Multiple Resonance Scattering

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Abstract

The previously proposed Multiple Resonance Scattering (MRS) theory is elaborated. In addition of predicting a radiationless fusion of two deuterium nuclei into a ^{4}He - nucleus in its ground state, the MRS theory is also shown to be in agreement with the experimental results concerning the transmutations of heavier nuclei. A form for the nuclear active environment is suggested and new experiments are proposed to verify the MRS theory.

1. Introduction

Condensed matter nuclear science (CMNS) has been mostly overlooked by the main physics community for years. The main reason for that has been the gab between the known experimental facts of CMNS and the presently existing theoretical knowledge. The aim of this article is to provide a bridge over this gab proposing an idea, how the results of CMNS can be consistent with the accepted theories of modern physics. The theory proposed here is an elaborated version of the idea proposed earlier in ref. [1].

We consider here a nano-size cluster of Pd atoms (or other host metal atoms). Furthermore, we assume that the deuterons absorbed (or adsorbed) to the Pd cluster are confined into the cluster but can otherwise move quasi-freely. Alternatively we can consider other kind of deuterium traps, voids in metal or acoustic cavities in heavy water.

Normally the deuterons inside the metal lattice are assumed to situate rigidly in the interstitial sites. However, the free particle approximation, which will be used here as a calculation tool, can be defended by two arguments. Firstly, the mobility of deuterons inside palladium and their thermal movement make the free particle approximation as a reasonable starting point. Secondly, consider the enhancement of the cross sections for the normal d + d fusion reactions found in the thick target experiments [2]. That enhancement can be understood by the screening caused by charged particles obeying classical Maxwell-Boltzmann distribution (leading to the Debye screening) as noted in ref. [2], where the electrons are assumed to have Maxwell-Boltzmann distribution although the known Fermi-Dirac distribution for the cold electron gas is noted. However, the enhancement can be understood by the screening caused by the background electron gas modulated by the moving quasi-free classical deuterons. The role of the mobility of deuterons in the enhancement of the fusion cross sections in thick target experiments have also been stressed in refs. [3,4]

2. Transition rate for the deuterium fusion

Consider a reaction of N+1 deuterons confined in the Pd cluster such that two of them fuse together and the other deuterons share the released energy by consecutive elastic interactions. We regard here the deuterons as free particles confined in the finite volume V_0. The final state consists confined (or escaping) deuterons and the fusion product(s). There will be N particles in the final state for ^{4}He -channel, and N+1 particles for tritium and ^{3}He -channels. The transition rate (per unit time and per cluster) is given by the Fermi golden rule

$$\lambda = \frac{2\pi}{\hbar} \left| \left\langle \Psi_f \left| \hat{H} \right| \Psi_i \right\rangle \right|^2 \rho \tag{1}$$

where ρ is the density of the states given by

$$\rho = \delta(E_f - E_i) \left(\frac{V_0}{(2\pi)^3} \right)^N \prod_{j=1}^{N} d^3 p_j \quad . \tag{2}$$

Note that the conservation of momentum does not require here the existence of a photon in the final state in the ^{4}He -channel, because the momentum conservation is fulfilled by the other deuterons in the final state. As the Hamiltonian that is responsible for the nuclear reaction, we use the Fermi pseudo-potential

$$\hat{H} = - \frac{4\pi\hbar^2}{m} a \delta(\mathbf{r}) \quad , \tag{3}$$

where a is the nuclear scattering length, which will be of the order of the nuclear diameter. The scattering lengths for all the d+d fusion channels can be assumed to be roughly equal. Note that the relative branching ratio (of order 10^{-7}) for the ^{4}He -channel in the vacuum reaction is mainly due to the electromagnetic coupling to the photon field, which is absent here.

For the elastic interaction between deuterons sharing the released energy we use a screened Coulomb potential such that, when the wave number of the transmitted momentum is larger than the Debye wave number, the potential is approximated by the pure Coulomb potential, and by a constant below that

$$\hat{H} = 4\pi\alpha\hbar c D(k) \quad , \tag{4}$$

where

$$\begin{aligned} D(k) &= \frac{1}{k^2} \quad , \quad k > k_D \\ &= \frac{1}{k_D{}^2} \quad , \quad k \le k_D \end{aligned} \quad . \tag{5}$$

The Debye wave number k_D is calculated in the first order approximation (RPA approximation) in ref. [1] and is for D/Pd $\cong 1$ of order $5 \cdot 10^{11}$ 1/m and twice of that provided that the value D/Pd $\cong 4$ for Pd nanoparticles is adopted.

