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Research Article

Electron Mass Enhancement and the Widom–Larsen Model

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Abstract

Widom and Larsen have put forth a model to describe excess heat and transmutation in LENR experiments. This model is the single most successful theoretical model that the field has seen since it started; it has served as the theoretical justification for a program at NASA; and it has accumulated an enormous number of supporters both within and outside of the condensed matter nuclear science community. The first step in the model involves the proposed accumulation of mass by electrons through Coulomb interactions with electrons and ions in highly-excited coupled plasmon and optical phonon modes. Historically for us this mass increase has been hard to understand, so we were motivated in this study to understand better how this comes about. To study it, we consider simple classical models which show the effect, from which we see that the mass increase can be associated with the electron kinetic energy. The basic results of the simple classical model carry over to the quantum problem in the case of simple wave packet solutions. Since there are no quantum fluctuations of the longitudinal field in the Coulomb gauge, the resulting problem is conventional, and we find no reason to expect MeV electron kinetic energy in a conventional consideration of electrons in metals. We consider the numerical example outlined in a primer on the Widom–Larsen model, and find that multiple GW/cm^2 would be required to support the level of vibrational excitation assumed in the surface layer; this very large power per unit area falls short by orders of magnitude the power level needed to make up the expected energy loss of the mass-enhanced electrons. We note that the mass enhancement of an electron in a transverse field is connected to acceleration, so that the electron radiates. A similar effect is expected in the longitudinal case, and a very large amount of easily detected X-ray radiation would be expected if an MeV-level mass enhancement were present even in a modest number of electrons.

Keywords: Electric field fluctuations, Increased electron mass, LENR theory, Weak interaction models, Widom–Larsen theory

1. Introduction

Excess heat in the Fleischmann–Pons experiment [1,2] has proven to be particularly vexing to theorists. The original experiment involved electrolysis in heavy water with a Pd cathode and a Pt anode, run in a configuration known to load deuterium into the Pd cathode. A very large amount of excess heat is seen in successful experiments, but there is no sign of commensurate chemical products, a situation which led to Fleischmann's conjecture that the energy produced was of nuclear origin. The absence of commensurate energetic nuclear products places the effect outside of conventional

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nuclear physics, and in addition prevents us from studying the reaction kinematics directly as we might with any normal nuclear reaction.

During the subsequent 24 years and more, much attention has been focused on figuring out how deuterons might tunnel, and then fuse, in a way that has something to do with the experiments. In spite of the many papers devoted to this approach, at present it has not won over a clear majority of the field. There are issues associated with how the Coulomb barrier might be overcome; why one should not expect to see the normal deuteron–deuteron fusion reaction products if the deuterons managed to get together; and finally the question of what happened to the 24 MeV gamma if helium is formed. Of course, we know now that the problem is even more perplexing; rather than being restricted to gammas, the issue is that as far as we know in the PdD experiments there are no commensurate energetic products of any kind.

The problem associated with overcoming the Coulomb barrier in the case of incoherent reaction schemes is sufficiently difficult that there have been many efforts to look to reaction schemes that do not involve fusion at all. Largely these have simply been ignored within the field (to be fair, nearly all theories have pretty much been ignored by the experimentalists, and the theorists have tended to focus only on their own approaches).

The situation changed considerably with the publication of a model by Widom and Larsen in 2006 [3]. While the theorists in the field have come from diverse backgrounds, this was the first major new theory in more than a decade from a mainstream active particle physicist (Widom), and published initially in a mainstream journal. It was not a fusion-based model, so there was no problem with the Coulomb barrier. In this model, electrons acquire sufficient mass so that electron capture produces the weak interaction mediated reaction [4]

$$W_{\text{electric}} + \text{e+p} \ \rightarrow \ \text{n} + \nu_{\text{e}}.$$

Here W_{electric} is energy contributed from the longitudinal electric field interactions with excitations in the local environment. Normally it is the neutron that undergoes a beta decay via

$$\text{n} \ \rightarrow \ \text{p+e} + \overline{\nu}_{\text{e}} + 0.782 \text{ MeV}.$$

To get the reaction to go in reverse sufficient energy (more than 0.782 MeV) must be supplied; the backward version of the reaction is important in astrophysics, for example in the conversion of electrons and protons to neutrons in a very strong magnetic field [5]. Once a neutron is created, then Widom and Larsen propose that it makes use of long wavelength states (that diffract) in a crystal, and are transferred to other nuclei resulting in energy production and transmutation effects.

Since it first appeared, this model has been remarkably successful. The existence of the model has been communicated to a very large number of people that follow the field but are not in it; a surprisingly large number of people within the scientific community in general are aware of the model, and many think highly of it; and by now there have been many presentations. Mike Melich reported receiving a widely favorable response in an unscientific polling of high-energy physicists at a Lake Louise Winter Institute meeting. This new model has received considerable attention at the New Energy Times web site, which some have suggested advocates for the theory. A success that cannot be similarly claimed by any other model in the field, is that Widom–Larsen has been deemed sufficient to justify an experimental LENR effort at NASA.

When we first encountered the model, our attention was drawn to the very large mass enhancement predicted for electrons, sufficiently large to be able to drive the neutron beta decay reaction backward. Since there are no obvious conventional mechanisms at play in the experiments done so far capable of doing this, we were under the impression that the model sought to take advantage of some new exotic mechanism related to quantum fluctuations of the electromagnetic field. For example, an argument can reasonably be made that quantum fluctuations in the transverse field produces a mass shift in the electron (as one term among many that appear in perturbation theory), and effect which has been verified both theoretically and experimentally [6]. After some thought, it became clear that if we work in the Coulomb gauge (and most applied physics is done in the Coulomb gauge) there are no quantum fluctuations in the longitudinal field [7]. The largest mass shift obtainable in a terrestrial laboratory is much too small to be relevant for the application under discussion.

This argument was criticized by Widom et al. [8], who suggested that we had omitted Coulomb interactions, and by doing so had thrown out the effects of interest in the Widom–Larsen model. >From our perspective, the issue was never whether Coulomb interactions should be included or not included (and we had included a brief discussion of the longitudinal field in our work), but that there were no quantum fluctuations associated with the Coulomb fields in the Coulomb gauge (while there are quantum fluctuations in the case of transverse fields). This may seem to be a minor technical issue, but it relates to how we might think about the effect, what models we might use, and whether we can use our intuition about the associated physics and applications. If there are no quantum fluctuations of the longitudinal fields in the Coulomb gauge, then the effects under discussion are purely conventional (and there would be no reason to expect mass increases at the MeV level in LENR experiments). In a sense the arguments of Ref. [8], and also of the subsequent primer [4], are helpful in clarifying the issue since the discussion and analysis is conventional.

Some years have passed now, and we have been motivated to return to the problem once again. The Widom–Larsen model has now more supporters by far than any other model in the field, and it is becoming clear that both the theorists and experimentalists in the field probably need to become more familiar with the ideas and the analysis of Widom and coworkers. According to the New Energy Times web site, this model explains LENR, and we should cease thinking about cold fusion altogether. And we can find some guidance in Larsen's slides as to how to think about the effect. For example, in the Lattice Energy LLC slide set from Feb. 14, 2009, we find written: "E-M radiation on metallic hydride surface increases mass of surface plasmon electrons." An arrow points to the reaction

$$(\text{radiation}) + \mathrm{e}^- \rightarrow \tilde{\mathrm{e}}^-, \tag{1}$$

where $\tilde{\mathrm{e}}^-$ is a "heavy-mass surface plasmon-polariton electron."

This, of course, is where our difficulties begin. We simply do not understand how an electron in a metal hydride can develop a mass enhancement as large as needed by the model. However, given the importance of the model, probably it is time to roll up our sleeves and try to understand how this might work. If electrons are going to gain mass, and a lot of mass, then we would like to understand how it works in simple terms if possible. We know that mass is energy, and vice versa, so this will take energy, and the energy has to come from somewhere. We would like to understand where the energy comes from, what form it is in according to a conventional perspective, and we would like to be able to develop intuition generally in terms of the standard microscopic picture that we use for other applied physics effects.

