Selective Resonant Tunneling through Coulomb Barrier by Confined Particles in Lattice Well

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ABSTRACT

A confined particle in a lattice potential well may tunnel through Coulomb barrier in terms of resonance. It will select the resonance which is of a specific life-time. In contrast to beam-target situation using an accelerator, the selected life-time of this resonance is $\tau_{life} \approx \theta\sqrt{\tau_N \tau_L}$ instead of $\tau_{life} \approx \theta^2 \tau_N$. Here θ is a large number as the square root of the reciprocal of Gamow penetration factor [1], τ_N is the flight-time of this particle inside the nuclear potential well; and τ_L is the flight-time of this particle inside the crystal potential well. Introducing an imaginary part in the lattice potential well has been found very useful to describe the deuterium-loading effect which has not been used in previous literature of tunneling [2].

1. Introduction—Imaginary Part of Potential Well

The selective resonant tunneling model has been successful in calculating three major fusion cross-sections (d+t, d+d, and d+^{3}He) in terms of an imaginary part in a nuclear potential well [3,4]**.** In this paper we introduce another imaginary part in the crystal potential well, in order to describe the deuterium-loading effect in experiments. It is found that this imaginary part in the crystal potential well facilitates the description of the current conservation, and the steady-state requirement of resonant tunneling.

"Excess heat" is a major anomalous phenomenon in deuterium-loaded metals. Various calorimeters have been used to detect this excess heat; however, all the calorimeters have a time constant which is on the order of 100 seconds or more. This means that if the excess heat is a result of resonant tunneling, we have to have steady-state resonant tunneling in order to detect the excess heat in a calorimeter. In the case of a beam-target, this steady-state is maintained by an injected beam. Any absorption of the injected wave in the nuclear potential well is balanced by the probability current of the plane wave which is coming from infinity. Now in the case of the confined particle in a lattice potential well, we have to supply the particle in the lattice potential well in order to compensate for the absorption in the nuclear potential well. In experiments, this supplement is maintained by deuterium-loading in terms of electrolysis or gas loading procedures. The simplest way to describe this deuterium-loading effect is to introduce an imaginary part in the crystal potential well. That is

$$\int_{nulear\ well} U_{1i} \left|\phi(r)\right|^2 dr = -\int_{crystal\ well} U_{2i} \left|\phi(r)\right|^2 dr \quad (1)$$

Here, U_{1i} and U_{2i} are the imaginary part of the potential in the nuclear well and the crystal well, respectively; and ϕ(r) is the reduced radial wave function of the confined deuteron wave (Fig. 1). In order to solve the Schrodinger equation for ϕ(r) it is necessary to assume the potential well in the regions I, II, and III respectively (Fig. 1). We shall find that a pair of specific values for the imaginary part of the potential well, U_{1i} and U_{2i}, will make the tunneling current maximized.

2. Model—Eigenequation

A deep square-well is assumed for the nuclear potential well in region I; and a down shifted Coulomb potential is assumed in region II in order to show the repulsion between two positively charged nuclei and the electron screening effect in metal. Beyond region II, the potential is assumed to be constant in order to confine the charged particle and show the effect of reflections in the lattice potential well.

