appropriate to apply Bose statistics to an assembly of partially ionized deuterons. It appears that Z approaches +1 (i) in the Fleischmann and Pons experiment at high current densities and (ii) in the experiments with DₓWO₃ crystals which give high fusion rates. Departure of Z from value +1 probably hinders the attainment of high fusion rates.

References


