LINT: A SEMI-CLASSICAL QUANTIZED THEORY OF LATTICE INDUCED NUCLEAR TRANSMUTATIONS

Robert W. Bass
Scientific Advisory Board, ENECO, Inc.
391-B Chipeta Way, Salt Lake City, UT 84108

ABSTRACT

In a frozen supersaturated or fully deuterated ("beta phase") $Pd{\cdot}D_{[1.0]}$ crystal lattice, an incoming deuteron of energy 5.1 eV [above the elsewhere-estimated zero-point energy level of 6.257 eV] will collide with a zero-point energy level E_0 = 0.052 eV [+ 6.257 = 6.31 eV] bound deuteron, losing 18 of its 31 quanta of linear momentum as measured by Duane's Rule (for Inelastic Collisions between a particle and a lattice), and knocking the bound deuteron, approximated as a quantized harmonic oscillator, into its 40th bound energy level above ground state E_1 = 0.156 eV [+ 6.257 eV = 6.41 Ev], i.e. E_{40} = 4.2 eV [+ 6.3 = 10.5 Ev] which, in the present admittedly over-simplified theory, ALSO coincides with a Resonant Transmission energy level, as in the Turner-Bush theory of transmission resonance, in which the de Broglie wavelength of an excited particle is in resonance with the lattice period. Elsewhere the WKB Approximation is used more realistically with a Madelung-Coulomb lattice potential to compute 88 Resonant Transmission energy levels for excited bound deuterons of energies between 6.32 eV and 13.74 eV, but no mechanism for exciting zero-point (or room-temperature!) deuterons to these resonant energy levels has been suggested. The present semi-classical scattering calculation exhibits an explicit quantal mechanism for exciting the bound deuterons and introduces two new integral quantum numbers (J,K) in terms of which all possible such resonant *scattering* scenarios may be enumerated.

INTRODUCTION

The theoretical analysis of high-energy $D + D$ collisional interactions in a near vacuum (e.g. a low-density fusion plasma made from deuterium gas) predicts two principal reactions, namely [with $D = {}_1^2H, T = {}_1^3H$]

$$D + D \rightarrow {}^3He + n + 3.27 \text{ MeV}, \tag{1}$$

$$D + D \rightarrow T + H + 4.03 \text{ MeV}. \tag{2}$$

Theory and experiment (cf. Miley [1]) predict and confirm that these two reactions occur with approximately equal probabilities, the ratio of which is the *branching ratio*. A third possibility, usually <u>ignored</u> because its branching ratio, in comparison to either of the preceding reactions, is about 10^{7} times smaller, is

$$D + D \rightarrow {}^4He + \gamma + 23.85 \text{ MeV}, \tag{3}$$

where the excess energy appears in the form of a gamma-ray photon γ.

The usual explanation for the rarity of (3) is that when one high-energy (say about 10 keV) D-nucleus approaches another high-energy D-nucleus the three-dimensional orientations and linear and angular momenta of the colliding nuclei are quite *asymmetrical*; the nucleon farthest from the system's center of mass escapes the strong nuclear force and is torn from its original partner, while the remaining nearest three particles coalesce.

But (recalling *Bloch's Theorem* that a solution of Schrödinger's equation inside of a periodic lattice is not valid unless its logarithmic derivative is a *spatially periodic function* of the same period) it is pure unreflective dogmatic incompetence to *assume* [in the absence of experimental evidence] that at <u>low energies</u>, and <u>inside</u> of a *PERIODIC* <u>lattice</u>, the established high-energy, near-vacuum branching ratio remains unaltered. In the prophetic words of Nobel Laureate Julian Schwinger [2A], the question demands "study, not fiat."

In particular, if two bound *D*-nuclei forming a harmonic-oscillator pair inside a lattice approach each other when the energy level of the oscillator is excited, their momenta and orientations could be relatively symmetrical, leading by tunneling to their transitory conjunction as an <u>excited</u> α-particle (or $^4He^*$ nucleus). Could such an excited $^4He^*$ somehow shed its excess energy and drop into its unexcited ground state ^4He?