The process, where the energy is collectively distributed among all the deuterons in the trap, contains N-1 elastic interaction between the deuterons (in the tree-diagram level). Hence, the rate will be of form

$$\begin{aligned} \lambda = & \left(\frac{4\pi\hbar a}{m}\right)^2 |\psi_{rel}(0)|^2 \left(\frac{1}{V_0}\right)^N \int \frac{d^{3N}p}{(2\pi)^{3N}} \left| \prod_{i=1}^{N-1} G(\omega_i, \mathbf{q}_i) \right|^2 \cdot \\ & (4\pi\alpha c)^{2(N-1)} \left| \prod_{l=1}^{N-1} D(\mathbf{k}_l) \right|^2 (2\pi)^4 \hbar \delta(E_i - E_f) \delta(\mathbf{p}_f) \end{aligned} \quad . \tag{6}$$

In Eq. (6) the functions $G(\omega_i, \mathbf{q}_i)$ are the propagators corresponding to the virtual lines (the spin indices have been deleted for simplicity).

$$G(\omega_i, \mathbf{q}_i) = \frac{1}{\omega_i - \frac{\hbar \mathbf{q}_i{}^2}{2m} - i\frac{\Gamma}{2\hbar}} \quad , \tag{7}$$

where ω_i and $\mathbf{q}_i$ are the frequency and the wave number vector of the virtual line (depending on the final state wave vectors $\mathbf{p}$ in such a way that the frequency and the momentum are conserved at all the verteces in the Feynman diagram). Γ is the line width corresponding the decay of the virtual deuteron to the lower energy states.

The dominant contribution to the transition matrix arises from the reaction, in which the energy released is distributed most effectively, so that the energies of the virtual particles split (roughly even) by consecutive interactions with the initial state deuterons. The corresponding diagram is shown in Fig. 1.

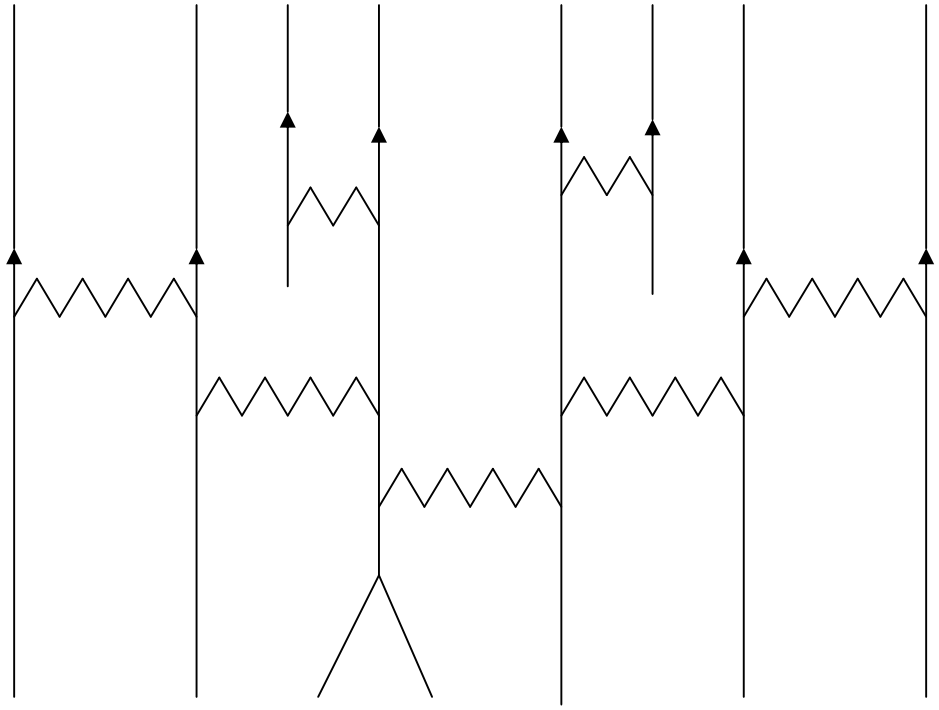

Figure 1. The diagram having largest contribution to the fusion rate.