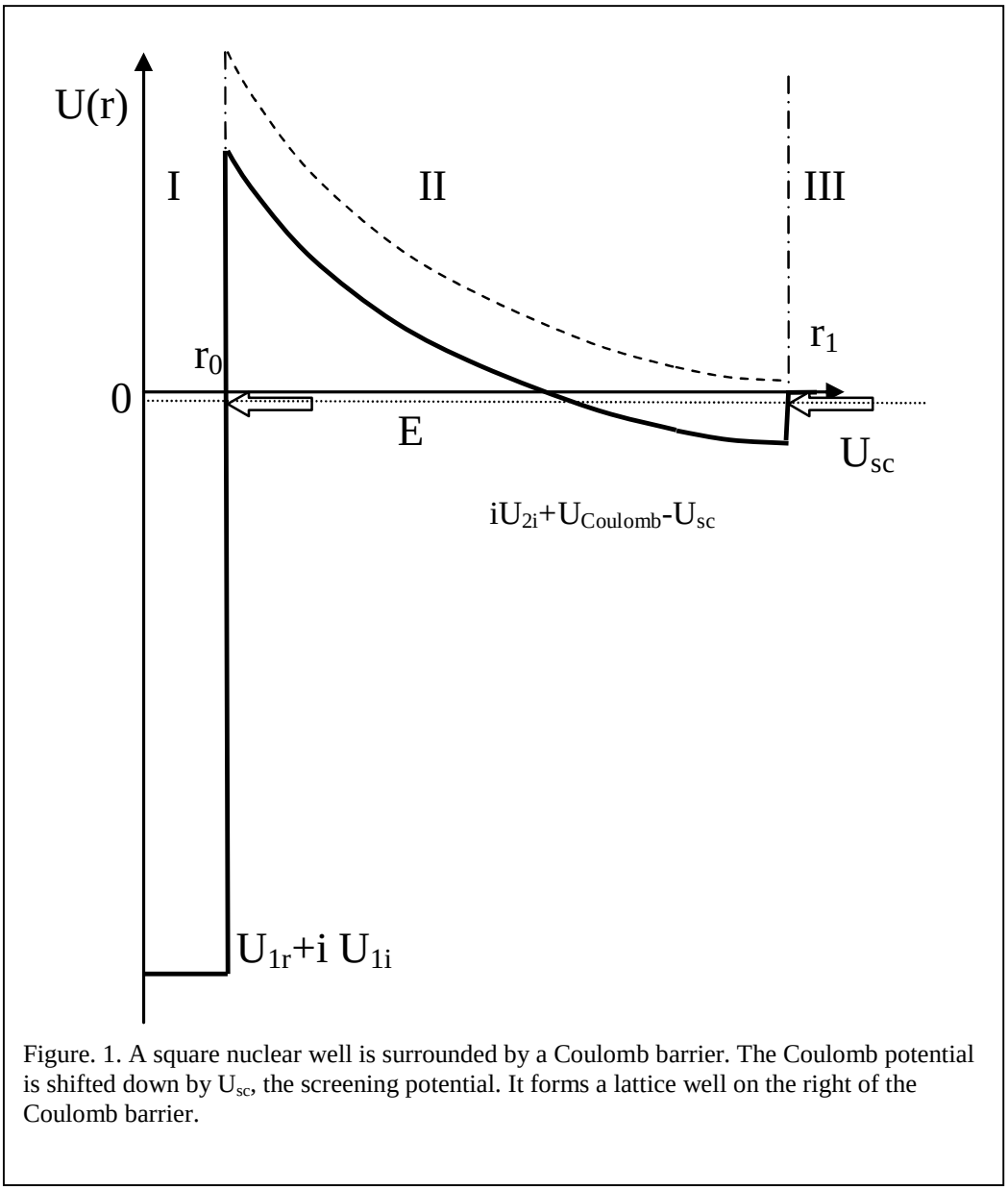

Figure. 1. A square nuclear well is surrounded by a Coulomb barrier. The Coulomb potential is shifted down by U_{sc}, the screening potential. It forms a lattice well on the right of the Coulomb barrier.

To satisfy the boundary conditions at r =0 and r =∞, the reduced radial wave function in region I, II, and III should be in the following form:

$$\phi_I(r) = ASin(k_1 r) \tag{2}$$

$$\phi_{II}(r) = B[F_0(r) + Tan(K)G_0(r)]/k \tag{3}$$

$$\phi_{III}(r) = DExp(-\beta r); \quad \mathrm{Re}(\beta) > 0 \tag{4}$$

Here, the wave number in each region is defined by the potential U_1, U_2, and U_3 as:

$$k_1 = \sqrt{\frac{2\mu}{\hbar^2}(E - U_1)} \;;\; k = \sqrt{\frac{2\mu}{\hbar^2}(E - U_2)} \;;\; \beta = \sqrt{\frac{2\mu}{\hbar^2}(U_3 - E)}\,, \tag{5}$$

$$U_1 = U_{1r} + iU_{1i};\; U_2 = U_{Coulomb} + iU_{2i} - U_{sc};\; U_3 = U_{3r} + iU_{3i}$$