According to Schwinger [2B], quantum electrodynamic (QED) selection rules now "<u>forbid</u>" the emission of a photon -- apparently (Park [3A]) because two S-state, spin 1 boson *D*-nuclei can only produce a $^4He^*$ of spin 0 or 1 or 2, but there are no photons of spin 0 or spin 2, and because parity must change in an allowed transition from the spin 1 state ("Laporte's Rule", [3B, p. 364]) there can be no photon emitted in that case either.

Park [3A] doubts that phonons can be emitted, for he expects that purely radial pulsations of the $^4He^*$ would have insufficient amplitude to affect the lattice. However, no one has ever suggested a detailed micro-physical "picture" of how mass is transformed into energy according to $E = m \cdot c^2$, and it has been suggested by R.D. Washburn ([4]) that the excess energy is shed by the two *D*-nuclei in the form of decreasing kinetic energy of angular momentum as they spiral around one another in shrinking orbits while "radiating" millions of phonons and coalescing (to use a classical picture) via the strong force. [Such a 3-dimensional problem is beyond the scope of the present point-particle viewpoint paper.]

At any rate, Hagelstein [5] and Schwinger [2B] have <u>postulated</u> the possibility that the excess 23.85 MeV can somehow be released into the lattice as heat energy via phonon excitations. Schwinger [2B] has assumed that the *D* lattice inside a fully loaded $Pd{\cdot}D$ or doubly-loaded $Pd{\cdot}D_2$ lattice absorbs 2.4 x 10^8 phonons of energy 0.104 eV each.

Some high-energy physicists, such as S.E. Jones [18] have attempted to refute the possibility of such a reaction by the following argument. According to Heisenberg, $\Delta E{\cdot}\Delta t \cong h$; assuming that photons or phonons or whatever is emitted cannot travel superluminally, and that the distance between nuclei is $L \cong 1$Å, one finds that $\Delta t = L/c = 10^{-8}$cm/(10^{10}cm/sec) = 10^{-18}sec. Thus, from Heisenberg's relationship in the form $\Delta E \cong h/\Delta t$ = (10^{-15}eV-sec)/10^{-18}sec = 10^3 eV = 1 keV, which is vastly less than 23.85 MeV. The incompetence of this argument is exposed by noting that a photon [or phonon] of energy $2h\nu$ can split into two photons [or two phonons] of energy each $h\nu$, and so there is no good reason for simply ASSUMING that the emitted energy has to have the characteristics of a single particle! (Also, in *non-relativistic* quantum mechanics the time-duration of transition of energy levels is treated as if it were either *instantaneous* or (Park [3B]) *non-analyzable*, and so light-cone arguments are irrelevant). Therefore Schwinger's postulate of 2.4 x 10^8 phonons being released into the *D* lattice merits further serious consideration.

However, the mean potential energy of a Pd-oscillator in the Pd lattice is only 22.8% of that of a *D*-oscillator in the *D*-lattice (because of the Virial Theorem for quantized harmonic oscillators, and because the phonon energy scale in a *Pd*-lattice is 0.0237 eV).

Accordingly, if the postulated 23.85 MeV release puts the *Pd·D* lattice into its lowest available energy state, the release would be absorbed by the *Pd* lattice rather than the *D* lattice. Consequently the present theory purports to improve and refine Schwinger's NEAL theory by postulating that instead of 2.4×10^8 phonons going into the *D*-lattice, about 1×10^9 phonons of size 0.0237 eV go into the Pd-lattice.

ANALYSIS

In the following theory, we shall borrow from (non-relativistic) quantum mechanics the proposition that *energy* and *linear momentum* are commuting observables, and therefore both can have *simultaneously sharp* observed values. Accordingly we shall assume that it is legitimate to change back and forth between the *kinetic energy E* of a particle of [constant] mass *M* and its *linear momentum p* by the transformations

$$E = p^2/2M, \;\; p = (2M \cdot E)^{1/2} . \tag{4}$$

Also, instead of utilizing quantum mechanics, we shall proceed as did Bohr, applying what Landé [6] calls *quantal postulates* in an *ad hoc* manner to a classical treatment. Note that Landé says that his three *quantal postulates*

$$\text{(PLANCK)} \quad \Delta E = n\cdot (h/T) \equiv n\cdot h\nu, \;\; (\nu = 1/T) , \tag{5a}$$

$$\text{(DUANE)} \quad \Delta p = n\cdot (h/L) \equiv n\cdot h\kappa, \;\; (\kappa = 1/L) , \tag{5b}$$

$$\text{(deBROGLIE, BOHR, SOMMERFELD-WILSON)} \quad \Delta H = n\cdot (h/[2\pi]) \equiv n\cdot\hbar , \tag{5c}$$