Now, consider a virtual deuteron that interacts with an initial state deuteron so that both of them become free particles at the final state. The conservation of the energy and the momentum determine that the propagator of the virtual particle, Eq. (7), has to be

$$G(\omega_i,q_i) = \frac{1}{\frac{\hbar p^2}{2m}+\frac{\hbar q^2}{2m}-\frac{\hbar(\mathbf{p}+\mathbf{q})^2}{2m}-\frac{i}{2}\Gamma/\hbar} \quad , \tag{8}$$

where **p** and **q** are the wave numbervectors of the two final state deuterons. From Eq. (8) we note that there will be a resonance, if the final state momenta are perpendicular to each other. Furthermore, there will be multiple resonances, if the final state momenta are such that after every vertex the momenta of the two (virtual or real) particles are perpendicular to each other. For this multiple resonance scattering to occur, N-3 components of the final state wave number vectors are restricted to be small. Moreover, the overall conservation of momentum gives three extra constraints. For the diagram shown in Fig. 1, the momenta of the internal and final state particles are shown in Fig. 2

for this multiple resonance situation. Note that in Fig. 2 the two first momenta have to be opposite to each other due to the conservation of momentum.

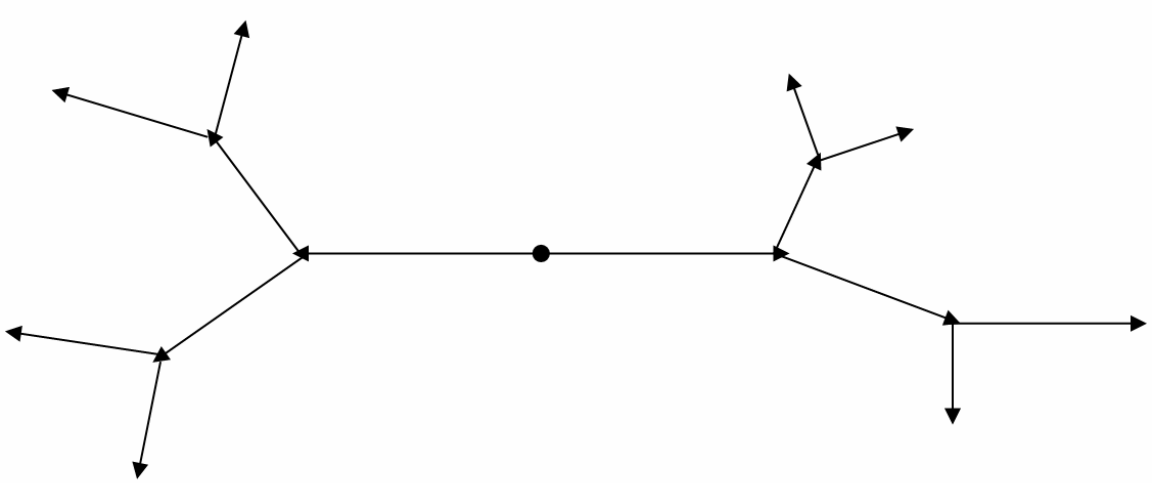

Figure 2. The momenta of the virtual and final state particles in the multiple resonance scattering corresponding to the diagram in Fig.1.

We neglect the N-3 small components of the wave number vectors everywhere else except in the propagators corresponding to the resonances. Then the integration over those N-3 wave number vectors can be performed yielding to

$$\lambda = \left(\frac{4\pi\hbar a_{nucl}}{m} \right)^2 |\psi_{rel}(0)|^2 (\alpha c)^{2(N-1)} \times \int \frac{d^{2N} p}{(2\pi)^{2N}} \frac{2\pi\hbar}{V_0^N} \delta\left(\sum_{i=1}^{2N} \frac{\hbar^2 p_i^2}{2 m_i} - Q \right) \left| \prod_{l=1}^{N-1} D(\mathbf{k}_l) \right|^2 \frac{\hbar^4}{Q^2 \Gamma^2} \prod_{j=1}^{N-3} \frac{m_j}{q_j \Gamma} , \quad (9)$$

where q_j are some combinations of the remaining 2N components of the final state wave number vectors. As a matter of fact, the line width Γ in Eqs. (7-9) is not a constant, but depends highly on the energy of the virtual line. However, for those internal lines, in which the energy is small, and which form the majority of the internal lines, the line width is small and essentially constant (arising from the decay to the bound deuteron pairs [1]). Consequently, considering the line width as a constant is not an essential error.