For simplicity, we have assumed U_{3r}=0, and U_{3i}=U_{2i}. $F_0(r)$, and $G_0(r)$ are the regular and the irregular Coulomb wave function, respectively [5]. For the low incident energy, only the lowest partial wave (S-wave) is considered here. The potentials are assumed to be isotropic. K is the phase shift, and $Tan(K)$ is the coefficient of the linear combination for the wave function in region II. A, B, and D are the normalization constants in each region, respectively. μ is the reduced mass of the two interacting charged particles, $\hbar$ is the Planck constant divided by 2π. The continuity of the logarithmic derivative at r = r_0, and r = r_1, will give two equations for $Tan(K)$. They are:

$$k_1 \cot(k_1 r_o) = \left.\frac{F_o'(r) + Tan(K)\, G_o'(r)}{F_o(r) + Tan(K)\, G_o(r)}\right|_{r=r_o} \;; \tag{6}$$

$$-\beta = \left.\frac{F_o'(r) + Tan(K)\, G_o'(r)}{F_o(r) + Tan(K)\, G_o(r)}\right|_{r=r_1} \tag{7}$$

Here, $F_o'(r)$ denotes the derivative of $F_o(r)$ with respect to r. Equations (6) and (7) give two expression for $Tan(K)$:

$$Tan(K) = -\frac{F_o(r)}{G_o(r)}\left[\frac{F_o'(r)/F_o(r) - k_1\cot(k_1 r)}{G_o'(r)/G_o(r) - k_1\cot(k_1 r)}\right]_{r=r_o} \;;$$

$$Tan(K) = -\left[\frac{F_o'(r) + \beta F_o(r)}{G_o'(r) + \beta G_o(r)}\right]_{r=r_1}$$

$Tan(K)$ represents the coefficient of the linear combination in wave function $\phi_{II}(r)$. It gives the eigenequation:

$$\left[\frac{F_o'(r)/F_o(r) - k_1\cot(k_1 r)}{G_o'(r)/G_o(r) - k_1\cot(k_1 r)}\right]_{r=r_o} = \frac{G_o(r_o)}{F_o(r_o)}\left[\frac{F_o'(r) + \beta F_o(r)}{G_o'(r) + \beta G_o(r)}\right]_{r=r_1} \;; \tag{8}$$

It is noticed that r_0 is much smaller than a_c. $a_c \equiv \frac{\hbar^2}{\mu z_1 z_2 e^2}$ is the Coulomb unit of length for the two charged particles with electrical charge of z_1e and z_2e, i.e. the Coulomb potential is much greater than the kinetic energy of the relative motion between two particles. Hence, $F_o(r_0)$ is exponentially small, and $G_o(r_0)$ is exponentially large. We may write [6]

$$F_o(r) = C\rho\;\Phi(r);$$

$$\Phi(r) \equiv \frac{1}{\sqrt{y}} I_1(2\sqrt{y}) = 1 + \frac{y}{1!2!} + \frac{y^2}{2!3!} + \frac{y^3}{3!4!} + ..., \tag{9}$$

$$G_o(r) = \frac{\Theta(r)}{C};$$

$$\Theta(r) \equiv 2\sqrt{y}K_1(2\sqrt{y}) = 1 + y[\ln(y) + 2\gamma - 1]\Phi - \sum_{s=1}^{\infty} \frac{y^{s+1}}{s!(s+1)!} \sum_{l=1}^{s} \left(\frac{1}{t} + \frac{1}{t+1}\right); \tag{10}$$