(which result from PERIODICITY in *time T*, in *space L*, and in *angular orientation* 2π) are in some sense more fundamental than quantum mechanics, because by simply assuming (5a,b,c) together with some innocuously unobjectionable postulates about *symmetry* and *invariance* in probability theory, Landé derives Schrödinger's equation!

In the following one-dimensional model of a collision between an incoming free D-nucleus and a bound D-nucleus which is half of a bound-oscillator pair, we shall replace (5a,b) *in the present context* by the more appropriate postulates (which can be derived quantum-mechanically from, respectively, the theory of Quantized Harmonic Oscillators and the theory of Transmission Resonance of a free particle in a lattice defined by a periodic potential):

$$E_n = (2n + 1)\cdot E_0, \;\; E_0 = (1/2)(h/T) \equiv (1/2)h\nu , \tag{6a}$$

$$p_m = (2m + 1)\cdot p_0, \;\; p_0 = (1/2)(h/L) \equiv (1/2)h\kappa , \tag{6b}$$

where the factor (½) accounts for the *zero-point* quantum level. The use of Duane's Rule (5b) in the form (6b) is equivalent to the most basic part of the Bush Transmission Resonance Model (TRM) [7], wherein he developed the concept suggested by Turner [8]; for more details concerning Duane's Rule in the context of quantum mechanics and relativity, see Bush [9]-[10]. The intent of the present paper could be described as an attempt to "combine the best features of the Bush TRM theory and the Schwinger NEAL theory."

In the case of a quantized bound-oscillator, we shall denote the rms oscillation amplitude Λ and rms linear momentum |p| by the symbols

$$\Lambda \equiv \langle x^2 \rangle^{1/2},\ |p| \equiv \langle p^2 \rangle^{1/2}\ , \tag{7}$$

where <·> denotes the quantum-mechanical expectation operator.

According to Duane's Rule, a free particle in a lattice of period L can only exchange linear momentum with the lattice in *integral* units of (h/L). Thus we have for a free particle of mass M_f and able to exchange linear momentum with a lattice:

$$p_{\mathrm{f,m}} = (2m + 1)\cdot p_{\mathrm{f,0}},\quad p_{\mathrm{f,0}} = \mathrm{h}/(2L)\ , \tag{8}$$

$$E_{\mathrm{f,m}} = (2m + 1)^2\cdot E_{\mathrm{f,0}},\ \ E_{\mathrm{l,0}} = [h/(2L)]^2/(2M_{\mathrm{f}})\ , \tag{9}$$

where (8) is equivalent to Duane's Rule (6b), and (9) follows by (4).

The theory of a quantized harmonic oscillator is well known and will not be repeated here; suffice it to say that Schwinger [2B] is being followed in supposing that the zero-point rms amplitude Λ_0 of a bound particle of mass M_b is known from x-ray measurements, and therefore the energy levels of what in the present context should be called a *Schwinger oscillator* are given by:

$$\mathrm{E}_{\mathrm{b,n}} = (2\mathrm{n} + 1)\cdot E_{\mathrm{b,0}},\quad E_{\mathrm{b,0}} = \{h/(2[2\pi\Lambda_0])\}^2/M_{\mathrm{b}}\ , \tag{10}$$

$$|p_{\mathrm{b,n}}| = (2n + 1)^{1/2}\cdot|p_{\mathrm{b,0}}|\ ,\quad |p_{\mathrm{b,0}}| \equiv (M_{\mathrm{b}}\cdot E_{\mathrm{b,0}})^{1/2} = h/(2[2\pi\Lambda_0])\ , \tag{11}$$

where (11) follows from (10) by (4), and where in (10) the mass M_b is used instead of $2M_b$ (as for a free particle having kinetic energy only) because of the Virial Theorem's result that the absolute values of the mean kinetic and mean potential energies of a harmonic oscillator are equal (and therefore the mean total energy is twice either one).