Noting that the final state wave vectors are of the order $p_i \approx \sqrt{2mQ/N}/\hbar$, the wave vectors q_j in Eq. (9) range from $\sqrt{2mQ/N}/\hbar$ to $\sqrt{mQ}/\hbar$. The product in Eq. (9) can then be approximated (assuming that the energy is of the virtual particles is split roughly even by consecutive interactions with the initial state deuterons):

$$\prod_{j=1}^{N-3} \frac{1}{q_j} \approx C \left(\frac{\hbar^2 N}{4mQ} \right)^{(N-3)/2} , \tag{10}$$

where C is some numerical constant of order unity. The product corresponding to the interaction propagators can be estimated respectively taking here into account Eq. (5), so that the number of unscreened elastic interactions is roughly Q/U_D while the number of screened interactions is $N - Q/U_D$, where we have define the Debye energy

$$U_D = \frac{\hbar^2 k_D^2}{2m} \quad . \tag{11}$$

In the delta function of Eq. (9) all the masses m_i are deuteron mass except the one(s) corresponding to the fusion product(s). The same is true also for the last product in Eq. (9). However, at the large N limit we can replace those masses by the deuteron mass without making any essential difference. The delta function in Eq. (9) can then be approximated as

$$\delta\left(\sum_{i=1}^{2N} \frac{\hbar^2 p_i^2}{2 m_i} - Q \right) = \delta\left(\frac{\hbar^2 p^2}{2 m} - Q \right) = \frac{m}{\hbar^2 p} \delta\left(p - \frac{\sqrt{2 m Q}}{\hbar} \right) . \tag{12}$$

where p is the length of the 2N dimensional wave number vector in Eq. (9). It should be noted that Eq. (12) is actually an approximation and not rigorous, because the wave numbers p_i in the sum inside the delta function are not one to one the integration wave numbers of Eq. (9), but combinations of them. Finally, using the Stirling's approximation

$$\Gamma(x) \approx x^{x-1/2} e^{-x} \sqrt{2\pi} \quad , \tag{13}$$

we can write the 2N dimensional integration volume element as

$$\prod_{j=1}^{2N} dp_j = d^{2N}p = \frac{2\pi^N}{\Gamma(N)} p^{2N-1}\, dp \approx \sqrt{\frac{2N}{\pi}} \left(\frac{\pi}{N}\right)^N e^N\, p^{2N-1}\, dp \quad . \tag{14}$$

(Note that there is a misprint in the corresponding equation in ref. [1].) The transition rate given by Eq. (9) becomes now

$$\lambda = A \cdot \frac{\hbar^3 a^2 |\psi(0)|^2}{m^2 Q N V_0} \left(B \cdot \frac{\alpha^2 mc^2}{\Gamma k_D{}^3 V_0} \sqrt{\frac{Q}{NU_D}} \right)^{N-1} 4^{-Q/U_D} , \tag{15}$$

where all the numerical factors (independent of N) have been embedded in the two constants *A* and *B*.

Eq. (15) greatly overestimates the transition rates for two reasons. Firstly, the deuterons are not free particles as assumed in the calculation, and consequently more sophisticated calculations are needed to obtain reliable quantitative results. However, the free particle approximation serves a reasonable starting point as discussed in the introduction. A more important thing is that the deuterons do not interact with all the other deuterons inside the trap, but only with their neighbours up to some distance, while the interaction with the deuterons far away is totally screened. It then means that the constant *A* in Eq. (15) will depend on the trap occupation *N* and the interaction distance *r*.

$$A = A(N, r) \quad , \tag{16}$$

in a way, which have to be studied in details in order to obtain quantitative predictions for the transition rates. Therefore, we can draw only qualitative conclusions from Eq. (15) at this stage.