As mentioned above, we are motivated in part by a responsibility to understand what has been argued to be the solution to a problem that has vexed the field for several decades. But there are other motivations as well. For example, applied physics in the areas of atomic physics and solid state physics are mature fields, and an effect in which an electron gains mass at the MeV level does not seem to be in the associated textbooks. One could argue that excess heat in the Fleischmann–Pons experiment is not in the textbooks either, so new textbooks need to be written. On the other hand, it would be nice to understand the new effects in conventional terms if possible in order to connect them to things that we have intuition about already.

So, we proceed with determination to understand the first step of the Widom–Larsen model, which involves the mass increase of the electron.

2. Electron Mass Increase in a Classical Model

Widom and Larsen [3] start from a description in terms of relativistic quantum mechanics, but our starting point is going to be with a much simpler description. Given the very close connection between quantum mechanics and classical mechanics, often it is the case that a quantum system acts very much like its classical equivalent. Our hope is that the same will be the case here, which motivates us to consider here the classical version of the model. One of our goals in this discussion is to familiarize ourselves with the the electron mass increase in a more familiar setting.

Unfortunately, some subtleties arise already. In the original model the effect seemed to be connected to quantum fluctuations of the electromagnetic field. However, as discussed above and in [7] there are no quantum fluctuations in the longitudinal field in the Coulomb gauge. Yet Widom and Larsen have formulas in terms of the associated power spectral density of the electric field, which again seems to point to fluctuations in the longitudinal fields. As there are no quantum fluctuations associated with the longitudinal field itself, all that is left is that there must be fluctuations in the charge density that sources the fields. There is no difficulty in extending this notion to a classical model. We might imagine that other charges move with some degree of randomness, producing random longitudinal electric fields. We are able to analyze the case of an electron in free space interacting with such a field; there are no associated difficulties.

2.1. Nonrelativistic model

We then begin with the simple case of a free electron interacting with a time-dependent longitudinal electric field that we will write as

$$\mathbf{E}(\mathbf{r}, t) = -\nabla\Phi(\mathbf{r}, t). \tag{2}$$

At this point in the discussion there is no reason not to assume that it is sinusoidal in time and uniform in space. Newton's laws are then

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{r}(t) = \frac{\mathbf{p}(t)}{m}, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{p}(t) = q\mathbf{E}(t). \tag{3}$$

We assert a time-dependence of the form

$$\mathbf{E}(t) = \mathbf{E}_0 \cos(\omega_0 t) = \mathrm{Re}\{\mathbf{E}_0 \mathrm{e}^{-i\omega_0 t}\} \tag{4}$$

and then solve to get a sinusoidal steady-state solution of the form

$$\mathbf{r}(t) = \mathrm{Re}\left\{-\frac{q\mathbf{E}_0}{m\omega_0^2}\mathrm{e}^{-i\omega_0 t}\right\}, \qquad \mathbf{p}(t) = \mathrm{Re}\left\{i\frac{q\mathbf{E}_0}{\omega_0}\mathrm{e}^{-i\omega_0 t}\right\}. \tag{5}$$

Our goal was to consider things in the simplest possible terms, and it is clear that things cannot get much simpler than this.

In this simple model, the electric field drives the free electron, and the electron moves. The total energy can be computed from the potential plus kinetic energy, but we will focus on the kinetic energy here. We compute

$$E_{\mathrm{K}}(t) = \frac{|\mathbf{p}(t)|^2}{2m} = \frac{q^2|\mathbf{E}_0|^2}{2m\omega_0^2}\sin^2(\omega_0 t). \tag{6}$$

2.2. Electron mass

If the electron is at rest, then the mass contribution to the energy is mc^2. However, here the electron is moving, so we can determine the (rest mass and kinetic) energy of the electron according to

$$m^*(t)c^2 = \sqrt{(mc^2)^2 + c^2|\mathbf{p}(t)|^2} \rightarrow mc^2 + \frac{|\mathbf{p}(t)|^2}{2m}, \tag{7}$$

where the approximate version is for the nonrelativistic case. The dynamical electron mass satisfies

$$\begin{aligned} m^*(t)c^2 &= \sqrt{(mc^2)^2 + \frac{c^2 q^2 |\mathbf{E}_0|^2}{\omega_0^2} \sin^2(\omega_0 t)} \\ &= mc^2 \sqrt{1 + \frac{q^2|\mathbf{E}_0|^2}{m^2c^2\omega_0^2} \sin^2(\omega_0 t)}. \end{aligned} \tag{8}$$

The associated dynamical mass enhancement factor is then

$$\frac{m^*(t)}{m} = \sqrt{1 + \frac{q^2|\mathbf{E}_0|^2}{m^2c^2\omega_0^2} \sin^2(\omega_0 t)}. \tag{9}$$

2.3. Constant electric field

Given this situation, it seems that the simplest way (consistent with this very simple approach) to develop a large electron mass increase is to accelerate the electron in a uniform field that is constant in time. In this case if we take the momentum to be zero initially, then the momentum evolves as

$$\mathbf{p}(t) = q\mathbf{E}_0 t. \tag{10}$$

We can write for the dynamical mass

$$m^*(t)c^2 = mc^2 \sqrt{1 + \frac{q^2|\mathbf{E}_0|^2}{m^2c^2} t^2} \tag{11}$$

with a mass enhancement factor given by

$$\frac{m^*(t)}{m} = \sqrt{1 + \frac{q^2|\mathbf{E}_0|^2}{m^2c^2} t^2}. \tag{12}$$

The thought here is that we can develop a mass enhancement factor that is substantially greater than unity by simply accelerating an electron up to a few MeV using a static electric field accelerator.

2.4. Random electric field

In the event that the electric field is stochastic, then we need to make use of the somewhat more complicated mathematical tools to describe it. The momentum can be related to the field according to

$$\mathbf{p}(t) = \int_{-\infty}^{t} q\mathbf{E}(t')\,\mathrm{d}t'. \tag{13}$$

Assuming that the electric field is wide-sense stationary the momentum autocorrelation function can be defined as

$$\begin{aligned} R_{\mathbf{pp}}(\tau) &= E[\mathbf{p}(t+\tau)\mathbf{p}(t)] \\ &= \int_{-\infty}^{t+\tau} \mathrm{d}t' \int_{-\infty}^{t} \mathrm{d}t'' \Big\{ q^2 E[\mathbf{E}(t')\mathbf{E}(t'')] \Big\} \\ &= \int_{-\infty}^{t+\tau} \mathrm{d}t' \int_{-\infty}^{t} \mathrm{d}t'' \Big\{ q^2 R_{\mathbf{EE}}(t'-t'') \Big\}. \end{aligned} \tag{14}$$

The corresponding power spectral density is

$$S_{\mathbf{pp}}(\omega) = \int_{-\infty}^{\infty} R_{\mathbf{pp}}(\tau)\mathrm{e}^{i\omega\tau}\,\mathrm{d}\tau = \frac{q^2}{\omega^2} S_{\mathbf{EE}}(\omega). \tag{15}$$

We can use this to evaluate the kinetic energy

$$\begin{aligned} E\left[\frac{|\mathbf{p}(t)|^2}{2m}\right] &= \mathrm{Tr}\{R_{\mathbf{pp}}(0)\} \\ &= \mathrm{Tr}\left\{\int_{-\infty}^{\infty} S_{\mathbf{pp}}(\omega)\frac{\mathrm{d}\omega}{2\pi}\right\} \\ &= \mathrm{Tr}\left\{\int_{-\infty}^{\infty} \frac{q^2}{\omega^2} S_{\mathbf{EE}}(\omega)\frac{\mathrm{d}\omega}{2\pi}\right\}. \end{aligned} \tag{16}$$