Here, $\rho=kr$, $y=2r/a_c$, $\gamma=0.577\ldots$(Euler's constant). C^2 is an exponentially small number which is called as Gamow penetration factor:

$$C = \sqrt{\frac{2\pi\eta}{Exp(2\pi\eta) - 1}}; \tag{11}$$

$$\theta^2 = \frac{1}{C^2} = \frac{Exp(2\pi\eta) - 1}{2\pi\eta} \tag{12}$$

$$\eta = \frac{e^2}{\hbar v}; \tag{13}$$

Here, e is the absolute value of the electron charge; v is the speed of relative motion between two particles. Using Eqs.(9) and (10) , the eigenequation (8) leads to a relation between two time scales τ_{life1} and τ_{life2} (See Appendix (A.6)):

$$\tau_{life1}\tau_{life2} \approx \theta^2 \tau_N \tau_L \tag{14}$$

Here, τ_{life1} and τ_{life2} represent the life-time of deuteron in the nuclear potential well and in the lattice potential well, respectively. They are defined as:

$$\tau_{life1} \equiv \frac{\int_{nuclear} |\phi|^2 \, dr}{\int_{nuclear} \frac{|U_{1i}|}{\hbar} |\phi|^2 \, dr} \tag{15}$$

$$\tau_{life2} \equiv \frac{\int_{lattice} |\phi|^2 \, dr}{\int_{lattice} \frac{|U_{2i}|}{\hbar} |\phi|^2 \, dr}. \tag{16}$$

τ_{life1} and τ_{life2} measure the rate of nuclear reaction in the nuclear potential well, and the rate of deuteron-loading in the lattice potential region, respectively.

τ_{N} and τ_{L} represent the flight-time of deuteron bouncing back and forth in the nuclear potential well and in the lattice potential well, respectively. They are defined as

$$\tau_N = \frac{r_o}{k_{1r}\hbar/\mu}; \tag{17}$$

$$\tau_L = \frac{r_1}{k_r\hbar/\mu}. \tag{18}$$

$k_{1\text{r}}$ and k_{r} are the real part of the wave number k_1 and k, respectively.

3. Probability Current Conservation—Requirement of a Steady State

Tunneling current is the most important physical quantity in the case of resonant tunneling through Coulomb barrier by confined particles in lattice well. The normalized tunneling current is defined as

$$j \equiv \frac{\iiint_{nuclear} \left|\frac{U_{1i}}{\hbar}\right| |\phi(r)|^2 \, dr}{\iiint_{nuclear} |\phi(r)|^2 \, dr + \iiint_{lattice} |\phi(r)|^2 \, dr}. \tag{19}$$

Because of the probability current conservation for a steady state, it may be written as:

$$j \equiv \frac{1}{\tau_{life1} + \tau_{life2}}; \tag{20}$$

Use has been made of the equation:

$$\iiint_{nuclear} \left|\frac{U_{1i}}{\hbar}\right| |\phi(r)|^2 \, dr = \iiint_{lattice} \left|\frac{U_{2i}}{\hbar}\right| |\phi(r)|^2 \, dr \, . \tag{21}$$

Consequently,

$$j = \frac{\tau_{life1}}{\tau_{life1}^2 + \theta^2 \tau_N \tau_L}. \tag{22}$$

This is the most important result in this paper which gives the normalized tunneling current in the case of resonance. It involves the selectivity of life-time also.

4. Selectivity in Resonant Tunneling—Matching Loading Rate with Life-Time

This normalized tunneling current shows clearly the selectivity in resonance: it has a peak

