GENERALIZED ISOTOPIC FUEL LOADING EQUATIONS

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ABSTRACT

The quasi-one-dimensional (**Q1D**) model [1] of isotopic fuel loading, including the coupling to a material [2], has offered insights into electrodes and materials filled by an isotopic fuel. Past examinations [1,2,3] have discussed both competitive gas-evolving reactions at the surfaces of the electrode, the impact of the ratio of the applied electric field energy to thermal energy [k_B*T], and other factors which appear decisive in controlling the loading. Following the demonstration of sonic-induced loading, and cold fusion reactions [4] and complexities at the metal-double layer junction [5] the derived fusion flux equation linking the deuteron loading flux from the region external to the material to potential reactions at that site must be extended to include any ordering force including those derived from sonic fields.

There may be implications upon calculations of the activities and fugacities [6,7] following aqueous loadings [8] and derivations of distributions of deuterium in the palladium [9] and the solution [1,10]. Our present calculations attempt to account for induced drift by the applied electric field [11], electric field distributions altered with complex conduction and polarization phenomena [12,13,14], secondary space charge polarization, propagation of solvated deuterons, deuterons in clathrates, and L-,D-deuteron defects with their ferroelectric inscription in the heavy water [11,13], and the formation of low dielectric constant bubbles [14]. The double layer between the solution and the metal is created both by the cathode fall of ions and other polarization reactions [6,7,11] and is significant. At this double-layer boundary intermolecular deuteron transfer from the solution to octahedral sites within the palladium may control the loading [11]. Within the metal, the deuteron diffusion [1,3,9] involves optical and acoustic phonon spectra [3], material defects, grain boundary dislocations, "zeolite"-like diffusion [9] and fissures.

INTRODUCTION

Classical calculations of the activities of an ionic electrolyte [1,2] adjacent to a metal electrode have been applied to cold fusion reactions following loading of isotopic fuel into a metal [3], and have been used to derive the distributions of deuterium in the palladium [4] and in the solution [5,6]. However, one premise is that the systems are at equilibrium which may not be true [7]. Therefore, a quasi-one-dimensional (**Q1D**) model for an electrode filled by the isotopic fuel was formulated [5]. The **Q1D** model offers insight into the processes because it indicates how both competitive gas evolving reactions at the metal electrode surface and the ratio of the applied electric field energy to thermal energy [k_B*T] are decisive in controlling the loading of the metal by the deuterium [5]. We now extend that model and correct the derived fusion equation [8] which links the deuteron loading flux from the solution into the metal for potential reactions at that site.

DEVELOPMENT OF THE MODEL

Fig. 1 shows the four regions of the electrochemical cold fusion cell. Within the heavy water solution, most deuterons are tightly bound to oxygen atoms. The power source generates the applied electric field intensity. The induced drift by the applied electric field is shown schematically in the figure; which does not mean that the deuterons actually are free to travel in such a simple fashion [9]. The electric field

distribution is altered as the solution and system each respond with complex conduction and polarization phenomena [10,11,12]. Ionic drift, secondary space charge polarization, propagation of solvated deuterons, deuterons in clathrates, and L-,D-deuteron defects with their ferroelectric inscription in the heavy water [9,11], and the formation low dielectric constant bubbles abutting the cathode are the minimum expected [12]. The double layer between the solution and the metal is created both by the cathode fall of ions and other polarization reactions [1,2,9].

In the absence of significant convection, the flux (J_i) of any i_{th} species (here deuterons) results from both diffusion down concentration gradients and electrophoretic drift [5,7,10].

$$J_D = - B_D * \frac{d[D+(z,t)]}{dz} - \mu_D * [D^+(z,t)] * \frac{d\Phi}{dz} \quad (1)$$

Three components of the deuteron flux (J_i) must be considered at the cathode. The first flux component is the entry of deuterons into the bulk of palladium (J_e). The second flux component is the loss of deuterons secondary to gas evolution (J_g). The third flux component is caused by those deuterons lost to any putative fusion reactions, and is represented as J_{fus}. J_{fus} is assumed to be 0 in the bulk solution. The mathematical solution for the time rate of change of the deuterium in any given volume is determined by these fluxes and Gauss' Theorem [5].