Note the analogy, between L in the case of the free particle and $2\pi\Lambda_0$ in the case of the bound particle, disclosed by comparison of (8) and (11). It may be a mere coincidence, but in the two numerical cases where we shall use measured values of L and Λ_0, they exhibit a commensurability to almost four decimal places. Accordingly this question merits further investigation. But we shall assume that it is simply a remarkable coincidence pertaining to the $Pd{\cdot}D$ and $Pd{\cdot}D_2$ lattices that there can always be found an integer n_σ such that a resonance in spatial periods (hence in wave numbers) exists, namely:

$$L = (n_\sigma + [1/2])\cdot (2\pi\Lambda_0) \equiv (2n_\sigma + 1)\cdot (\pi\Lambda_0)\ . \tag{12}$$

In the First Example below, $n_\sigma = 4$; but in the Second Example below, $n_\sigma = 1$.

However, (12) alone is insufficient to allow us to derive Diophantine equations for the quantum-transitions involved in the postulated collision between a free D-nucleus and a bound D-nucleus.

QRT PRINCIPLE

For this purpose we need the *Quantum Resonance Triggering* (QRT) *principle,* which assumes that *certain* energy levels of incoming free particles have ground-state *momenta* which are in resonance with the ground-state rms-momentum of a bound particle, in the sense that there exist *incommensurable* positive integers *(r,s)* such that

$$r\cdot |p_{b,0}| = s\cdot p_{f,0}\ . \tag{13}$$

Now suppose that a free *D*-nucleus of momentum-level *m* collides with a bound D-nucleus which is part of a Schwinger oscillator of energy-level *j*, and that after their interaction the free D-nucleus remains free but has *lost* (*m-k*) quanta of linear momentum, while the oscillator has *gained* (*n-j*) quanta of energy. In an obvious notation,

$$D_{f,m} + D_{b,j} \rightarrow D_{f,k} + D_{b,n}\ . \tag{14}$$

Since conservation of both energy and linear momentum are obeyed in quantum mechanics, we may then conclude that (14) implies that

$$\text{(CONSERVATION OF MOMENTUM)}\quad p_{f,m} + |p_{b,j}| = p_{f,k} + |p_{b,n}|\ , \tag{15a}$$

$$\text{(CONSERVATION OF ENERGY)}\quad E_{f,m} + E_{b,j} = E_{f,k} + E_{b,n}\ . \tag{15b}$$

In order to demonstrate the quantum-level transitions determined by (15a,b), we rearrange them as

$$p_{f,m} - p_{f,k} = |p_{b,n}| - |p_{b,j}|\ , \tag{16a}$$

$$E_{f,m} - E_{f,k} = E_{b,n} - E_{b,j}\ . \tag{16b}$$

Therefore, using (8), (9), (10), (11), we have from (16a,b):

$$p_{f,m} - p_{f,k} \equiv 2(m-k)\cdot p_{f,0} =$$

$$= |p_{b,n}| - |p_{b,j}| \equiv \{(2n+1)^{1/2} - (2j+1)^{1/2}\}\cdot |p_{b,0}|\ , \tag{17a}$$

$$E_{f,m} - E_{f,k} \equiv \{(2m+1)^2 - (2k+1)^2\}\cdot E_{f,0} \equiv 4(m-k)\,(m+k)\cdot E_{f,0} =$$

$$= E_{b,n} - E_{b,j} \equiv 2(n-j)\cdot E_{b,0}\ . \tag{17b}$$

Thus conservation of momentum and energy are equivalent to

$$(2n+1)^{1/2} - (2j+1)^{1/2} = 2(m-k)\cdot\{p_{f,0}/\ |p_{b,0}|\ , \tag{18a}$$

$$n - j = 2(m-k)\,(m+k+1)\cdot\{E_{f,0}/E_{b,0}\}\ . \tag{18b}$$

Recall that, from the **QRT** hypothesis of *momentum resonance* (13), together with the *oscillation-amplitude resonance* "coincidence" [?] (12),

$$r/s = p_{f,0}/\,|p_{b,0}| \equiv (2\pi\Lambda_0)/L = 2/(2n_\sigma + 1), \tag{19}$$