$$j_{\max} = \frac{1}{2\theta\sqrt{\tau_N \tau_L}} \tag{23}$$

at

$$\tau_{life1} = \theta\sqrt{\tau_N \tau_L} \, . \tag{24}$$

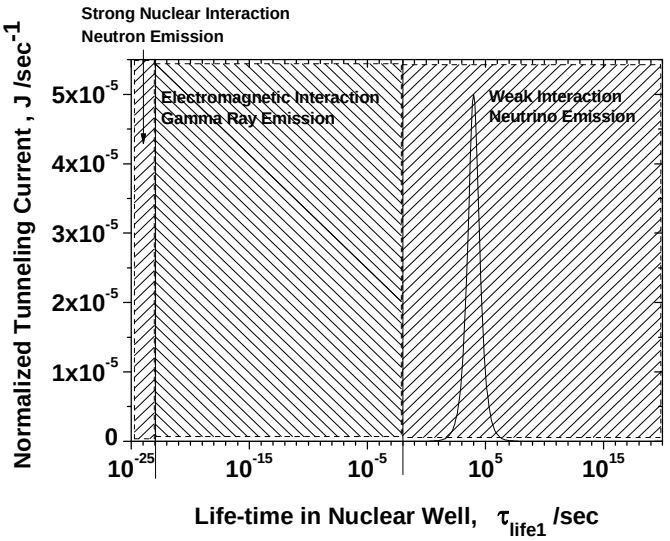

Figure 2. The normalized tunneling current has a very sharp peak at the specific life-time in nuclear well with $\tau_{life1} \approx \tau_{life2} \approx \theta\sqrt{\tau_N \tau_L}$

When $\tau_{life1} \ll \theta\sqrt{\tau_N \tau_L}$ or $\tau_{life1} \gg \theta\sqrt{\tau_N \tau_L}$, this normalized tunneling current j would approach to $j \approx \dfrac{1}{\theta^2 \tau_N \tau_L}$. It would be much smaller than the peak value $j_{\max}$.

The resonance would increase the tunneling current by a factor of θ at the specific life-time, $\tau_{life1} = \theta\sqrt{\tau_N \tau_L}$, with a narrow width. Figure 2 shows the peak-wise behavior for the case of $\tau_N = 10^{-23}\,\text{sec.}, \tau_L = 10^{-13}\,\text{sec.}$, and $\theta = 10^{23}$. [7]

It is noticed that this long life-time reaction rate is in the range of weak-interaction ($\tau_{life1} \sim 10^4$ sec.). The electromagnetic interaction and the strong nuclear interaction are too rapid to have any detectable tunneling current (Fig. 2).

5. Discussion

Based only on the general Principle of Quantum Mechanics, we have proved that:

(1) Resonant tunneling selects not only the right energy level, but also the reaction rate in the nuclear potential well. Only if the life-time in the nuclear potential well is on the order of

$$\tau_{life1} = \theta\sqrt{\tau_N \tau_L} \quad (25)$$

There will be a maximized tunneling current which will be greater than that of non-resonance case by a great factor of θ. θ is on the order of 10^{23} or more. It is different from the beam-target case, where the resonant tunneling would select the life-time in the nuclear potential well on the order of

$$\tau_{life1} \approx \theta^2 \tau_N \,. \quad (26)$$

(2) In the case of confined deuterons in the lattice potential well, the resonant tunneling selects the loading-time in the lattice potential well also in order to keep a steady state. This means that there is a matching loading rate which allows a resonant tunneling in a steady state. **This point is important in carrying out any experiment with detectable excess heat**.

(3) This selectivity in resonant tunneling implies a specific linear combination of wave functions. It is neither a pure molecular state in lattice potential well (i.e. not the pure $F_o(r)$), nor a pure excited nuclear state of $^4He^*$(i.e. not the pure $G_o(r)$). It is a mixture of both molecular state in lattice potential well and the pure excited nuclear state of $^4He^*$ (i.e. the linear combination: $F_0(r) + Tan(K)\, G_0(r)$). This coefficient, *Tan(K)*, cannot be zero; but it also cannot be too large. It should be on the order of ($1/\theta$) in order to have a maximized resonant tunneling current. This long-life-time state has been confirmed in experiment by 3-deuteron fusion reactions. [8,9,10,11]

(4) The electron screening effect is dependent on the temperature of the deuterium-loaded metal [12,13]; hence, the θ factor is a function of temperature as well. Hence, in the experiment we should control not only the loading rate but also the temperature in order to maintain the resonant tunneling state.