Deuteron entry to the cathode is electron limited with all entry occurring at the cathode-double layer interface. At this boundary intermolecular deuteron transfer from the solution to octahedral sites within the palladium may control the loading [9]. Within the metal, the deuteron diffusion has been considered by several models [5,4,13]. Optical and acoustic phonon spectral [13], material defects, grain boundary dislocations, "zeolite"-like diffusion [4] and fissures, all may influence the deeper loading of the metal.

CRITICAL LOADING FLUX

The **Q1D** model links the deuteron flux from the solution into the pericathodic volume, and includes deuteron flow into the metal, deuterons involved in gas evolution, and deuterons consumed in any potential fusion reactions.

$$\frac{d[D^+(z,t)]}{dt} = \left[B_D * \frac{d^2(D^+)}{dz^2}\right] + \left[\mu_D * (D^+) * \frac{d^2\Phi}{dz^2}\right] + \left[\mu_D * \frac{d\Phi}{dz} * \frac{d(D^+)}{dz}\right] + \left[\frac{d(D^+)}{dz} * \frac{dB_D}{dz}\right]$$
$$+ (D^+) * \frac{d\Phi}{dz} * \frac{d\mu_D}{dz} - \frac{d[\Sigma J_i]}{dz} \quad (2)$$

The mathematical solution of equation 2 is determined both by the boundary conditions and by conservation of mass. There is assumed conservation of deuterons with the exception of a loss (J_{fus}) to all putative fusion reactions, extremely small compared to either most loading rates or gas evolving reactions[5]. As discussed previously [5], examination of the solution indicates that the deuteron loading rate into the electrode is critically linked to gas evolution and is also first order on [μ_D * E].

$$\kappa_e = (\mu_D * E) - (\kappa_g + \kappa_{fus}) \quad (3)$$

This loading rate equation relates deuteron availability (secondary to the applied electric field) to the losses of deuterons to both gas evolution and the fusion reactions. One simple but important corollary

is that the evolution of D_2 gas and deuteron loading to the palladium cathode are mutually exclusive for any given applied electric field.

Quantity of Deuterons Contributing (Ψ_{fus})

In a successful cold fusion system J_{fus} is not zero. Therefore, the non-dimensional parameter, Ψ_{fus}, is defined as the fractional amount of intrapalladial deuterons which actually contribute to the desired reactions. When the filling of the palladium with deuterium is complete, J_e would be on the order of J_f [5]. This fusion rate equation can be examined for its relation to thermal processes by substitution using further non-dimensional parameters and the Einstein relation.

$$\frac{B_D}{\mu_D} = \frac{k_B * T}{q} \tag{4}$$

The Loading Ratio ($\Lambda_{Pd,D}$)

The non-dimensional parameter $\Lambda_{Pd,D}$, is defined as the ratio of the two largest and most important pericathodic fluxes; the loading flux (J_e) to the gas evolution (J_g). It is very much a function of the isotope and the material, hence the paired subscript.

$$\Lambda_{Pd,D} = J_e / J_g \tag{5}$$

Thus if $\Lambda_{Pd,D}$ is .01, most of the current is going to gas electrolysis, whereas $\Lambda_{Pd,D} = 100$ would indicate more efficient loading. Substitution of the transolution voltage, and $\Lambda_{Pd,D}$ as the loading factor, and the Einstein relation yields equation 6.

$$J_{fus} = \frac{[2\Lambda_{Pd,D}]}{[2\Lambda_{Pd,D} + 1]} * \frac{[B_D * <D_i>]}{L_c} * \frac{1}{[1 - \exp\left(\frac{q * V}{k_B * T}\right)]} * \frac{[q * V]^2}{[k_B * T]^2} * \Psi_{fus} \tag{6}$$

This fusion flux equation (equation 7) contains five terms after separation of variables. The first term results from gas evolution. The second term is composed of geometric and material factors. The next two terms reflect the applied electric field intensity and k_BT and are dominated by the ratio of the applied electrical energy which are organizing the deuterons to the energy causing their random thermal disorganization. The final term is the fraction of deuterons which actually partake in any potential fusion process(es). Ψ_{fus} is the fractional amount of intrapalladial deuterons which actually contribute to the desired reactions. Introducing ζ, the electric order/thermal disorder ratio, then simplifies this fusion flux equation.