Since we assumed that the electric field is wide-sense stationary, the expectation value of the mass in the nonrelativistic limit is time-independent

$$E[m(t)c^2] \;\rightarrow\; E\left[mc^2 + \frac{|\mathbf{p}(t)|^2}{2m}\right] = mc^2 + \frac{q^2}{2m}\mathrm{Tr}\left\{\int_{-\infty}^{\infty} \frac{S_{\mathbf{EE}}(\omega)}{\omega^2}\frac{\mathrm{d}\omega}{2\pi}\right\}. \tag{17}$$

The expectation value of the mass increase in the nonrelativistic limit is then

$$E\left[\frac{m(t)}{m}\right] \;\rightarrow\; 1 + \frac{q^2}{2m^2c^2}\mathrm{Tr}\left\{\int_{-\infty}^{\infty} \frac{S_{\mathbf{EE}}(j\omega)}{\omega^2}\frac{\mathrm{d}\omega}{2\pi}\right\}. \tag{18}$$

The generalization of this to the relativistic case looks to be problematic since we need to evaluate higher-order moments. A way to circumvent this is to work instead with

$$\sqrt{E\left[\frac{m^2(t)}{m^2}\right]} = \sqrt{1 + \frac{E[\ |\mathbf{p}(t)|^2]}{m^2c^2}}$$
$$= \sqrt{1 + \frac{q^2}{m^2c^2}\mathrm{Tr}\left\{\int_{-\infty}^{\infty} \frac{S_{\mathbf{EE}}(\omega)}{\omega^2}\frac{\mathrm{d}\omega}{2\pi}\right\}}. \tag{19}$$

This is consistent with the relevant equations given by Widom and Larsen [3]. Based on the discussion here, we will require very large random fields to accelerate an electron to the MeV scale. The underlying problem is very little different from the deterministic case, except perhaps that the acceleration will be more efficient in the case of constant or sinusoidal fields. In the stochastic case we would expect cancellation effects since the fields may point in random directions, or reverse sign randomly in a preferred direction.

2.5. Discussion

If the dynamics are nonrelativistic then we can use Newton's laws to understand the motion that results. If the dynamics are relativistic, then the trajectory is more complicated, but we would get the same solution for the momentum. An electron in motion has kinetic energy because it moves. Finally, we can evaluate the mass increase once we know the kinetic energy, since in this way of looking at things the kinetic energy and mass provide two ways of describing the same thing. We find that the mass increase is dynamic (as expected because the momentum is dynamic). The situation in the presence of a stochastic field is very closely related; if anything, we would expect a somewhat weaker mass increase due to cancellation effects.

The mass increase comes about due to a momentum increase, which means in the classical version of the free electron problem, terms containing the electric field only arise because we have solved for the momentum in terms of the electric field. The mass enhancement from this perspective is only due to the contribution of the momentum (a point we will think about again later on).

Perhaps the most useful result here is that we have a picture now which we can understand simply, and that shows us precisely what the mass shift is all about. The mass shifts because the electron acquires kinetic energy when it is accelerated by the electric field. There is no mysterious quantum effect here that conspires to arrange for collective effects to increase the mass of the electron. In essence, knowing the mass increase of the electron in this case is equivalent to knowing its kinetic energy.

Given this, then if the electron mass in free space is increased by ten percent for example, we know that this is because it has gained kinetic energy equal to ten percent of the rest mass.

3. Electron Mass Increase in a Quantum Mechanical Model

At this point in the discussion we are generally pleased that mass shifts similar to what is discussed by Widom and Larsen can be understood simply in the classical case. In turning our attention to the quantum mechanical case, once again there are several issues to be thought about. Perhaps the first is the establishment of a suitable starting place for the discussion, which motivates us to consider a generalized Foldy–Wouthuysen transformation. Although not pursued by Widom and Larsen, it is clear that a sizable mass shift can be developed in the case of a bound electron; this becomes of interest to us in connection with the issue of the electron mass shift in general, but it will lead to a clarification of whether we should focus on the energy or the mass shift (or whether there is a difference). This discussion generally will put us in a better position to think about mass shift for an electron in a metal.

3.1. Electron interacting with a potential and a field

We consider now a Hamiltonian of the form

$$\hat{H} = \boldsymbol{\alpha} \cdot c\hat{\mathbf{p}} + \beta mc^2 - e\boldsymbol{\alpha} \cdot \hat{\mathbf{A}}(\mathbf{r}) + V(\mathbf{r}). \tag{20}$$

If the potential is equal to zero

$$V(\mathbf{r}) \ \rightarrow \ 0, \tag{21}$$

then we can carry out a Foldy–Wouthuysen transformation leading to a rotated Hamiltonian of the form

$$\hat{H}' = \hat{\mathcal{U}}^\dagger \left[\boldsymbol{\alpha} \cdot c\hat{\mathbf{p}} + \beta mc^2 - e\boldsymbol{\alpha} \cdot \hat{\mathbf{A}}(\mathbf{r}) \right] \hat{\mathcal{U}} = \beta \sqrt{(mc^2)^2 + c^2 \left| \hat{\mathbf{p}} - \frac{e}{c}\hat{A} \right|^2}. \tag{22}$$

If we include the potential, things quickly become more complicated; we may write

$$\hat{H}' = \hat{\mathcal{U}}^\dagger \hat{H} \hat{\mathcal{U}} = \beta \sqrt{(mc^2)^2 + c^2 \left| \hat{\mathbf{p}} - \frac{e}{c}\hat{A} \right|^2} + V(\hat{\mathbf{r}}'), \tag{23}$$

where

$$\hat{\mathbf{r}}' = \hat{\mathcal{U}}^\dagger \mathbf{r} \hat{\mathcal{U}}. \tag{24}$$

The transformation of the position coordinate is straightforward, but untangling the results leads to all kinds of terms. From our perspective, we will view the rotated version of the model as

$$\hat{H}' = \beta \sqrt{(mc^2)^2 + c^2 \left| \hat{\mathbf{p}} - \frac{e}{c}\hat{A} \right|^2} + V(\mathbf{r}) + \cdots \tag{25}$$

The idea here is that in the Coulomb gauge the transverse field operator transforms as the momentum operator, but to the potential remains outside the square root. This is consistent with our earlier arguments [7] that it is the transverse fields that contribute to a mass shift, while the longitudinal fields in this way of thinking do not.

Note that in light of the arguments of the previous section, it is still the case that we can think of a mass shift resulting from longitudinal interactions, but this additional analysis will be required (since the mass enhancement is still due to an increase in the kinetic energy, which may depend on the potential).

In light of our earlier work [7], the largest mass shift that can be developed from transverse field fluctuations in the absence of an intense laser field are very small. Correspondingly, either we can absorb them into the mass here and continue, or else we can simply neglect them. In either case there is no reason to carry transverse fields further in this discussion.

3.2. Mass effect due to localization in a bound state

If we seek a sizable mass shift for an electron in a potential, it seems the place to look for it is in the case of a bound electron, and the more strongly bound the better. In the case of a hydrogenic ion, the deeply bound $1s$ electron has a (nonrelativistic) kinetic energy given by

$$\frac{\langle|\hat{\mathbf{p}}|^2\rangle}{2m} = Z^2 I_{\mathrm{H}}. \tag{26}$$

The corresponding (nonrelativistic) mass enhancement factor is

$$\frac{m^*}{m} = 1 + \frac{\langle|\hat{\mathbf{p}}|^2\rangle}{2m^2c^2} = 1 + \frac{Z^2 I_{\mathrm{H}}}{mc^2} = 1 + \frac{1}{2}(Z\alpha)^2. \tag{27}$$

At large Z the electron has sufficient kinetic energy that relativistic effects are important and one can see the effects of the increase mass on the wavefunction itself.

In this case we can generate a substantial mass increase, however this is under conditions where the electron is tightly bound. The mass increase in this case corresponds to a great deal of energy that can contribute in a weak interaction, but there is also a corresponding potential energy that is twice as large and opposite in sign

$$\langle V\rangle = -2\frac{\langle|\hat{\mathbf{p}}|^2\rangle}{2m} \tag{28}$$

in the nonrelativistic problem.

