THE UPPER BOUND OF HOT-SPOT TEMPERATURES INDUCED BY SUPERSONIC FIELD

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ABSTRACT

The phenomenon of sonoluminescence has attracted an intense attention of researchers in a variety of fields of science and technology. Recently direct measurements of temperatures of a hot spot, which was created in a liquid exposed to a supersonic field, were carried out and very large values, for example $T \approx 6eV$, were obtained.

It seems thereby to be an urgent, interesting scientific problem to determine the upper bound of temperatures and densities realizable in the hot spot, particularly in connection with the possibility of so called cold fusion.

In this paper we calculate the maximum of hot-spot temperature allowable for a particular set of parameters, by use of the bubble dynamics which has been developed for sixty years.

1. Introduction

Let us start with a brief description of the phenomenon of sonoluminescence. A supersonic field applied to a liquid is known to yield vapor and/or gas-filled cavities in the liquid, which then starts to oscillate almost in phase with the applied supersonic pressure field. In the contraction phase the content of the cavity is greatly, adiabatically compressed so that the temperature is highly raised and atoms or molecules in it are excited to eventually irradiate photons.

Recently Flint and Suslick [1] succeeded in directly measuring the temperature of hot spots in a silicon oil by analyzing the luminescence spectra from C_2 molecules and obtained the value

$$T = 5{,}075 \pm 156K.$$

After a year Hiller, Putterman and Barber [2] raised the upper bound to

$$T \approx 6eV$$

by fitting the spectra of the luminescence from an air bubble in water to those of black body radiation.

The aim of this work is to calculate the maximum value of hot-spot temperatures which is allowable for a suitably chosen set of parameters.

2. Fundamental equation

The sonoluminescence has a long history of research of about sixty years and constitutes a well established branch of science and technology. We now have a lot of review works, by which one may easily get familiar with the theories and experiments on sonoluminescence. For example, see References [3] and [4].

For simplicity, we at first ignore the compressibility and bulk viscosity of a liquid and assume the spatial uniformity of a gas content, i.e., we are here considering a gas-filled cavity. Furthermore we neglect the thermal irradiation and mass diffusion from or to the cavity. The spherical symmetry of the cavity is also assumed throughout the present paper.

Let a cavity with radius R_o be initially in equilibrium with a liquid of hydrostatic pressure P_o and temperature T_o and a supersonic field

$$P^e(t) = -P_A \sin \omega_A t \tag{1}$$

be applied to the liquid at t=0. By use of the conservation laws of mass and momentum for the liquid, we can get the dynamical equation of motion

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = -\frac{1}{\varrho R}\left(P(R,t) - P(\infty,t)\right) \tag{2}$$

for the radius R of the cavity, [3,4] where P(R,t) and P(∞,t) are pressures in the liquid at the wall and at a remote point from the cavity, respectively. The former should be in equilibrium with the pressure of the gas content, i.e.,

$$P(R,t) = \left(P_0 + \frac{2\sigma}{R_0}\right)\left(\frac{R_0}{R}\right)^{3\kappa} - \frac{2\sigma}{R} - \frac{4\mu\dot{R}}{R}\,, \tag{3}$$

where ϱ, σ and μ are, respectively, the density, surface tension and viscosity coefficient of the liquid. In the above the equation of state of an ideal gas is assumed for definiteness. The generalization to a van der Waals gas is straightforward. The κ is the polytropic index, which verifies between the specific heat ratio γ and unity; adiabatic and isothermal limits, respectively. On the other hand

$$P(\infty,t) = P_0 + P^e(t) \tag{4}$$

3. Maximum of hot-spot temperatures

In this section we try to solve the equation (2). It has been well known that there are two types of solutions, i.e., a stable solution and transient one, depending on values of parameters, P_A and w_A, the amplitude and frequency of the applied supersonic field. For the purpose of obtaining high hot-spot temperatures, transient cavities are preferable. In the present paper, however, we do not optimize parameters P_A and w_A for that purpose. Instead we try to get the solution for a particular point of the parameter space which lies in the transient region and then examine the response of the solution to the variation of the initial radius R_o.