$$J_{fus} = \frac{[B_D * <D_i>]}{[L_c * [1 + (2\Lambda_{Pd,D})^{-1}]]} * \frac{[\zeta^2]}{[1 - \exp[-\zeta]]} * \Psi_{fus} \tag{7}$$

This relationship is demonstrated in Fig. 2 and Fig. 3 which show the impact. In figure 2, for simplicity, J_{fus} is assumed to be 0. The loading flux of deuterons into the palladium at the cathode surface (J_e) is shown as a function of the electric field intensity, for various rates of gas [D_2] evolution rates (J_g). The series of parametric curves indicates how the loading rates are sensitively dependent both upon the electric field energy as well as the competing gas evolving reactions. Examination of equation 7 indicates that

although $\Lambda_{Pd,D}$ has major effects for every ζ, however, that importance requires a level of $\Lambda_{Pd,D}$ approx. > 1 to plateau its importance as is shown in Fig. 3.

The critical term previously did reflect the applied electric field intensity and k_BT and are dominated by the ratio of the applied electrical energy (or generally, other organizing energy) which are organizing the deuterons to the energy causing their random thermal disorganization. The final term is the fraction of deuterons which actually partake in any potential fusion process(es). We now expand the analysis with ξ which is the generalized order/thermal disorder ratio. Θ is the term interrelating the build up of the isotopic fuel to the desired final pathways.

$$J_{fus} = \frac{[B * <D_i>]}{[L_c * [1 + (2\Lambda_{Pd,D})^{-1}]]} * \frac{[\xi]^2}{[1 - \exp[-\xi]]} * \Psi_{fus} * \Theta$$

Ψ_{fus} is the fractional amount of intrapalladial deuterons which actually contribute to the desired reactions. $\Lambda_{Pd,D}$ the loading flux ratio.

Summary

The quasi-one-dimensional (**Q1D**) model of isotopic fuel loading has been modified using three non-dimensional factors, $\Lambda_{Pd,D}$ the loading flux ratio, Ψ_{fus} the fractional amount of intrapalladial deuterons which actually contribute to the desired reactions, and ξ the generalized order/thermal disorder ratio.

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TABLE OF VARIABLES

B_D	diffusivity of deuterons [cm^2/sec]	q	electric charge
$[D^+]$	deuteron concentration	T	absolute temperature (Kelvin)
$<D_i>$	initial deuteron concentration [t=0]	V	voltage = - Φ the potential
E	electric field intensity	κ_f	first order deuteron fusion rate
F	the Faraday	κ_g	deuteron gas evolution rate
I	electrical current	κ_e	first order deuteron entry rate
J_e	flux of deuterons entering Pd cathode	$\Lambda_{Pd,D}$	flux loading/gas evolution ratio
J_g	flux of deuterons evolving to gas	μ_D	electrophoretic mobility
J_f	flux of deuterons in fusion reaction(s)	η_D	electrical transference ratio
k_B	Boltzmann constant	Ψ_{fus}	fraction of deuterons involved
L	length	ζ	electric order/thermal ratio