The transition rate, Eq. (15), is extremely sensitive function of the trap occupation N and the energy released Q. This expression leads to measurable transition rates provided that N is large enough and that the factor

$$\beta = B \cdot \frac{\alpha^2 mc^2}{\Gamma k_D{}^3 V_0} \sqrt{\frac{Q}{NU_D}} \tag{17}$$

is greater than unity. This factor also explains, why the enhancement of the reaction rates by the multiple resonances has not been found in any other circumstances. What is

needed, is large number of heavy but (quasi-)free particles confined in a small volume. Electrons are too light, and lattice atoms cannot be considered as free particles in any reasonable approximation. Hence, the enhancement can appear only in the hydrogen loaded metals or other systems, where hydrogen atoms are confined in a small volume. Because the factor of Eq. (17) is proportional to the mass, the deuterium loading will give much higher transition rates compared to the hydrogen loading. It should, however, take into account that the difference between these two hydrogen isotopes is not as huge as could be assumed from Eq. (15). Protons, as lighter particles, can be approximated better as free particles than the heavier deuterons. Hence, their greater mobility compensates the mass difference up to some extent.

Furthermore, because of this mass dependence, the heaviest isotope, tritium would be the best catalyst for the low energy nuclear reactions. However, some mixture of tritium and deuterium should be used, because the deuterium is needed as fuel while tritons give the largest enhancement to the transition rate due to the larger phase space for the resonance. One should take into account that the metal lattices absorb hydrogen better than deuterium [6] and presumably similarly deuterium better than tritium. Hence, the tritium content in the electrolyte should be very large in order to essential improvement of the excess heat can be expected in the electrolysis experiment.

Moreover, the transition rate is a monotonic and rapidly increasing function of the released energy Q, provided the occupation N is large enough. This shows that practically only the process having the largest possible releasing energy could take place. In the case of a fusion of two deuterons considered here, this is the $^4\mathrm{He}$-channel, directly to the ground state.

As the dominant fusion process will be $\mathrm{d} + \mathrm{d} \rightarrow {}^4\mathrm{He}$ (to the ground state and without any final state photon) there will be no radiation except possible soft secondary radiation caused by the free deuterons in the final state.

Proton-proton fusion in the light water experiments will not give detectable transition rates for two reasons. Firstly, (and more importantly) the released energy is too small. Moreover, the transition rate is smaller, because the protons are lighter and consequently the phase space of the resonances is smaller.

If we consider the fusion rate as a function of the trap volume V_0, we note that there will be a “window”, where the fusion can take place. If the trap volume is too small, the trap cannot maintain such a high deuteron occupation that the multiple resonances could overcome the Gamow factor and the factor $A(N,r)$ of Eq.(16) arising from the limited interaction distance. On the other hand, for too large volume the factor of Eq.(17) becomes less than unity for reasonable values of the trap occupation N preventing the fusion to occur.

3. Transmutation processes

The MRS mechanism discussed here can also be applied to the transmutation reactions. The essential difference is that the overlapping of the deuteron wave functions in Eq. (15) has to be changed to the overlapping of the wave functions of the heavy nucleus and one or more interacting hydrogen nuclei. The transition rate will be much smaller than in the d + d fusion process due to the smaller Gamow factor corresponding to the overlapping of the wave functions. Again the transition rate depends very sensitively on the released energy. That explains the stabilising effect, because the higher released energies are obtained when the final state nucleus is as stable as possible.

Besides the released energy, also the electron structure of the heavy atom affects the transition rate through the screening of the positive charges, changing the overlapping of the wave functions. Similarly as in the case of the deuteron-deuteron fusion we can draw only qualitative conclusions on the transmutation processes at this stage.

Because the transition rate depends sensitively on the released energy, the outcome of transmutation processes will naturally show unnatural isotopic abundances.

As an example of the prediction of the mechanism proposed here, consider the gas permeation experiment by Iwamura et al [7], where they deposited barium on the Pd-CaO complex. The most abundant isotope of barium, $^{138}\mathrm{Ba}$, was found to transmute to the isotope $^{150}\mathrm{Sm}$ by absorbing six deuterons. If we consider all the stable isotopes of barium, the released energy are largest for similar processes for the isotopes $^{132}\mathrm{Ba}$ and $^{134}\mathrm{Ba}$. Because the structure of the core electrons are the same for all the isotopes of a given element and cannot then affect the relative transition rate, the released energy is the only factor that differs the isotopes. Hence, we can expect that the lighter isotopes will consume more rapidly compared to the heaviest isotope, $^{138}\mathrm{Ba}$. Considering for example the isotope $^{134}\mathrm{Ba}$, this does not mean that it will transmute by absorbing six deuterons:

$$6\ {}^{2}\mathrm{D} + {}^{134}\mathrm{Ba} \rightarrow {}^{146}\mathrm{Sm} + 71.6\,\mathrm{MeV}$$

It can have also some other more probable transmutation channel, for example to neodymium

$$4\ {}^{2}\mathrm{D} + {}^{134}\mathrm{Ba} \rightarrow {}^{142}\mathrm{Nd} + 50.1\,\mathrm{MeV}$$

or some fission channel. No matter, which one of these possible channels actually take place, it will have higher rate than the transmutation rate of the heaviest isotope $^{138}\mathrm{Ba}$.