Let the point of the parameter space be

$$P_A = 4\text{bar and } \omega_A = 15\text{kHz}. \tag{5}$$

The solution corresponding to (5) is examined in detail by Walton [4]. Namely in the first three quarters of a period of the supersonic field the cavity abnormally expands mainly due to the inertia of the surrounding liquid, in spite of the fact that P(∞,t) which decreased from zero to $P_o - P_A$ turns to increase up to around $P_o + P_A$. Let the maximum value of the radius be R_{max}. In the last quarter of a period the cavity collapses very rapidly towards the minimum radius R_{min}, where the gas achieves the maximum temperature T_{max}.

The expansion phase is describable by numerically solving (2). Fig. 1 depicts R_{max} against R_o. There is no appreciable difference between an ideal gas and a van der Waals one. In calculating Fig.l the isothermal process (κ = 1) is assumed since the expansion takes place slowly.

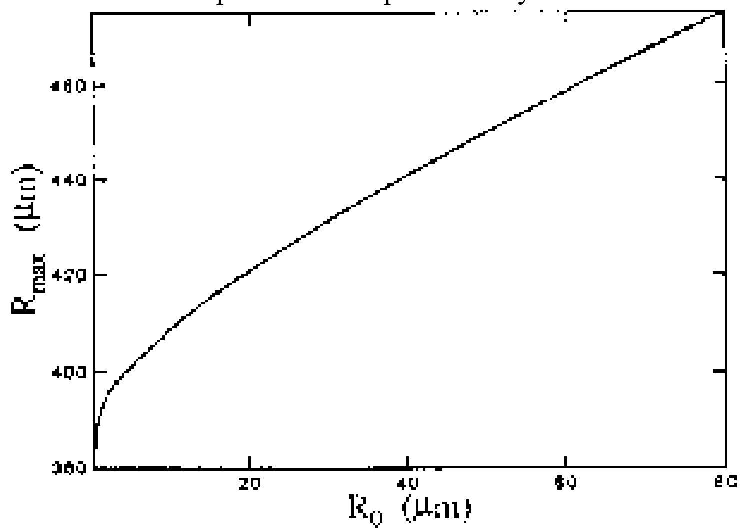

Fig. 1. The maximum radius R_{max} vs. the initial radius R_0. There is no appreciable difference between an ideal gas and a van der Waals one for R_{max}. The values of parameters are shown in (5).

Next turn to the contraction phase. It turns out to be a very tough job to numerically calculate the collapse process because it occurs in a very short time.

So let us try to analytically solve (2) by use of the fact that the collapse takes place around

$$P(\infty,t) \approx P_m \equiv P_0 + P_A,$$

as Walton [4] did. Multiplying $R^2\dot{R}$ and integrating over t from the time when R reaches the maximum value R_{max}, (2) may reduce to

$$\dot{R}^2 = \frac{2}{3\rho}\left[\frac{Q}{1-\gamma}\left(\left(\frac{R_m}{R}\right)^{3\gamma} - \left(\frac{R_m}{R}\right)^3\right) - \frac{3\sigma}{R}\left(1-\left(\frac{R_m}{R}\right)^2\right) - P_m\left(1-\left(\frac{R_m}{R}\right)^3\right)\right] ,$$

where the adiabatic process (kappa = gamma) is assumed since the contraction occurs very rapidly and Q is the pressure when $R=R_{max}$.

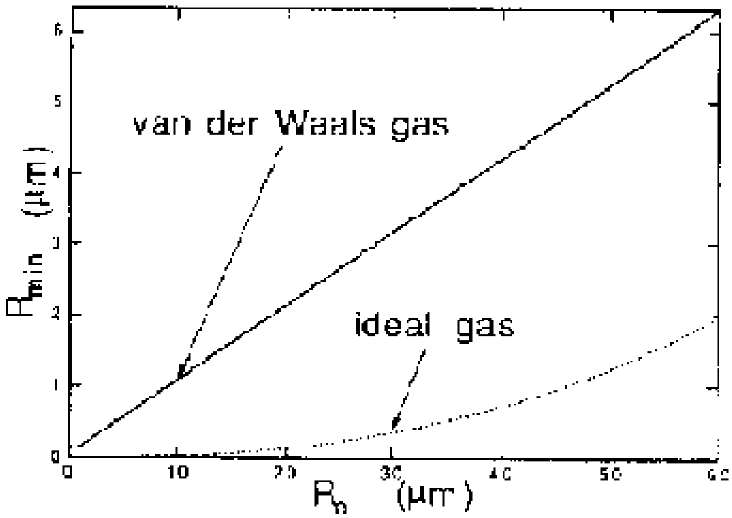

Fig. 2. The minimum radius R_{min} vs. R_0. The broken line and solid one correspond to an ideal gas and a van der Waals one (D_2), respectively.