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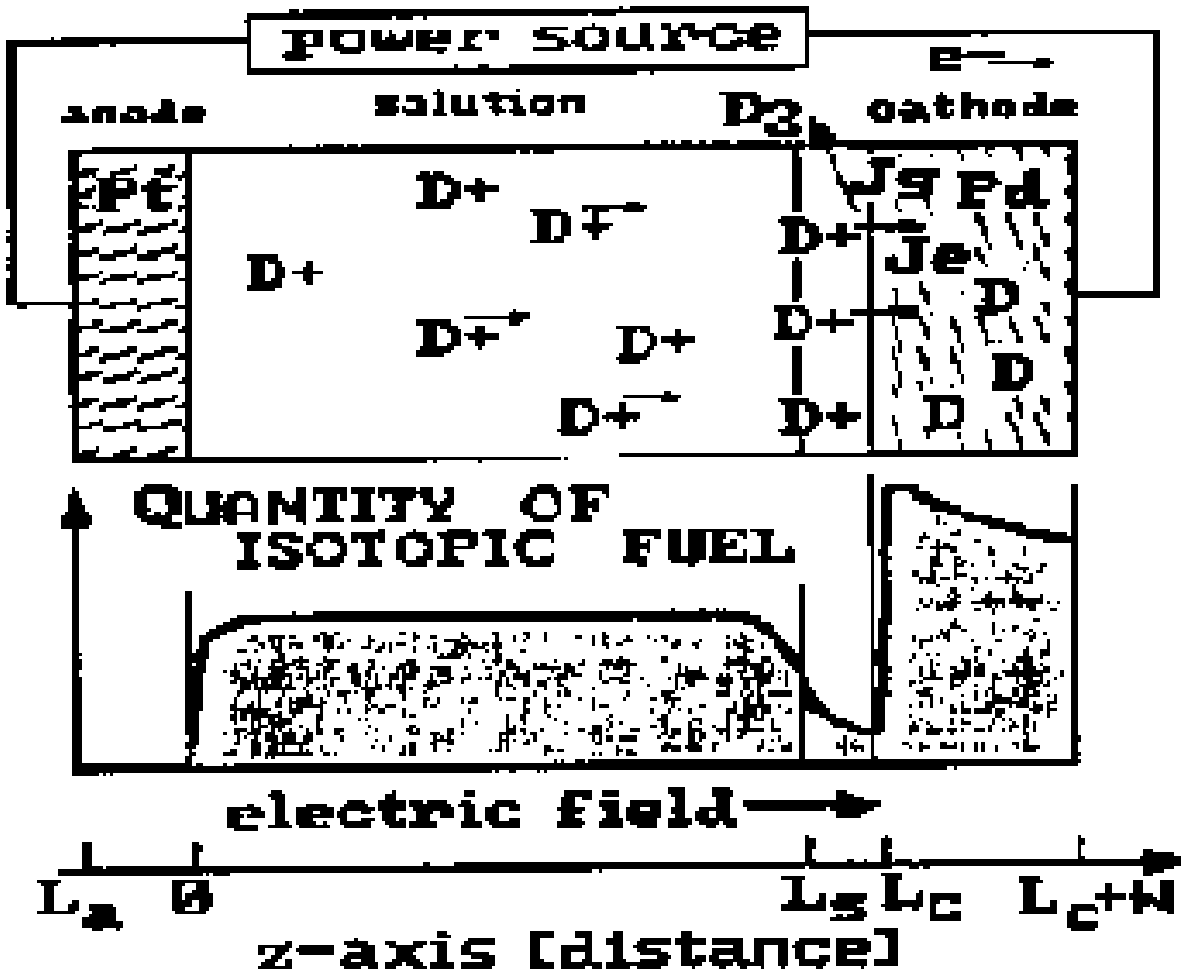

FIGURE 1 - THE Q1D MODEL OF ISOTOPIC FUEL LOADING

The applied electric field influences the spatial distribution of deuterons in aqueous solution. There are four compartments considered outside of the material (palladium electrode in this case) to be loaded with the isotopic fuel. The first is the anode. The induced drift in the second compartment (heavy water) by the initial applied electric field is schematically shown. There is a double layer region, the "width" of which is greatly exaggerated in the figure. The last compartment is the gas volume outside of the material.