3.3. Mass and energy

We know that mass and energy are the same thing; however, probably here we need to think about the issue for a bound electron. The mass of the electron and nucleus as a composite in the $1s$ state is reduced by the binding energy $Z^2 I_{\mathrm{H}}$, of this we can be sure. Should we think of the electron as having a reduced mass in this case, instead of an increased mass as we have argued above? In the case of a free electron things seemed to be much clearer, since we could identify the mass increase of the electron with the kinetic energy.

But on second thought perhaps things should not have been so clear. For the classical problem of the last section, we might have written for the total energy the sum of the kinetic and potential energy

$$E(t) = \frac{|\mathbf{p}(t)|^2}{2m} + V(\mathbf{r}(t), t). \tag{29}$$

We might have sought to equate mass with the total energy, which might be problematic. For example, in one way of thinking the potential here is not specified to within a constant, so the total energy in such a calculation could be negative. Clearly under these conditions, it would make sense to consider the kinetic energy in connection with the mass increase. On the other hand, it might be the case that the electron is attracted to ions that are moving by slowly. In this case, we know that if the electron is (weakly) bound to these moving ions that the mass of the electron and ions would be reduced by the amount of the binding energy. In such a picture the notion of mass in connection with the potential becomes sensible, but perhaps only when thinking about the total mass of the electron and ions.

3.4. Electron wave packet in free space and uniform field

In the classical version of the problem (in free space with a uniform dynamic external field), once the longitudinal field is given, we can solve for the electron momentum directly in terms of the field, and then solve for the electron mass. In the quantum mechanical version of the problem it would be most elegant to rotate out the uniform field, and end up with a Hamiltonian with an explicit dynamical mass. Unfortunately, as yet we have not found publications where this kind of unitary transformation is studied (but we would expect it to exist). So instead, we will rely on the less elegant use of wave packet solutions.

For the nonrelativistic case we consider the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\psi(\mathbf{r},t) = \left[-\frac{\hbar^2\nabla^2}{2m} - q\mathbf{E}(t)\cdot\mathbf{r}\right]\psi(\mathbf{r},t). \tag{30}$$

We can develop a Gaussian wave packet solution of the form

$$\psi(\mathbf{r},t) = \frac{1}{(\pi L)^{3/4}}\frac{1}{(1+i\hbar t/mL^2)^{3/2}}\ \mathrm{e}^{-i\Theta(t)}\ \mathrm{e}^{i\mathbf{P}(t)\cdot[\mathbf{r}-\mathbf{R}(t)]}\ \mathrm{e}^{-|\mathbf{r}-\mathbf{R}(t)|^2/2L^2(1+i\hbar t/mL^2)}, \tag{31}$$

where $\mathbf{R}(t)$ and $\mathbf{P}(t)$ satisfy the classical equations of motion

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{R}(t) = \frac{\mathbf{P}(t)}{m}, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{P}(t) = q\mathbf{E}(t). \tag{32}$$

The phase factor satisfies

$$\hbar\frac{\mathrm{d}}{\mathrm{d}t}\Theta(t) = -\frac{|\mathbf{P}(t)|^2}{2m} - q\mathbf{E}(t)\cdot\mathbf{R}(t). \tag{33}$$

The kinetic energy in this case is

$$\frac{\langle|\hat{\mathbf{p}}|^2\rangle|}{2m} = \frac{|\mathbf{P}(t)|^2}{2m} + \frac{\hbar^2}{4mL^2}. \tag{34}$$

The nonrelativistic mass enhancement is then

$$\frac{\langle m^*(t)\rangle}{m} = 1 + \frac{|\mathbf{P}(t)|^2}{2m^2c^2} + \frac{\hbar^2}{4m^2c^2L^2}. \tag{35}$$

In this case we see that the mass shift includes the classical contribution as well as a contribution due to the original localization of the wave packet. This result demonstrates that for a Gaussian wave packet we can take the classical results over directly to the quantum version of the problem. If we consider a random external electric field, and ignore the localization energy, then the nonrelativistic mass increase for this wave packet solution would be

$$E\left[\frac{\langle m\rangle}{m}\right] \ \to\ 1 + \frac{q^2}{2m^2c^2}\mathrm{Tr}\left\{\int_{-\infty}^{\infty}\frac{S_{\mathbf{EE}}(j\omega)}{\omega^2}\frac{\mathrm{d}\omega}{2\pi}\right\}. \tag{36}$$

The construction of wave packets in the relativistic case is technically more complicated, but can be done as discussed in Ref. [9]. We would expect to recover a comparable relativistic relations between the mass enhancement and the classical momentum in this case.

3.5. Discussion

This discussion helps us in developing intuition about the electron mass increase generally. We know that the contribution of the transverse fields is small in laboratory experiments that do not involve an intense laser field, and we see that the longitudinal fields rotate differently than transverse fields under a Foldy–Wouthuysen transformation. A modest mass enhancement can be developed for a tightly bound electron, but in this case we need to focus on the total electronic energy (including the potential) rather than on the mass shift, unless we wish to work with the total mass of the composite.

Since the analysis for the quantum version of the problem is more complicated than the classical version, we probably need to think some about the connection between the classical and quantum versions of the problem. A reasonably general unitary transformation capable of rotating out a field uniform in space would be very useful in this context, and would allow for an elegant treatment of the problem. Probably such a rotation exists, in spite of our not having found a relevant publication. Instead we can work with wave packet solutions, which exhibit explicitly the classical relations in the context of the quantum problem. This is very useful because it shows that both the classical and quantum mechanical models describe essentially the same physics of interest to us.

4. Electrons in a Metal

At this point we have developed a sufficient foundation to begin considering the situation of an electron interacting with other electrons in a metal. Widom and Larsen have proposed that there is a mass enhancement in this case, so this motivates us to consider longitudinal Coulomb interactions between a reference electron and other electrons in the metal. >From the arguments of Section 2, the picture that is suggested is one in which the electron in the metal is in some ways like a free electron, and that it experiences longitudinal electric fields due to Coulomb interactions with the very large number of other electrons in the metal. These interactions are random, so if we think classically we end up with a situation very much like the one considered above. In this case we found that the momentum resulting from a stochastic electric field comes into the problem as a mass shift, and we were were able to recover a formula analogous to Widom and Larsen's equation for the mass enhancement.

There are of course issues to be considered in such a picture. For example, the electrons are not free electrons; but instead are either Bloch electrons, or quasi-particles, or perhaps something more complicated depending on the theoretical perspective adopted. In the original version of the problem there appear Coulomb interactions which all of the other electrons in the metal; however, we know that conduction electrons are effective at screening, so that interactions are only significant with a few at a time. We know that large electric fields can be generated with strong plasmon excitation. In this case the plasmon fields are coherent (and not stochastic), which given the discussion above presents no particular difficulty in thinking about or analyzing.

4.1. Conduction in a static field

The simplest model for conduction in a metal is one in which the electrons are treated as classical, are accelerated by a field, and lose momentum by scattering treated as friction. This is the metal version of the uniform constant accelerator field problem from Section 2. In this case we may write

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\mathbf{p}}{\tau} = q\mathbf{E}. \tag{37}$$

The momentum in this case is constant

$$\mathbf{p} = \tau q \mathbf{E}. \tag{38}$$

The kinetic energy is then

$$\frac{|\mathbf{p}|^2}{2m'} = \frac{\tau^2 q^2 |\mathbf{E}|^2}{2m'}. \tag{39}$$

The electron mass in a metal in connection with the Bloch picture, the quasi-particle picture, or in connection with experiment, is in general different from the free electron mass. It is usually denoted as m^*, which in the context of the present discussion is a notational headache. Consequently, we will use m' instead in this section. If we decide that the mass enhancement is consistent with this kinetic energy, then we end up with

$$\frac{m^*}{m} = \sqrt{1 + \frac{\tau^2 q^2 |\mathbf{E}|^2}{m m' c^2}}. \tag{40}$$

This mass enhancement in general is small, unless the applied electric fields are enormous (in which case the mass enhancement will still be small but but plasma formation would be expected follow).

