J. Condensed Matter Nucl. Sci. 12 (2013) 41–53

Research Article

Neutrino Equation of Motion and Neutrino–electron Bound Pairs in LENR

Burke Ritchie *

Lawrence Livermore National Laboratory, Livermore, CA 94550, USA

Abstract

The long-established electron-capture reaction $e^- + p^+ = n + \nu$ may be considered to be a prototype reaction in the nascent field of physics known as low-energy nuclear reactions (LENR) since it involves an interface between electron and atomic physics (EAP) on the left-hand side and nuclear physics on the right-hand side of the reaction. It is a form of inverse beta decay $n = p^+ + e^- + \nu$, which is understood using a conceptual and mathematical methodology (forces mediated by the exchange of bosons known as force carriers and specifically for beta decay the W^- boson as the force carrier for the electroweak force) which is totally foreign to EAP but well-supported by copious nuclear experimental data. Since no such established experimental database exists in LENR, an equation of motion (EOM) is proposed for the neutrino in analogy to Dirac's equation, which is the EOM for the electron. The combined electron and neutrino EOM's support temporary neutrino-electron binding and discover the mass and length scales of a nucleon on an *ab initio* basis. It is believed that the bound pair is a form of W boson, symbolized here by $W_s^{\pm}$ for binding of a neutrino to a positron or electron (±) and for spin (s) equal to 0 or 1. It is also believed that $W_s^{\pm}$ bosons may be useful as building blocks in constructing models in the LENR regime which may be physically equivalent to quarks and the known $W^{\pm}$ boson in the high-energy regime.

Keywords: Electron, Neutrino, Nucleon, Positron, W-boson

1. Introduction

A small theoretical and experimental community exists in low-energy nuclear reactions (LENR); however it appears that a overwhelming majority of the general theoretical-physics community has reached a consensus that LENR observations cannot be explained by quantum mechanics using Coulombic forces [1]. Remarkably a prototype of a LENR experimental observation, namely electron capture by a nucleus, has long been known,

$$e^- + p^+ = n + \nu, \tag{1}$$

where ν represents a neutrino. Pauli first postulated the existence of the neutrino in beta decay,

*E-mail: ritchie@lstc.com

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$$n = p^+ + e^- + \nu \tag{2}$$

and Fermi successfully calculated the rate of beta decay using first-order perturbation theory (Fermi's Golden Rule) with a phenomenological nuclear interaction for the nuclear internal-conversion transition. Modern theory has postulated a fundamental electroweak interaction which causes a nuclear internal conversion from an up to a down quark, which cancels the net unit positive charge of the proton's two up and one down quarks. The electroweak interaction is supported by experimental observation and is understood as a force mediated by the W^- boson, which decays immediately into an electron and a neutrino.

A fundamental difference in the mathematical and conceptual methodologies between high-energy physics and electron and atomic physics (EAP) is already obvious from the description I have just given of beta decay such that it is no mystery that never the twain shall meet between the high-energy and low-energy communities, thereby leading to the negative conclusions of particle theorists regarding the validity of LENR experiments. In particle physics forces are mediated by the exchange of bosons or force carriers. Taking this picture to EAP the Coulombic force is mediated by the exchange of photons. The force-carrier formalism is based on perturbation theory with free-particle zeroth-order basis and is therefore largely inapplicable to bound states. The nonperturbative theory appropriate for bound states is lattice quantum chromodynamics (QCD), whose set of difficulties include its totally numerical nature such that there are no analytic test problems available to mitigate numerical uncertainties as in other computationally-intensive fields of physics. Curiously one could not easily, if at all, repeat the highly accurate nonperturbative calculations of atomic, molecular, and condensed-matter physics using the force-carrier formalism, even though remarkably lattice QCD is the primary tool available for elucidating the internal structure of nucleons and of nucleon–nucleon forces generally.

Of greater importance however is the need in high-energy physics to understand the nature of forces as mediated by intermediate particles which are bosons and are known as force carriers. Using this picture we may attempt to formulate the decay of an excited atom which lies above one or more ionization thresholds such that the atom spontaneously emits an electron. The forces are Coulombic such that the intermediate particle is a photon. The photon is unobserved so that it must be considered to be a virtual particle. In beta decay however the intermediate particle, the W^- boson, must be considered to be a real since the experimental observation is its decay into an electron and a neutrino. The fragmentation of the W^- boson is "immediate" compared to the electroweak up to down quark transition mediated by the W^- boson, which requires nearly 15 min.