whence, incidentally, there must exist a positive integer n_{QRT} such that

$$r = 2n_{QRT}\ , \tag{20a}$$

$$s = (2n_\sigma + 1)\cdot\, n_{QRT}\ . \tag{20b}$$

Similarly

$$E_{f,0}/E_{b,0} = [M_b/(2M_f)]\cdot\, (r/s)^2\ . \tag{21}$$

Henceforth we shall concentrate on the case (14), in which

$$M_f = M_b\ . \tag{22}$$

PROBLEM STATEMENT

Then the conditions of conservation of momentum and conservation of energy become, after inserting (19) and (21)-(22) into (18a,b),

$$(2n + 1)^{1/2} - (2j + 1)^{1/2} = 2(m - k)\cdot\, (r/s)\ , \tag{23a}$$

$$(n - j) = 2(m - k)\ (m + k + 1)\cdot\, (r/s)^2\ . \tag{23b}$$

These are *Diophantine equations*, in that the only solutions sought are to be in *integers*. Clearly this is impossible unless both $(2n + 1)$ and $(2j + 1)$ are perfect squares, which will emerge in the following. For the moment it is essential to note that even if the left hand side of (23a) were the difference of two integers, the equation has no solutions unless $(m - k)$ is divisible by s. Accordingly we assume that there exists an integer l such that $(m - k) = s\cdot l$. Then the conditions of *conservation of momentum, momentum resonance, and conservation of energy* become

$$(2n + 1)^{1/2} = (2j + 1)^{1/2} + 2lr\ , \tag{24a}$$

$$m = k + s\cdot l\ , \tag{24b}$$

$$n - j = 2sl(2k + 1 + sl)\cdot\, (r/s)^2 \equiv [(2k + 1)/s]\cdot\, lr^2 + l^2r^2\ . \tag{24c}$$

These three Diophantine equations can be somewhat simplified by squaring both sides of (24a), simplifying, and dividing by 2 to obtain

$$n - j = 2(2j + 1)^{1/2}\cdot\, lr + 2l^2r^2\ . \tag{25}$$

Now equate the right-hand sides of (25) and (24c), simplify, and divide by lr^2 to obtain

$$(2/r)\cdot (2j + 1)^{1/2} = [(2k + 1)/s] + l\,, \tag{26}$$

which, of course, is equivalent to (24a). Thus, finally, we may conclude that the conditions of *conservation of momentum, momentum resonance, and conservation of energy* are equivalent to the three Diophantine equations

$$l = [(2k + 1)/s] - (2/r)\cdot (2j + 1)^{1/2}\,, \tag{27a}$$

$$m = k + l\cdot s\,, \tag{27b}$$

$$n = j + [(2k + 1)/s]\cdot lr^2 + l^2r^2\,. \tag{27c}$$

The lengthy algebra required in deriving a complete solution of (27a,b,c) will be omitted in favor of specifying the results, which can be verified by substitution to be the *completely general solution* of (27a,b,c), in that they convert these equations into identities.

PROBLEM SOLUTION

Let (J,K) be an arbitrary pair of non-negative integers. Then define

$$n_{\mathrm{QRT}} = 2J + 1 \tag{28a}$$

$$n_J = n_\sigma + (2n_\sigma + 1)\cdot J\,, \tag{28b}$$

$$r = 2(2J + 1) \geq 2\,, \tag{28c}$$

$$s = 2n_J + 1 \equiv (2n_\sigma + 1)\,(2J + 1)\,, \tag{28d}$$

$$j = 2J(J + 1)\,, \tag{28e}$$

$$k = n_J + (2n_J + 1)\,(K + 1)\,, \tag{28f}$$

$$l = 2(K + 1) \geq 2\,, \tag{28g}$$

$$m = n_J + 3(2n_J + 1)\,(K + 1)\,, \tag{28h}$$

$$n = 2J(J + 1) + 8(K + 1)\,(4K + 5)\,(2J + 1)^2 \geq 40\,. \tag{28i}$$

Using the definitions (28a-i), and some algebra, it is easy to verify that, also,