The problem is straightforward, but ultimately we would imagine that a better approach might be to use an external high-current accelerator to produce a beam of MeV electrons, and drive them into the metal. At least until they degrade, a situation will be produced in which there will be a large number of electrons with increased mass.

4.2. ARPES and correlation contributions

In textbooks on solid state physics, one can find discussions of the problem of an electron in a periodic potential, which have periodic Bloch solutions. The energy eigenvalues associated with these solutions depend on the wave vector, and people use the results to construct band diagrams. The effective mass tensor of the Bloch states can be determined from the energy bands through

$$\frac{1}{m'_{i,j}} = \frac{1}{\hbar^2} \frac{\partial^2}{\partial k_i \partial k_j} E(\mathbf{k}). \tag{41}$$

In the case of insulators and semiconductors, once an energy band has been established (fitted to experimental data), one can use it to determine the effective mass for modeling.

In the case of metals, there are electron-electron and electron-ion interactions which are present beyond the effects of a periodic potential. The importance of these additional effects became particularly evident once angular-resolved photoelectron spectroscopy (ARPES) began to be used to study electron bands in metals [10–12]. Although electronic band structures in metals had been calculated for decades, there had not been a good way to extract an electron band directly from a set of experimental measurements. Once data was taken in the case of simple metals, and band diagrams began to be constructed, it became evident that the experimental results differed considerably from the theoretical band diagrams. This motivated the theorists to bring to bear more powerful theoretical tools, which led to the adoption of the G-W formalism and the quasi-particle picture [13,14], and other techniques [15]. Correlation effects (which include exactly the effect described by Widom and Larsen, among others) and phonon exchange are well modeled in this approach, with the result that the band diagrams from quasi-particle models are in good agreement with experiment.

The results can be understood in this case as O(eV) corrections to the band diagram. Effective masses can be developed from the corrected band diagrams which differ from those obtained from Bloch theory. Substantial corrections to the effective masses are found.

4.3. Plasma oscillations

Plasma oscillations are known to occur as organized longitudinal modes in metals, and have been proposed to contribute to the enhancement of the electron mass. It seems useful to consider this briefly here.

Collective oscillations in conduction electrons in metals was described many years ago by Pines and Bohm [16]. The plasma frequency (in MKS) is

$$\omega_{\mathrm{p}} = \sqrt{\frac{ne^2}{\epsilon_0 m'}}. \tag{42}$$

It varies from a few eV to more than 10 eV in different metals. Since the electric field varies sinusoidally, the dynamical mass increase can be computed directly from

$$\frac{m^*(t)}{m} = \sqrt{1 + \frac{q^2|\mathbf{E}_0|^2}{(m')^2c^2\omega_{\mathrm{p}}^2}\sin^2(\omega_{\mathrm{p}}t)} \tag{43}$$

in the case of a long-wavelength plasmon wave. Generally the associated momentum enhancement will be much smaller than the Fermi momentum, so that the incremental kinetic energy of an individual electron is much less than 1 eV (which does not get us close to the regime of interest for a reverse beta decay reaction).

However, the plasmon resonance in metals can be at a sufficiently high energy to result in observable physical effects. One example of interest in the literature is enhanced surface desorption resulting from plasmon excitation (see e.g. Hoheisel, Vollmer, and Träger [17]). In this case, the average electron kinetic energy is small, but the energy exchange with an electron involved in the plasmon excitation occurs near the plasmon energy $\hbar\omega_{\mathrm{p}}$.

It is possible to drive plasmon oscillations in a metal sufficiently hard that much higher electron energies are produced using intense laser excitation. An example of this is reported by Fennel et al. [18], in which electron energies up to a few hundred eV were detected at a laser intensity near 10^{14} W/cm^2 incident on metal clusters. Presumably we might expect MeV-level electron acceleration at much higher incident laser intensity.

4.4. Electron–phonon interactions

In [4] a discussion is given as to how energy exchange between moving ions in a metal might interact with electrons in order to contribute to a mass enhancement in the electrons. As might be expected, the interaction between vibrating ions in a metal and conduction electrons has been of interest for a very long time and there is a corresponding extensive literature.

In the earliest of these models, a uniform positive charge distribution up to a fixed radius away from the nucleus is used. The corresponding potential is

$$V(r) = \begin{cases} -\dfrac{Z^*e^2}{4\pi\epsilon_0}\left(\dfrac{3r_s^2 - r^2}{2r_s^3}\right) & r \leq r_s, \\[2ex] -\dfrac{Z^*e^2}{4\pi\epsilon_0 r}, & r_s \leq r. \end{cases} \tag{44}$$

This is consistent with the electron–phonon interaction used by Bardeen [19], and by many other authors up until about 1960. This interaction is much softer than an unscreened ion potential, which was required to obtain agreement with experimental conductivity data. The interaction comes about when the ion is displaced, so that [20]

$$V(\mathbf{r}-\mathbf{R}) = V(\mathbf{r}-\mathbf{R}^{(0)}) + (\mathbf{R}-\mathbf{R}^{(0)}) \cdot \Big[\nabla V(\mathbf{r}-\mathbf{R})\Big]_{\mathbf{R}_0} + \cdots \tag{45}$$

It was found to provide at least a plausible description for the phonon dispersion relation of Na by Toya [21,22]. Subsequently, people have relied on pseudopotentials for the electron–phonon interaction. In the case of hydrogen, one might imagine that a bare Coulomb potential is appropriate. However, a free-electron picture is used in a description such as in Refs. [21,22] under conditions where the electrons are better described by Bloch waves. As such, an electron–phonon matrix element calculated with free-electron wavefunctions and a Coulomb potential for conduction electrons will greatly overestimate the coupling, as compared to an electron–phonon matrix element computed with Bloch waves.

We would expect electron–phonon coupling (for conduction electrons) in a metal hydride to be much weaker than would result from the picture assumed in Ref. [4]. On the other hand, very energetic electrons would see the full Coulomb field of a proton at close range well inside of the $1s$ radius.

4.5. Discussion

Metals are made up of individual atoms, and most of the electrons in a metal are tightly bound to the nuclei in the lattice. Bloch conduction electrons, even in the case of alkali metals which most closely match a free-electron model, have localized components which are for the purpose of the discussion more like bound electrons than free. Consequently, our focus really should be on the electron energy, rather than on the mass enhancement determined from the rest mass and kinetic energy.

Nevertheless, free-electron models have been very important for describing conduction electrons in metals, and there is no reason not to consider the mass increase associated with the kinetic energy. For electron conduction it is straightforward to determine the momentum and corresponding mass enhancement as above, and we find that the mass enhancement is above unity by only a trivially small amount (expect perhaps in exploding wire experiments, and other fast electron plasma systems).

The analysis of electron-electron interactions in metals is a worthy topic of discussion in its own right. Electron-electron correlations are most readily treated in jellium [23], and come about due to potential contributions (rather than indirectly through the resulting momentum as we have interpreted the Widom–Larsen formalas). These contributions are included in modern GW calculations, and in related models; corrections are at the eV level (and not at the MeV level).

Bulk and surface plasmon modes are well studied in metals, and are interesting for all kinds of reasons in solid state physics and for applications. We would not expect plasmon excitation to result in MeV electrons unless driven by a very intense laser.

5. Mass Enhancement in PdH

In contrast to the possibly discouraging conclusions one might come to in light of the arguments outlined above, we can find in Ref. [4] a discussion of the mass enhancement in PdH where the authors come to a decidedly optimistic conclusion. This motivates us to examine the arguments, given that we would not have expected an optimistic conclusion in this case.