In practice the lifetime of the force carrier is determined experimentally by the range of the interaction which varies from infinite (photons) to very short ($W^\pm$ bosons). The conceptual difference from the nonperturbative calculations of EAP, in which the force can be expressed mathematically without the need to postulate an intermediate particle at all, is striking. Most importantly in the low-energy theory of matter the particles are the *same* before and after a transition, whereas in nuclear theory the particles are different, for example quarks and $W^\pm$ bosons initially and quarks, electrons, and neutrinos finally. The vague description of an "immediate" decay of the $W^\pm$ intermediate particle into a positron and neutrino or an electron and neutrino is a "black box" with respect to further physical elucidation of the internal structure of the $W^\pm$ boson before its fragmentation. The darkness of our knowledge of any possible $W^\pm$ boson internal structure is mandated by the binomial logic of particle methodology in which particles are created or destroyed with respect to a particle field which possibly comprises an incomplete physical description of the particle itself.

I discuss this point at length in the appendices of [1] in which I point out that that radiation-free matter does not exist in nature. But yet theoretical physics has evolved, reflecting the separate developments historically of mechanics and electrodynamics, into a radiation-free quantum theory of matter, a matter-free quantum theory of radiation, and a theory of the mutual interaction of radiation and matter. This piecemeal approach leads to an infinite energy for the Lamb shift and other "radiative corrections" of the electron in absence of the use of physical argument and mathematical adjustments to "renormalize" theory in order to obtain a finite result which remarkably agrees with high accuracy with experimental observation. One may question however if perfect theoretical agreement with a specific set of experiments should be accepted with uncritical acclaim in absence of a theory which explains the source of the infinities and provides a divergence-free result. It is hard to imagine that renormalization theory with its mathematical recipes for the removal of divergent contributions could be a general theory of nature, not withstanding its high degree of accuracy.

Indeed one may say that particle fields for matter-free photons or photon-free electrons represent incomplete physical descriptions of these particles. This is the lesson which we may take away from Lamb's experiments, which definitively demonstrate the existence of permanent radiative shifts in atomic energy levels, namely that radiation-free matter does not exist in nature such that a photon- free material particle field or a particle-free photon field, however neat and pleasing this it is mathematically, is not a complete picture either of the material particle or of the photon. The renormalization scheme itself confirms this view since infinities are removed from radiation-matter calculations by postulating that photons are always present in the structure of a free electron such that when the free-electron radiative shift is added back to bound-electron calculations the unphysical infinities are removed.

In this paper I return to the fully relativistic, Lorentz-invariant potential methodology of relativistic quantum mechanics as a powerful tool available for use in particle theory. As an application I propose a neutrino equation of motion (EOM).

It is shown that a neutrino–positron or neutrino-electron pair can form a mutually bound state with lifetime against breakup into a free neutrino and positron or a free neutrino and electron of the same order as the lifetime of the neutron against beta decay. The temporarily bound pair is given the symbol $W_s^{\pm}$ for a positron-neutrino (+) or electron-neutrino (−) boson where the subscript denotes the two possible spin states 0 or 1. It is possible therefore to postulate that the unstable $W_s^{\pm}$ bosons aggregate to give a physical equivalency for the proton. One such aggregate might be $\mathrm{e}^- W_0^+ W_0^+ n\nu \leftrightarrow$ UUD for the two up and one down quarks of the proton. Since the zero-mass, zero-charge neutrinos would not be expected to undergo mutual repulsion or add mass or charge, it is possible that a large number of neutrinos ($n \gg 1$) could be dragged along as internal constituents of the proton, where in a spin unpolarized state the net spin of the aggregate of n neutrinos would be zero such that the net spin and charge of the proton is given by the two $W_s^{\pm}$ bosons and the electron. Then it can be further postulated that electron capture given by Eq. (1) proceeds as follows,

$$\mathrm{e}^- W_0^+ W_0^+ n\nu + \mathrm{e}^- = \mathrm{e}^- W_0^+ W_0^+ W_1^- (n-2)\nu + \nu \tag{3}$$

such that an equivalency exists for the neutron

$$\mathrm{e}^- W_0^+ W_0^+ W_1^- (n-2)\nu \quad \leftrightarrow \quad \mathrm{UDD}$$

for the one up and two down quarks of he neutron. The neutron decay is then determined by the decay of W_1^-,

$$\mathrm{e}^- W_0^+ W_0^+ W_1^- (n-2)\nu = \mathrm{e}^- W_0^+ W_0^+ (n-2)\nu + \mathrm{e}^- + \nu \tag{4}$$

such that our postulate for the proton (first term on the right-hand side of Eq. (4)) holds if $(n-2)\nu \cong n\nu$. Neutrino emission in LENR has also been proposed by authors using a "selective resonant" theory [2] in which the rate of tunneling through the deuterium–deuterium Coulomb barrier matches the rate of decay of an excited state of the helium nucleus by the electroweak interaction accompanied by neutrino emission [3].