$$2j + 1 = (2J + 1)^2\,, \tag{29a}$$

$$2n_J + 1 = (2n_\sigma + 1)\,(2J + 1)\,, \tag{29b}$$

$$2k + 1 = (2n_\sigma + 1)\,(2J + 1)\,(2K + 3) \geq 3\,, \tag{29c}$$

$$2m + 1 = (2n_\sigma + 1)\,(2J + 1)\,(6K + 7) \geq 7\,, \tag{29d}$$

$$2n + 1 = [(2J + 1)\,(8K + 9)]^2 \geq 81\,, \tag{29e}$$

Now, upon inserting the definitions (29a-i) and identities (29a-e) into the three Diophantine equations (27a,b,c) one finds (after some algebra which is left as an exercise for the reader) *identities* in (J,K). Hint: Use the identity

$$(8K + 9)^2 \equiv 1 + 16(4K + 5)\cdot (K + 1)\,. \tag{30}$$

The *new quantum number J* specifies the level at which the **QRT** scenario begins. The *new quantum number K* specifies the number of quanta which are gained and lost in the transition (14). In fact,

$$p_{f,m} - p_{f,k} = 2n_p \cdot p_{f,0} \,, \tag{31a}$$

$$n_p \equiv m - k = 2(2n_\sigma + 1) \cdot (2J + 1) \cdot (K + 1) \geq 2(2n_\sigma + 1) \geq 2 \,, \tag{31b}$$

which displays the fact that in the collision the free D-nucleus *loses* $n_p \geq 2$ quanta of linear momentum. Likewise, making use of (30), it is not difficult to verify that

$$E_{b,n} - E_{b,j} \equiv 2n_E \cdot E_{b,0} \,, \tag{32a}$$

$$n_E \equiv \mathrm{n} - \mathrm{j} = 8(K + 1)\,(4K + 5)\,(2J + 1)^2 \geq 40 \,, \tag{32b}$$

which displays the fact that in the collision the bound D-nucleus harmonic oscillator *gains* $n_E \pm 40$ quanta of energy.

In the theory of Schwinger oscillators (Bass [13]) it is proved that

$$E_{b,n} = M_b(\omega\Lambda_n)^2,\ \Lambda_n = (2n + 1)^{1/2}\Lambda_0, \tag{33a}$$

$$\omega = h/\{2M_b\,[2\pi(\Lambda_0)^2]\} \tag{33b}$$

wherein the frequency $v \equiv \omega/(2\pi)$ is completely determined by the empirically measured rms oscillation amplitude Λ_0. In his theory of Nuclear Energy in an Atomic Lattice (NEAL), Schwinger [2B] computes that the tunneling between two bound D-nuclei in a harmonic oscillator is such that the fusion rate for reaction (3) is given roughly in the case $n = 0$ by

$$1/T \cong C_n \cdot \exp(-(\tfrac{1}{2})\sigma^2{}_n),\ \ C_0 = 1, \tag{34a}$$

$$\sigma_n \equiv L/\Lambda_n = \sigma_0/(2n + 1)^{1/2},\ \ \sigma_0 \equiv L/\Lambda_0 \tag{34b}$$

where it is appropriate to call σ_0 the *Schwinger Ratio,* inasmuch as he has proved in his NEAL theory ("albeit crudely") that this ratio summarizes the results of ALL of the forces at work in the lattice. (The mistaken conjecture that $C_n \cong C_0$ for $n \geq 1$, which is not correct, is hereinafter called *naive extrapolation*; actually C_n is a function of both n and σ_0 *and* the GLOBAL nature of the potential, and for every bound state above zero-point state of the deuteron in a Madelung-Coulomb potential considered as an isolated potential well, there is a corresponding *Transmission Resonance Transparency* if the well is just one cell of an endless *periodic* linear chain of cells; therefore, in a more realistic model [12]-[15], one would take $C_n = +\infty$ for every $n > 1$.)

Moreover, in the case of the numerical value of the ratio at $n = 0$ for a doubly-loaded $Pd{\cdot}D_2$lattice, Schwinger has shown that if Λ_0 were only 25% bigger than it is in the zero-point state, then the fusion rate (34a) would be large enough to account for the Fleischmann-Pons (FP) Effect's measured excess enthalpy.