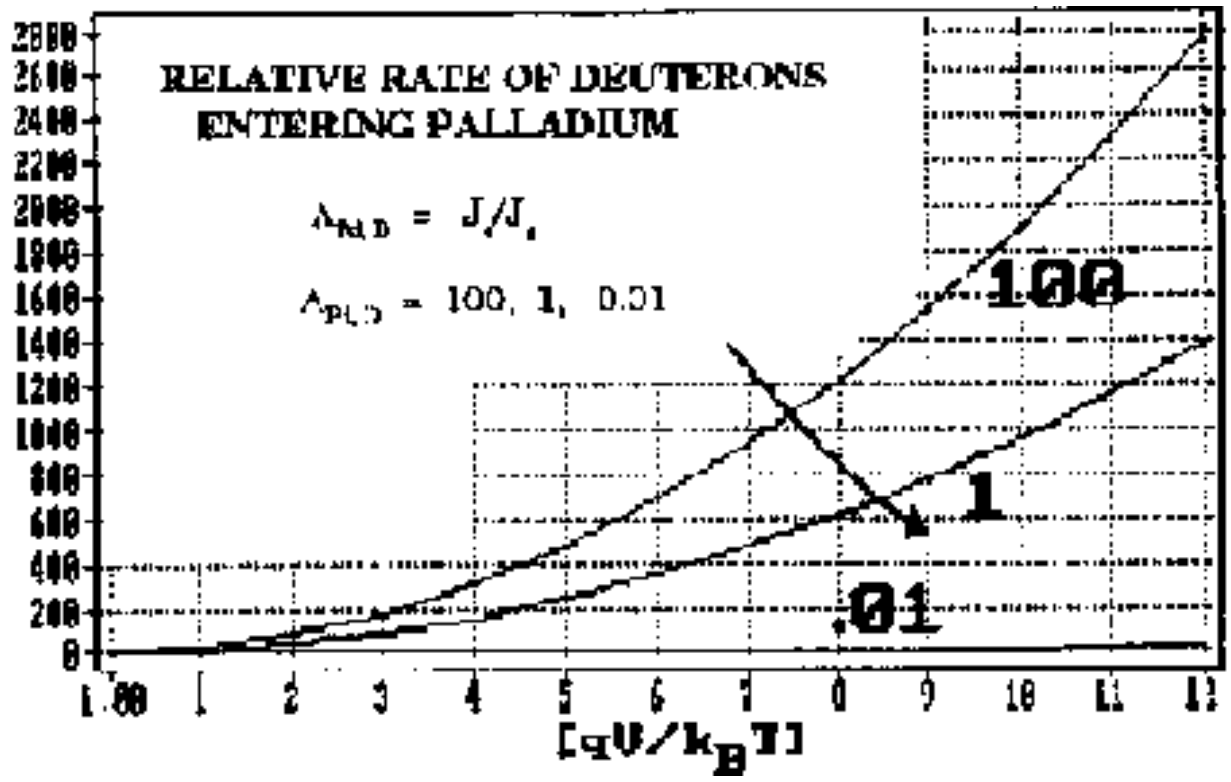

FIGURE 2 - LOADING RATE OF PALLADIUM
The relative values for the loading flux (J_e) is shown as a function of the electric field intensity, parametrically for various rates of gas [D_2] evolution rates at the cathode (characterized as J_g). In this example, J_{fus} is zero. The curve is shown as a function of ζ, the electric order/thermal disorder ratio.

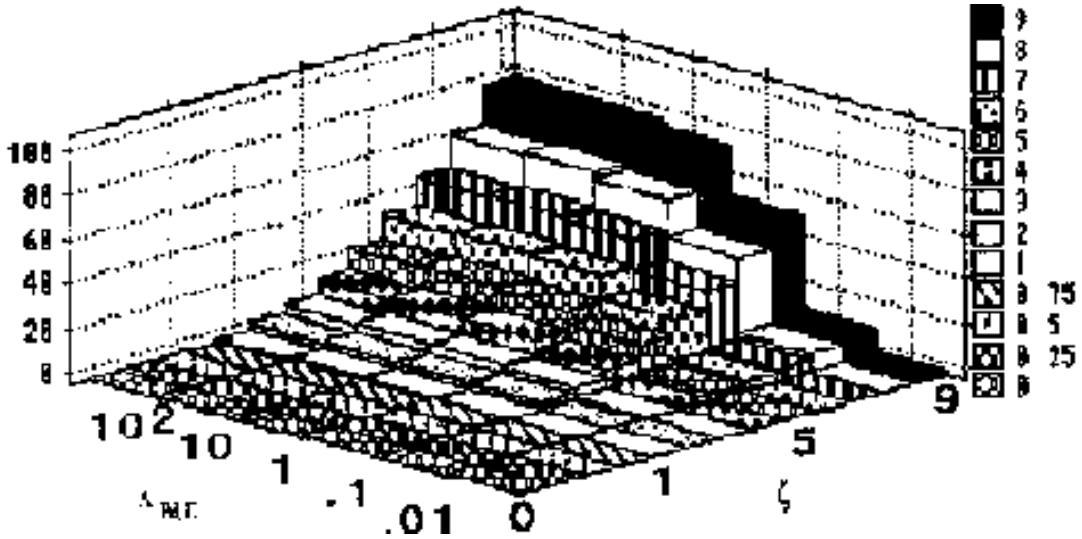

Figure 3 - Parametric Examination of Q1D Fusion Flux
This 3-D parametric graph represents an examination of the fusion flux equation based upon ζ and $\Lambda_{Pd,D}$ (see eq. 7).