(5) In the early literature after 1989, the concept of fusion cross-section was used without any consideration of the difference between the beam-target case and the lattice confined case. In this paper we introduced an important physical quantity, tunneling current j, to replace the concept of the fusion cross-section. In terms of maximization of this tunneling current, we obtain the matching condition for resonant tunneling. This matching condition includes both the energy level and the reaction rate.

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Appendix A. Eigenequation

The eigenequation of this potential well in Fig. 1 will lead to both the constraints of the energy level, E, and the constraints of the life-time of the nuclear state.

Let us first write the eigenequation in the form of

$$\frac{F_o'(r_o)/F_o(r_o)-k_1\cot(k_1r_o)}{G_o'(r_o)/G_o(r_o)-k_1\cot(k_1r_o)}=\frac{G_o(r_o)}{F_o(r_o)}\left[\frac{F_o'(r_1)+\beta F_o(r_1)}{G_o'(r_1)+\beta G_o(r_1)}\right]; \qquad (A.1)$$

The advantage of this form is: (1) the left-hand-side is mainly a function of k_1 only, and the right-hand-side is a function of k and k_3 only; (2) it clearly shows the big factor,

$\frac{G_o(r_o)}{F_o(r_o)} \approx \theta^2 \square 1$, which determines the constraints on the solution of this eigenequation. In order to satisfy this eigenequation, we have to have:

$$\left|[G_o'(r_o)/G_o(r_o) - k_1 \cot(k_1 r_o)]\right|\left|[F_o'(r_1) + \beta F_o(r_1)]\right| \approx \left|\frac{1}{\theta^2}\right| \to 0. \tag{A.2}$$

When we introduce the imaginary part of potential well in nuclear region and in crystal region, it can be expand as

$$\begin{aligned}&\{\left|[G_o'(r_o)/G_o(r_o) - k_1 \cot(k_1 r_o)]_o + i\frac{\tau_N}{\tau_{life1}}\frac{d}{d(k_1 r_o)}[G_o'(r_o)/G_o(r_o) - k_1 \cot(k_1 r_o)]_o\right|\} \\ &\square\{\left|[F_o'(r_1) + \beta F_o(r_1)]_o + i\frac{\tau_L}{\tau_{life2}}\frac{d}{d(kr_1)}[F_o'(r_1) + \beta F_o(r_1)]_o\right|\} \approx \left|\frac{1}{\theta^2}\right| \to 0.\end{aligned} \tag{A.3}$$

The suffix "o" after the square-brackets means the real part of the content in the square-brackets.

Now we may find the condition of resonance: i.e.

$$\begin{cases}[G_o'(r_o)/G_o(r_o) - k_1 \cot(k_1 r_o)]_o = 0, \\ [F_o'(r_1) + \beta F_o(r_1)]_o = 0.\end{cases} \tag{A.4}$$

And the constraint on life-time:

$$\frac{\tau_N}{\tau_{life1}}\frac{\tau_L}{\tau_{life2}} \approx \left|\frac{1}{\theta^2}\right|. \tag{A.5}$$

or

$$\tau_{life1}\tau_{life2} \approx \left|\theta^2\right|\tau_N\tau_L. \tag{A.6}$$

If only one of the two factors in (A.4) equals zero; then, we have

$$\tau_{life1} \approx \left|\theta^2\right|\tau_N \tag{A.7}$$

or

$$\tau_{life2} \approx \left|\theta^2\right|\tau_L. \tag{A.8}$$

In both cases, we have

$$j \approx \frac{1}{\theta^2} \square \frac{1}{\theta}. \tag{A.9}$$

Consequently, only if the energy level satisfies the resonant condition, (A.4), and the loading rate satisfies the matching condition,

$$\tau_{life1} = \tau_{life2} = \theta\sqrt{\tau_N\tau_L}, \tag{A.10}$$

We shall have the maximized tunneling current

$$j_{\max} = \frac{1}{2\theta\sqrt{\tau_N \tau_L}} \approx \frac{1}{\theta}\,. \qquad \text{(A.11)}$$