The present QRT theory purports to be a refinement and an improvement of Schwinger's NEAL theory, since it provides a *specific mechanism* for raising the energy level above the zero-point level considered by Schwinger (and so, in consideration of a WKB-based theory presented elsewhere, of *ASSURING* fusion). Moreover the result (32b) shows that even if $j = 0$, after the collision $n \geq 40$ and so $(2n + 1)^{1/2} \geq 9$, so that instead of being increased by a mere factor of 1.25, the oscillation amplitude Λ_0 is increased *AT LEAST* by a factor of nine! And in a more realistic theory, the deuteron actually finds the Coulomb "barrier" transparent when it is in any excited level above the zero point; this suggests that perhaps a better theory is the wave-mechanical theory of the Chubbs [19] in which the deuteron is regarded as a *delocalized* entity which fills the entire crystal and occupies all cells simultaneously.

Elsewhere I suggested (with a naive extrapolation of Schwinger's zero-point energy level fusion rate calculation which I now perceive as a mistake) that (34a) predicts the availability of 19.7 kW of excess enthalpy per cm^3 of $Pd{\cdot}D_2$ in an *aneutronic, radiationless* reaction (3) if even the first energy level $n = 1$ can be attained. Afterwards Bush & Eagleton [11] measured values of the order of 1 kW/cm^3 in an F-P type of cell using *thin-film Pd* coated on a gold cathode.

The obvious basic explanation for this discrepancy between my now-discarded naive theory and experiment is that the zero-point energy-level fusion rate computed by Schwinger in the case $n = 0$ needs to be redone using the higher-order harmonic-oscillator wave functions with *multiple nodes* to provide $C_n \neq 1$ for $n \geq 1$, in the case of an isolated or *local* potential well; and it needs to be done keeping in mind the known theory of Transmission Resonance (Bohm [20]) in the case of a *globally periodic* well.

An additional obvious contributor to this discrepancy between theory and experiment is simply that the approximation of the equilibrium *D*-nuclei in a *D*-lattice contained in a *Pd*-lattice as being in *quadratic* potential wells is only good for *SMALL* departures from equilibrium. When the *D*-nuclei in a Schwinger oscillator move a significant fraction of the distance between themselves, the electron screening (which had made the *quadratic* potential wells and the *harmonic* oscillators and the theory of the *Schwinger Ratio* a viable theory) is less effective in reducing the Coulomb repulsion between the *D*-nuclei, and a more refined theory of quantized *anharmonic* oscillators based on a Madelung-Coulomb potential (Bass [14]) must be used; however, the preceding theory appears to be qualitatively correct.

Moreover, even if a more accurate form of potential were used, the resultant oscillator could still be quantized, and a theory leading to Diophantine equations similar to those above could still be developed; cf. Bass [12]-[13]-[14]-[15].

Finally, even if the zero-point oscillation amplitude Λ_0 were not raised to the magnitude $\Lambda_n = (2n+1)^{1/2}{\cdot}\Lambda_0 \geq 9{\cdot}\Lambda_0$ indicated by the quadratic approximation to the potential (which is valid only near equilibrium), it would only have to be increased to a level $\Lambda_{0,NEW} \geq 1.25{\cdot}\Lambda_{0,OLD}$ in order to "explain" the FP Effect quantitatively.

However, the preceding parameter-sensitivity (which Schwinger has aptly characterized as "verging on chaos") is not the most striking result of the present QRT theory. What appears to merit that description is the fact that in the preceding theory the *minimum resonant energy* of an entering *D*-nucleus is independent of the nature of the particular spatial resonance (12) which is available, and depends *ONLY* upon h, Λ_0 and the deuteron mass M_b! To see this, note that

$$E_{res} = E_{f,m} = (2m + 1)^2{\cdot}E_{f,0} = 2n_{res}{\cdot}E_{b,0}\ , \tag{35a}$$

$$n_{\rm res} \equiv [(2m + 1)/(2n_\sigma + 1)]^2 = [(2J + 1)\cdot(6K + 7)]^2 \,. \tag{35b}$$

Thus

$$E_{\rm res} = 2[(2J + 1)\cdot(6K + 7)]^2\cdot E_{b,0} \geq 98\cdot E_{b.0} \,, \tag{36}$$

where, as mentioned, $E_{b,0}$ depends *only* upon upon h, Λ_0 and M_b. Whether it is a mere happy coincidence, or whether further research on Schwinger oscillators reveals it to be a universally valid relationship, the established values of Λ_0 for $Pd{\cdot}D$ and for $Pd{\cdot}D_2$ are almost identical; accordingly the *minimum* resonant triggering energy of $98\cdot E_{b,0}$ will be almost the same in each case!