It should be noted that the MRS mechanism does not necessarily require any particle to penetrate the nucleus for a fission process to take place. That possibility has to be taken

into account, when interpreting the results of the transmutation experiments. In that kind of process the Gamow factor arises from the tunnelling of a charged particle out of the nucleus. One particular example of this kind of process is the enhancement of the decay rate of radionuclides. Consider for example the nuclei having mass number A greater than about 150. By the energy consideration all of them are unstable against alpha decay. However, the decay constant of those isotopes in vacuum is too low by several orders of magnitude in order to their decay could be measured in the cases, where the decay energy is smaller than about 3 MeV. In hydrogen, deuterium or tritium loaded metals the decay can, however, be enhanced by the MRS mechanism. For example, consider the tungsten isotopes. Among all the tungsten isotopes the two lightest isotopes $^{180}\mathrm{W}$ (0.12%) and $^{182}\mathrm{W}$ (26.50%) have the largest decay energies:

$$^{180}\mathrm{W} \rightarrow {}^{176}\mathrm{Hf} + {}^{4}\mathrm{He} + 2.51\,\mathrm{MeV}$$
$$\downarrow$$
$$^{172}\mathrm{Yb} + {}^{4}\mathrm{He} + 2.26\,\mathrm{MeV}$$

$$^{182}\mathrm{W} \rightarrow {}^{178}\mathrm{Hf} + {}^{4}\mathrm{He} + 1.77\,\mathrm{MeV}$$
$$\downarrow$$
$$^{174}\mathrm{Yb} + {}^{4}\mathrm{He} + 2.08\,\mathrm{MeV}$$

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It could be possible that (at least part of) the hafnium and ytterbium signals found by Savvatimova [8] in the deuterium irradiation of tungsten are consequences of the enhanced alpha decay of these tungsten isotopes. If that is the case, it would represent the first measurement of the decay of these "stable" tungsten isotopes. However, further study would be needed to solve this issue.

4. Proposal for further experiments

The nuclear active environment (NAE), a term introduced by Edmund Storms, is by the prediction of the MRS mechanism a highly loaded correct size deuterium (or hydrogen or tritium) trap, for example a nano-cluster of the host metal atoms. Therefore, a detailed study of the size dependence of the Pd cluster to the excess heat and to the reproducibility would be very important.

As discussed in chapter 2, the highest transition rate will be obtained if the released energy is shared by tritons (and not by protons or deuterons). Therefore, a combination of deuterium and tritium will be the best choice in electrolysis experiment, because the deuterium is anyway needed as fuel. The optimum concentration of tritium should be investigated. Also a mixture of tritium and deuterium gas should be used in gas

permeation experiments for the same reason. One may expect that the optimum concentration of tritium would be less for the deuteron absorption processes involving four or six deuterons than for the deuteron-deuteron fusion.

Consider for a while the absorption processes – not involving deuterons – but tritons. For the heavy nuclei the binding energy per nucleon is typically 7.5-8.5 MeV. Consequently, the energy released per triton is about 14-17 MeV. That can be compared to the energy released per deuteron, which is about 13-15 MeV respectively. It is not clear, whether the larger energy (for the triton absorption processes) can compensate the smaller (mass dependent) Gamow factor. Although a definite prediction cannot be given here, possible transmutation involving several tritium nuclei should be explored.

As discussed in chapter 3, the rate of the similar transmutation processes of the different isotopes of the same element depend only on the energy released as far as the parity and spin conservation does not make a specific process less probable. Considering the transmutation of barium discussed there, we could propose here an experiment, where the abundances of the barium isotopes is measured as a function of time to see, whether the lightest isotopes consume more rapidly as predicted in chapter 3.

Furthermore, considering the enhancement of the decay rate of the radioactive nuclei, it would be interesting to study this by Iwamura type gas permeation experiment. Especially, one could deposit tungsten (which was discussed above) on the Pd complex to see whether the decay of the tungsten can be seen.

References

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