The minimum value R_{min} is obtainable by equating $\dot{R}$ to zero in (6) and it may follow that

$$Z \equiv \left(\frac{R_{\mathrm{max}}}{R_{\mathrm{min}}} \right)^3 \approx \left(\frac{(\gamma - 1) P_m}{Q} \right)^{\frac{1}{\gamma - 1}} \tag{6}$$

The maximum temperature and pressure are achieved when $R=R_{MIN}$ and may result in

$$T_{\mathrm{max}} = T_0 Z^{\gamma - 1} \approx \frac{(\gamma - 1) P_m}{Q}$$

$$P_{\mathrm{max}} = Q Z^{\gamma} \approx Q \left(\frac{(\gamma - 1) P_m}{Q} \right)^{\frac{\gamma}{\gamma - 1}} \tag{7}$$

Figs. 2 and 3 depict R_{min} and T_{max} against R_0, where we have used values of R_{max} calculated in Fig. l. The dashed and solid lines in Fig. 2 correspond to an ideal gas and a van der Waals one (D_2), respectively. No appreciable difference is recognized between both kinds of gas in Fig. 3.

As seen from Fig. 3, it is remarkable that for cavities which have radii of a few microns the maximum temperatures reached are high enough for nuclear fusion to take place.

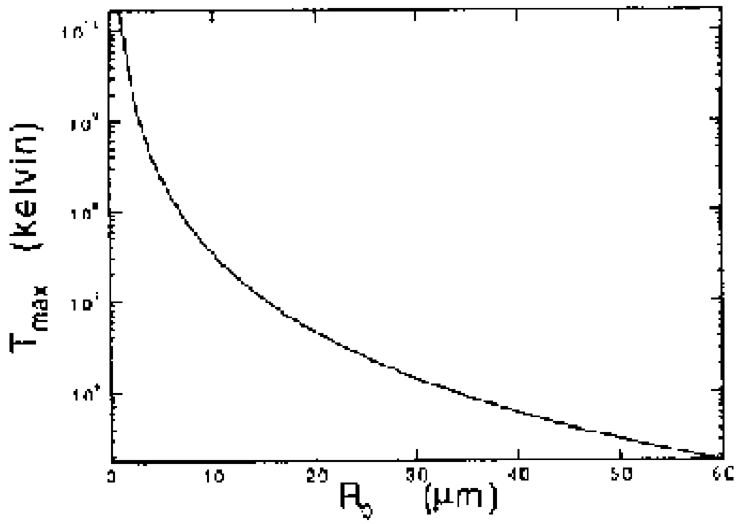

Fig. 3. The maximum temperature T_{max} vs. R_0. Both types of gas yield no appreciable difference for T_{max}.

DISCUSSIONS

As mentioned in Section 2, we have made a lot of simplifications to calculate the maximum temperature. Among them the assumption of the incompressibility seems crucial, since the speed of wall of the cavity exceeds the sound velocity of water in the collapse process. This fact means the invalidity of the incompressibility assumption.

Furthermore we neglected the diffusion of mass, which is usually ignored in the application of the bubble dynamics because the speed of wall is very large as compared to the diffusion velocity and then the effect is negligible in a few periods of the oscillation. The linear response theory however, would not be valid for such extreme condition that is realizable for cavities with initial radii of order of micron. We have to take account of the thermal radiation too.

Lastly the phase transition from a gas phase to a plasma phase and the instability of a spherical cavity should be taken into account.

After all, to obtain quantitatively acceptable results, we have to examine the above-mentioned in detail and then seek for the optimum condition for the realization of the upper bound of hot-spot temperatures. The present work, therefore, constitutes a preliminary step of the whole task and detailed calculations will appear in another place.

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