In our view there are a host of issues that could be touched on in connection with this single example. Rather than attempting to deal with even a subset of them, we will follow a simple line of thought. Our first goal is to orient ourselves with an example of an electronic vibration which would correspond to a mass increase sufficiently large to

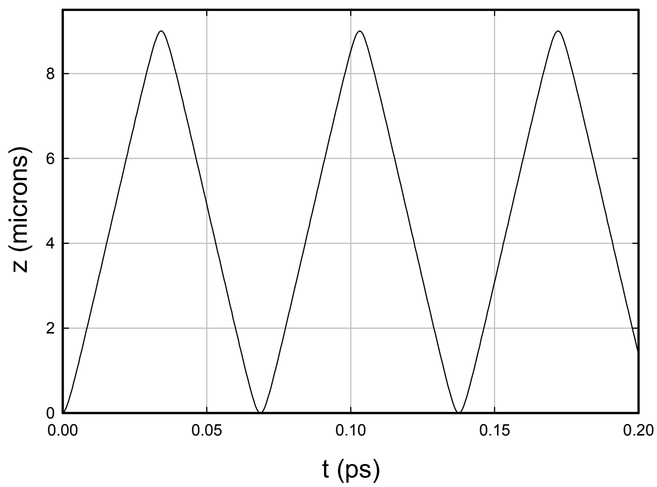

Figure 1. Trajectory of a relativistic electron with $\hbar\omega_0 = 60$ meV and maximum $\beta = 5$.

be relevant for neutron production from a proton. Once we have develop some intuition about the problem, we turn our attention to the practical issues of energy, power, and the transfer of energy from the vibrations to the electrons.

5.1. Model electron oscillation trajectory

It would seem that the place to start is with the end result, specifically the electron oscillations, which are already enormously interesting. Suppose that we consider free-space oscillations with $\beta = 5$, using the frequency consistent with the phonon frequency of 60 meV assumed in [4]. The trajectory that results is illustrated in Fig. 1; we see that the electron in this model travels close to 10 μm and has a roughly saw-tooth appearance since it spends most of the time moving close to c.

In Fig. 2 is shown the derivative of the kinetic energy with respect to time, which we see goes from about -30 to 30 W for one electron. The rate of energy exchange in this case for a single electron is prodigious. This brings out the biggest headache associated with the development of a mass shift through either uniform field that is sinusoidal in time, or random Coulomb fields; that an extremely large amount of power exchange is required to produce or maintain the electron energy.

5.2. Electron energy loss in Pd

However, in the Widom–Larsen model it is proposed that the electrons experience a mass enhancement in a metal hydride, rather than in free space. Energetic electrons lose energy rapidly through collisions and by radiative in solid matter, and it is of interest here to examine the rate of this energy loss. In Fig. 3 is shown the stopping power for energetic electrons in Pd (from the NIST ESTAR online database). From this stopping power we can estimate the rate of energy loss for an electron executing the above trajectory in Pd (the contribution of the H to this energy loss is minor); the results are shown in Fig. 4. We see that energy loss is minimized when the electron has acquired an energy near 1

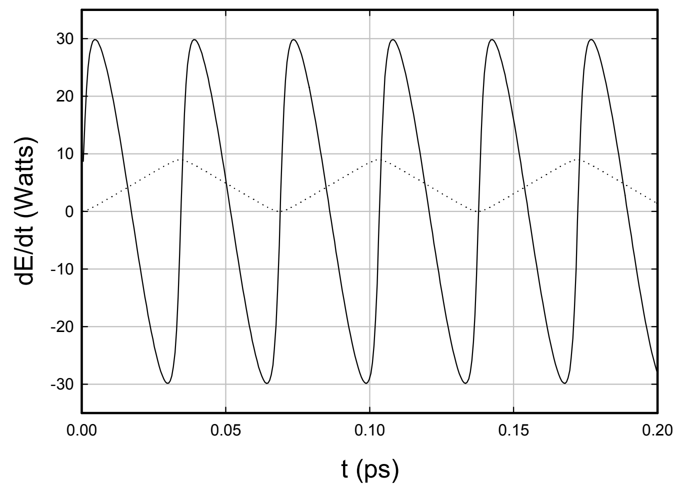

Figure 2. Rate of energy exchange dE/dt in Watts as a function of time (*solid line*); trajectory as in Fig. 1 (*dotted line*).

MeV, so that the electron loses only about 70 mW. The loss is very much larger when the electron energy is much less than 1 MeV.

An addition issue of interest is radiative loss, which would be expected to lead to penetrating X-rays that could be

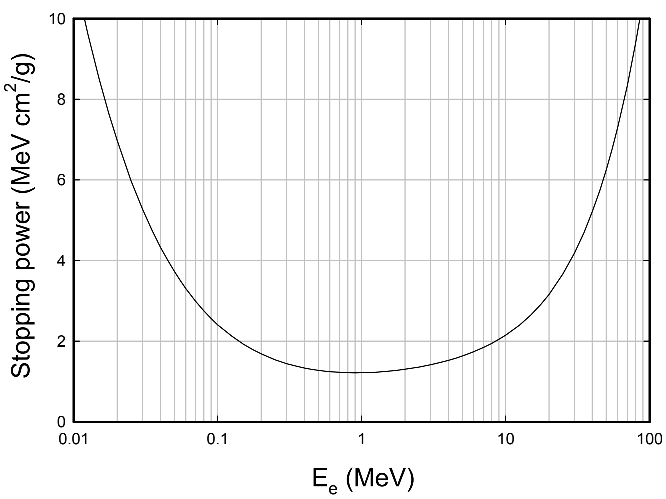

Figure 3. Electron stopping power in Pd.

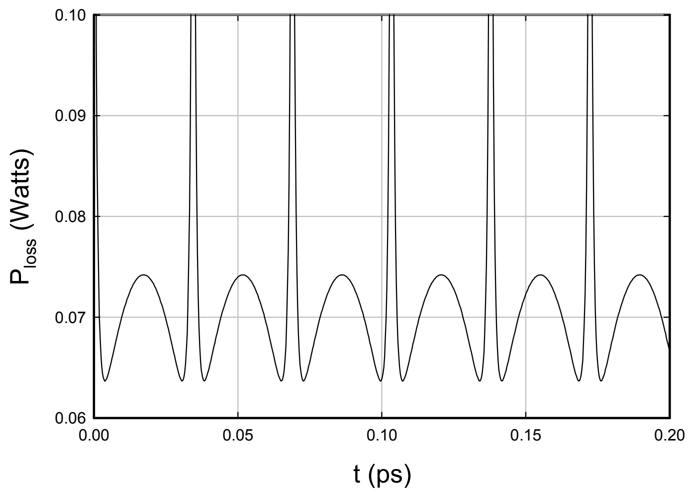

Figure 4. Rate of energy loss due to collisions and radiation for the trajectory above in Pd.

measured readily outside of the cell. For an MeV electron in the vicinity of 1 MeV, the fraction of the total rate of energy loss is about 6% of the total power loss due to collisions and radiation. Hence we would expect the radiated power to be near 4 mW per electron for at the 1 MeV electron minimum. This effect would seem to be useful in this context, as a positive signature for this radiation could be used to prove the existence of the effect.

5.3. Energy and power for phonon–electron coupling

It was suggested in [4] that a very highly excited optical phonon mode might couple with electrons in such a way as to produce MeV-level electron mass shifts. The arguments above suggest that maintaining an electron mass shift at the MeV level will involve substantial energy exchange, so this motivates us to examine the associated energy per unit area and exchanged power per unit area.

For simplicity we will consider a harmonic lattice model. The hydrogen motion is dominant for the optical phonon modes in PdH, so we can develop a reasonable estimate by neglecting the motion of the Pd atoms. In this case we can write for the vibrational energy

$$\sum_j \frac{1}{2} M\omega_0^2 \langle |\mathbf{R}_j|^2 \rangle \ \rightarrow \ N_{\mathrm{H}} \frac{1}{2} M\omega_0^2 \langle |\mathbf{R}|^2 \rangle. \tag{46}$$

We can evaluate the average vibrational energy of one hydrogen atom according to

$$\frac{1}{2} M\omega_0^2 \langle |\mathbf{R}|^2 \rangle \ \rightarrow \ 430 \text{ meV} \left(\frac{\langle |\mathbf{R}|^2 \rangle}{1 \text{ Å}^2} \right). \tag{47}$$

The total energy would be twice this in a harmonic lattice model. Note that vibrational amplitude in the bulk at the 1 Å level in this case could be considered to be positively heroic; certainly the vibrations would no longer be in a linear regime, and the hydrogen atoms would have no difficulty moving from site to site. This is relevant because surface vibrations will couple to near surface layers, so most of the vibrational energy will be away from the surface.