This seemingly *universally valid minimum resonant triggering energy* (36) will now be estimated in detailed presentations of two specific examples of the QRT theory.

FIRST EXAMPLE

In this example we use values of L and Λ_0 from Chubb [16]. Consider a fully loaded $Pd{\cdot}D$ lattice, in which

$$E_{b,0} = 0.052 \text{ eV}, \;\; L = (3.89/2^{1/2})\otimes 1.03 \text{ Å}, \;\; \Lambda_0 = 0.1002 \text{ Å}, \;\; \sigma = 9\pi. \tag{37a}$$

Thus, a spatial resonance is available with

$$n_\sigma = 4. \tag{37b}$$

The lowest-order examples of the QRT principle are obtained by taking $J = 0$ and $n_j = 4$ and

$$r = 2,\;\; s = 9,\;\; j = 0,\;\; k = 13 + 9K,\;\; l = 2(K + 1),\;\; m = 31 + 27K, \tag{37c}$$

$$n = 8(K + 1)\cdot(4K + 5),\;\; (K = 0, 1, 2, 3, ...). \tag{37d}$$

The absolutely lowest order example is then at $K = 0$:

$$r = 2,\;\; s = 9,\;\; j = 0,\;\; k = 13,\;\; l = 2,\;\; m = 31,\;\; n = 40, \tag{37e}$$

$$E_{\rm res} = 98\cdot E_{b,0} = 3969\cdot E_{f.0} = 5.1 \text{ eV}. \tag{37f}$$

(To make this example more realistic, one must note that according to the more accurate quantum-mechanical theory (Bass [12]-[15]) the zero-point bound deuterons are at the bottom of Madelung-Coulomb energy wells at about 6.257 eV, and so this should be added to the above over-simplified point-lattice theory of $E_{b,0}$ = 0.052 to give an adjusted zero-point of 6.309 eV, and likewise the energy level $E_{\rm res}$ = 5.1 eV should be replaced by 11.4 eV.)

SECOND EXAMPLE

Here we follow Schwinger [2B] in his theory of Tritium production from a $D + D$ reaction in an (extremely overloaded) doubly-deuterated $Pd{\cdot}D_2$ lattice, in which, according to Sun and Tomanek [17],

$$E_{b,0} = 0.052 \text{ eV}, \; L = 0.94 \text{ Å}, \; \Lambda_0 = 0.0997 \text{ Å}, \; \sigma = 3\pi. \tag{38a}$$

Thus, a spatial resonance is available with

$$n_\sigma = 1. \tag{38b}$$

As in the preceding example, we have, for $J = 0$ and $n_j = 1$, the results

$$r = 2, \; s = 3, \; j = 0, \; k = 4 + 3K, \; l = 2(K + 1), \; m = 10 + 9K. \tag{38c}$$

Accordingly, the lowest-order example of the QRT principle, for $K = 0$, is now

$$r = 2, \; s = 3, \; j = 0, \; k = 4, \; l = 2, \; m = 10, \; n = 40, \tag{38d}$$

$$E_{res} = 98 \cdot E_{b,0} = 441 \cdot E_{f,0} = 5.1 \text{ eV}. \tag{38e}$$

(As in the previous example, for greater realism this should be adjusted upward by a few eV to account for the fact that the present oversimplified theory treats the bound deuterons as points in a rigid lattice rather than as particles at the bottom of periodic energy wells; however, this case has not yet been computed by the more accurate WKB approximation.)

CONCLUSION

The preceding reasoning seems at least as plausible as that by which Bohr arrived at his model of the hydrogen atom. Accordingly, unless someone who is skeptical of the reality of cold fusion can point out a *specific flaw* in the preceding arguments, it would seem to be mandatory to proceed to an experimental test of the *principal result* derived above, namely that for each deuteron of energy 11.4 eV entering a fully-loaded *Pd·D* lattice perpendicular to a plane of the *D* lattice there will be produced exactly one α-particle which was not previously present.

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