In Ref. [4] an estimate for the number of surface electrons and protons involved is taken to be 10^{16} per cm^2. We can use this to obtain a vibrational energy estimate of

$$\frac{E_{\rm vib}}{\rm area} = 1.4 \ \frac{\rm mJ}{\rm cm^2} \ \left(\frac{\langle |\mathbf{R}|^2 \rangle}{1 \ \text{Å}^2} \right). \tag{48}$$

This perhaps seems like a manageable number; however, since optical phonons lose vibrational energy very rapidly, the power per unit area required to sustain this level of vibrations is roughly

$$\frac{P_{\rm diss}}{\rm area} = \left(\frac{\omega_0}{2\pi \ 10} \right) \frac{E_{\rm vib}}{\rm area} = \frac{10^{10}}{2\pi} \ \frac{\rm W}{\rm cm^2} \ \left(\frac{\langle |\mathbf{R}|^2 \rangle}{1 \ \text{Å}^2} \right). \tag{49}$$

This is a very high surface power.

Unfortunately, based on the estimates for power loss above, this falls far short of what is required to sustain an electron mass shift large enough to do what is proposed. Let us take the 70 mW power loss number per electron, and turn this into a surface power density using 10^{16} electrons/cm^2 (and for this estimate not worry about how such energetic electrons would manage to remain localized to a few Angstrom surface layer); the result is

$$\frac{P_{\rm loss}}{\rm area} = 70 \ {\rm mW} \times 10^{16} \ {\rm cm}^{-2} = 7 \times 10^{14} \ \frac{\rm W}{\rm cm^2}. \tag{50}$$

In the electrochemical experiments the power dissipation at the surface is on the general order of 10 W/cm^2, which falls far short of being able to provide sufficient power to the electrons or the hydrogen atoms required by the example of Ref. [4].

5.4. Number of excited electrons

We might expect some of the input electrochemical energy to find its way into exciting optical phonon modes, and there is no question that some of this energy will couple to conduction electrons. In a conventional picture individual electrons would be promoted with an energy increment of one phonon (60 meV in this example). We could begin to estimate how many individual electrons would be excited through this mechanism at a time; we estimate

$$N_{\rm e} = \eta_{\rm p} \eta_{\rm e} \left(\frac{P_{\rm electrochem}}{\hbar \omega_0} \right) \tau_{\rm e}, \tag{51}$$

where $\eta_{\rm p}$ is the fraction of the electrochemical energy going into optical phonon excitation; where $\eta_{\rm e}$ is the fraction of the phonon energy going into electron excitation; and where $\tau_{\rm e}$ is the relaxation time of an excited electron. We can parameterize to obtain

$$N_{\rm e} = 10^5 \ \eta_{\rm p} \eta_{\rm e} \left(\frac{P_{\rm electrochem}}{1 \ {\rm W}} \right) \left(\frac{\tau_{\rm e}}{1 \ {\rm fs}} \right). \tag{52}$$

We conclude from this that we might expect fewer than 1000 electrons to retain the 60 meV phonon energy at any particular time per watt of electrochemical power.

5.5. Discussion

A significant issue in working with the ideas put forth by Widom and Larsen is the clarification of what the mass shift is. For example, if we regard the effect as some benign interaction of the electron with its environment that simply makes the electron heavier, then we would be talking about a new physical picture very different from what is commonly understood as how electrons work in applied physics applications. On the other hand, if we understand the mass shift as connected with the electronic kinetic energy in a free-electron model, then there is nothing new or exotic to be worried about, and we can use standard pictures and analysis to bear.

In this case, independent of how a large mass enhancement is produced, we know that a great deal of energy must be involved. The purpose of the oscillating electron example was to allow us to study this in the case of a simple example where we can quantify every relevant number. The consideration of this example showed us that there is no particular difficulty in specifying a trajectory which has a large associated mass enhancement, but the rate at which energy much be exchanged to maintain it is astronomically large, and if we try to maintain such oscillations in PdH the power loss is prodigious.

If we pursue the numerical example of Ref. [4], we find that even though the example restricts consideration to only the outer monolayer of an idealized metal hydride, the energy involved even in the atomic vibrations are orders of magnitude higher than what is available. The amount of power needed to maintain a sizable mass shift in the number of electrons under consideration are even more orders of magnitude larger. How energetic electrons with MeV level mass increase might remain localized to a surface layer is problematic. Were we to carry out a conventional analysis of electron promotion in a this kind of highly idealized model for the the outer skin of a cathode, we would conclude that fewer than a thousand electrons on average would retain even one phonon of energy per watt of electrochemical power.

6. Mass Enhancement and Radiation

An accelerating charge accelerates, so we would expect an electron which is accelerated sufficiently to develop a mass increase on the order of the rest mass to radiate. For linear motion the radiated power is

$$P_{\mathrm{rad}} = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\frac{1}{m^2c^3}\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)^2. \tag{53}$$

6.1. Linear oscillations

In the case of our example with a field uniform in space and sinusoidal in time, this becomes

$$P_{\mathrm{rad}} = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\frac{e^2|\mathbf{E}_0|^2}{m^2c^3}\cos^2(\omega_0 t). \tag{54}$$

It seems useful here to relate the time averaged radiated power to the time average mass shift; we may write

$$\overline{P}_{\mathrm{rad}} = \frac{1}{3}\frac{e^2}{4\pi\epsilon_0}\frac{e^2|\mathbf{E}_0|^2}{m^2c^3} \tag{55}$$

and

$$\overline{\left(\frac{m^*(t)}{m}\right)^2} = 1 + \frac{q^2|\mathbf{E}_0|^2}{2m^2c^2\omega_0^2}. \tag{56}$$

We can use this to write for a single electron

$$\overline{P}_{\mathrm{rad}} = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\frac{\omega_0^2}{c}\left[\overline{\left(\frac{m^*(t)}{m}\right)^2} - 1\right]$$
$$= 0.592\ \mu\mathrm{W}\left[\overline{\left(\frac{m^*(t)}{m}\right)^2} - 1\right]. \qquad (57)$$

Even a modest number of electrons so excited would give a very clear signal that is detectable.

6.2. Discussion

It is possible to develop an exact solution for a Dirac electron in a classical electromagnetic field which exhibit a mass shift due to interactions with the field. From our perspective, the mass shift in this case is due to the electron acquiring considerable kinetic energy. The accelerating electron in this case is known to radiate [24]. Consequently, we would expect generally that when an electron interacts with a dynamic electric field, whether it is transverse or longitudinal, that it will radiate. The only question then is how much radiation is expected. We have estimated here the radiated power in the case of periodic linear oscillations.

Although the radiated power for a single electron is modest (about half a microwatt), in the example considered in Ref. [4] there are 10^{16} electrons/cm^2 with a substantail mass enhancement assumed. In this case the radiated power that could be associated with the electron acceleration is in the range of 10^{10}–10^{11} W/cm^2. This radiation is readily detectable if present, and it could be taken advantage of to verify the presence of enhanced mass electrons.

7. Summary and Conclusions

Widom and Larsen have put forth a new model to account for excess heat and transmutation in LENR experiments. As mentioned in the Introduction this model has proven to be the most successful model since the field started; it has an enormous number of supporters within and without the CMNS community; and it has been judged of sufficient merit to justify an experimental effort at NASA.

The model itself is complicated, with a great many individual pieces. Our focus in this work has only been on issues associated with a single issue in the model; the proposed increase of the electron mass. When we started this study, one of our goals was to understand this issue. Now, after many pages of discussion the conclusion is at best mixed. Inasmuch as the effect under discussion is conventional, we can understand it in terms of relatively simple classical and quantum mechanical models; in this case we would not expect a significant mass enhancement under conditions relevant to experiments in the field. On the other hand, it may be that there remain subtleties associated with the approach, which somehow allows a benign accumulation of mass through a large number of random low-energy Coulomb interactions. If so, then Widom, Larsen, and coworkers have some work left to do in explaining how such a thing might happen.

In our earlier work, the main conclusion relevant to this work was that the longitudinal field comes into the problem conventionally (since there are no quantum fluctuations for longitudinal electric fields in the Coulomb gauge). At the time we considered this to be significant, with the implication that no MeV-level mass increases would be expected due to conventional physics under the experimental conditions of the Fleischmann-Pons experiment and related experiments. As our work was criticized in Ref. [8], it seems that we did not succeed in making the point, or that perhaps there were aspects of the model that we did not understand. In the end, we were motivated to return to the problem once again, determined to understand things better.

We first considered the classical problem, where things are perhaps simplest and clearest. For an external field uniform in space, the electron acquires momentum, which in turn produces an increase in the electron mass. In this problem we are able to take advantage of this relation to express the mass increase in terms of the electric field. This was demonstrated for a constant field, a sinusoidal field, and a random field. Some of the formulas that we ended up with seem to be very close to those given by Widom and Larsen; where in our case we have very simple pictures and models that go with the formulas.

Ultimately we expect the quantum version of the problem to be very close to the classical problem, due to the close connections that exist between classical mechanics and quantum mechanics. We are able to develop a quantum wave packet solution in which the average position and momentum variables satisfy Newton's laws, and which the kinetic energy is related to the expectation value of the momentum very much like in the classical problem (with an additional contribution from the localization of the wave packet). This connection works for constant external longitudinal fields, sinusoidal fields, and fields with random time-dependence on equal footing. In all cases, classical and quantum mechanical, the mass increase of the electron is a result of its kinetic energy, acquired in interactions with the longitudinal fields. We find a modest increase in the electron mass for tightly bound electrons, where we conclude that the electron energy is more important if we focus on the electron (although mass is still relevant if we work with the composite instead of the electron).

In conventional models of electrons in metals, a great deal of work has been done, and there is no MeV-level mass increase of electrons. We can generate an increase in the electron kinetic energy in conduction, in Coulomb interactions, and in the case of plasmons; in all cases the problems are well known. But in no cases do we find MeV energy increases. We did note that in the case of an intense laser interacting with the plasmon mode of a metal cluster that electrons with several hundred eV energies result.

Given the rather dismal expectation for MeV mass enhancement from the conventional models, we turned our attention back to the example described in the primer of Ref. [4]. We find immediately that if we make use of the simple models from earlier in the paper, that an enormous amount of energy exchange would be required to support an electron trajectory that reaches a mass increase of 5 in free space. The power loss that such an energetic electron would sustain in a metal due to scattering and radiating is on the order of 70 mW per electron over most of its cycle. When we considered the proton vibrations in the model, then we find that GW/cm^2 would be required to sustain the assumed motion of a monolayer of protons at the surface assuming the loss occurs at a rate comparable to optical phonon loss rates in metal hydrides. Even though the power level associated with the assumed vibrations is astronomical, it is short by orders of magnitude of what would be needed to sustain the requisite electron mass shift of the small number of surface electrons in the model.

In the interaction of electrons with an electromagnetic traveling wave field, an exact analytic solution is available in which the dressed electron motion is described simply in terms of an effective mass (which is increased by interactions with the field). In a sense, this problem is the reference problem which is then generalized greatly by Widom and Larsen, which motivates us to consider it in similar simple terms. The interaction with the transverse fields causes the electron to execute a complicated trajectory, and to acquire kinetic energy as a result of the field interactions. This situation is similar to the one described in this work, in which the electron gains kinetic energy due to interactions with longitudinal fields. An electron interacting with an intense laser field radiates, and this radiation is well accounted for from a classical model where the radiation is computed by averaging the square of the acceleration over the trajectory. Similarly, an electron accelerated by longitudinal fields is going to radiate, simply because it has a charge that undergoes acceleration. The amount of radiation that would be produced by an electron with a 5-fold mass increase is in the vicinity of a microwatt. The large amount of penetrating X-rays that would result if even a modest number of electrons had this much kinetic energy could be readily measured. The absence of such signals in the few cases where relevant X-ray measurements have been done under conditions where excess heat has been observed would seem to preclude this as an explanation.

Our goal in this study was to understand the electron mass increase in the Widom and Larsen model. In the end, we have considered a variety of scenarios under which an electron mass enhancement can occur, but none of these have anything to do with LENR experiments. We have considered the example in the case of PdH which they have put forth to support the approach, but the example assumes an astronomically large level of vibrational excitation, which falls orders of magnitude short of being able to provide the amount of power needed to maintain the target level of electron kinetic energy required by the model. Electrochemical power at the watt/cm^2 level could support in a conventional analysis the promotion of less than 10^3 electrons/cm^2 at a time with one 60 meV phonon worth of excitation, rather than the several MeV level excitation in 10^{16} electrons/cm^2 discussed in the model. We conclude that we do not understand how Widom and Larsen's model could account for anomalies in LENR experiments. Since accelerated electrons radiate, if electrons were present according to the Widom and Larsen model, there would be a readily detected X-ray signal. The absence of such hard X-ray signals in experiment argues against this model.

We note that we are not alone in noticing issues in connection with the Widom-Larsen model; Vysotskii has earlier commented on a variety of issues with the model [25,26]. Although we had seen Vysotskii's presentation at ICCF17, we had clearly not appreciated the points which he made (points which became clear when a reviewer provided encouragement to look at his papers). One issue is that the large electric fields in the vicinity of a nucleus are localized, and do not persist over a large spatial region; so the use of the large local atomic field strength to estimate the coherent extended plasmon field strength is an overestimate (our arguments are consistent with Vysotskii on this point). Vysotskii makes an argument concerning the development of a mass enhancement without an associated momentum increase in a static electric field; an argument with seems qualitatively inconsistent with our conclusions (since the mass increase in the dressed description in our view arises from the momentum increase). Vysotskii argues that the efficiency of the electron-proton reactions in the Widom–Larsen model will be reduced because energy loss through ionization and radiative losses by orders of magnitude from that claimed by Widom and Larsen; we note here that by implication Vysotskii has already recognized the rapid energy loss of energetic electrons in the Widom–Larsen model. Finally, Vysotskii pointed out the very high surface power requirements implied in the Widom–Larsen model; qualitatively we are in agreement with Vysotskii on this, but the contributing factors are somewhat different, and our estimates are higher than those of Vysotskii.

The biggest issue in the consideration of the part of the Widom–Larsen model from our perspective is the origin of the mass enhancement in the Coulomb gauge when transverse fields are not present. Consequently, our focus initially was on examining how such a mass enhancement could come about, since in the Coulomb gauge there are no quantum fluctuations associated with the longitudinal field. Our conclusion is that when the electron is accelerated, the momentum that results can be described through a mass enhancement (a result that should persist in a dressed picture). Vysotskii for the most part appears to have for the most part presumed this to be the case based on the starting point for his discussion. Widom and Larsen seem to consider the mass enhancement to be associated with mass renormalization, perhaps with the implication that the excess mass is benign in the same sense as the rest mass. However, the formulas which they obtain to evaluate the mass shift in the case of a longitudinal field are very similar to ours, and the mass enhancement in our formulas come about from a momentum increase.

In connection with the last issue, one might assert that mass renormalization in QED itself is modified in the presence of an external longitudinal field. Probably there is literature on this issue that could be dug up which would shed light on things. On the other hand, the issue is in a sense moot, since all of the longitudinal fields in the problems at hand originate from other electrons and ions (there is no significant "external" field if we include all of the relevant electrons and nuclei in our description). Hence if there is a mass renormalization effect due to an external field, then we would expect that if we replaced the external field with a larger description so that no external field were needed, we should be able to make use of conventional mass renormalization. In this case, there would be no physical basis for a benign electron mass shift (i.e. one that is unconnected with an enhanced momentum). Presumably Widom and Larsen will clarify things in their future publications.

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