We close this section by reminding readers of the Lorentz covariance of Dirac's equation even in the presence of the potentials [4] by writing Dirac's equation in manifestly covariant form using the Clifford algebra of 4×4 γ-matrices,

$$(\gamma_0, \vec{\gamma}) \cdot \left(\left(i\hbar \frac{1}{c}\frac{\partial}{\partial t} - \frac{e}{c}\Phi \right), \left(i\hbar\vec{\nabla} + \frac{e}{c}\vec{A} \right) \right) \psi_{\mathrm{D}}(\vec{r}, t) = mc\psi_{\mathrm{D}}(\vec{r}, t), \tag{5a}$$

$$\left[\gamma_0\left(i\hbar\frac{1}{c}\frac{\partial}{\partial t}-\frac{e}{c}\Phi\right)+\vec{\gamma}\cdot\left(i\hbar\vec{\nabla}+\frac{e}{c}\vec{A}\right)-mc\right]\psi_{\mathrm{D}}(\vec{r},t)=0, \tag{5b}$$

where Eq. (5b) follows on carrying out the 4-vector operations indicated in Eq. (5a), $(\Phi,\vec{A})$ is the electromagnetic 4-potential, and $\psi_{\mathrm{D}}(\vec{r},t)$ is Dirac's 4-component vector wave function. Notice in Eq. (5a) Dirac's equation has been written formally as the scalar product of 4-vectors (scalar product of γ 4-vector and the electron's 4-momentum) operating on $\psi_{\mathrm{D}}(\vec{r},t)$ on the left-hand side equal to the Lorentz constant mc times $\psi_{\mathrm{D}}(\vec{r},t)$ on the right-hand side. Readers are reminded that

$$(\gamma_0,\vec{\gamma})=\left(\begin{pmatrix} I & 0\\ 0 & -I\end{pmatrix},\begin{pmatrix} 0 & \vec{\sigma}\\ -\vec{\sigma} & 0\end{pmatrix}\right),$$

where I is the 2×2 identity matrix, $\vec{\sigma}$ is Pauli's vector,

$$\vec{\sigma}=\hat{i}\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}+\hat{j}\begin{pmatrix} 0 & -i\\ i & 0\end{pmatrix}+\hat{k}\begin{pmatrix} 1 & 0\\ 0 & -1\end{pmatrix},$$

and

$$\psi_{\mathrm{D}}(\vec{r},t)=\begin{pmatrix}\psi(\vec{r},t)\\ \chi(\vec{r},t)\end{pmatrix}$$

in which $\psi(\vec{r},t)$ and $\chi(\vec{r},t)$ respectively are the "large" and "small" two-component spinors. As pointed out in [4] the transformation properties of the γ-matrices guarantee the Lorentz covariance of Eqs. (5a) and (5b) even in the presence of the 4-potential since the frame transformation is now carried out on the vector difference of the 4-momentum and the 4-potential, namely

$$\left(ih\frac{\partial}{c\partial t}-\frac{e}{c}\Phi,\ ih\vec{\nabla}+\frac{e}{c}\vec{A}\right)$$

rather than on the free-particle 4-momentum by itself, and the vector difference is still a 4-vector such that Lorentz covariance is preserved.

I believe that it is possible to use first-quantized relativistic quantum mechanics and potential theory to understand nuclear reactions such as electron capture, $e^- + p^+ = n + \nu$, which take place in the low-energy regime. This belief is supported by the success of Dirac's equation to give an exact *ab initio* description of the hydrogen atom in absence of radiative corrections and nuclear recoil. It is necessary then to discover neutrino-matter forces on an *ab initio* basis, which has been accomplished here by deriving a neutrino EOM which is compatible with the electromagnetic equation of continuity, in analogy to the electron EOM (Dirac's equation with electromagnetic 4-potential), which is compatible with the material equation of continuity. The latter criterion was essential to Dirac in his program to derive a correct Lorentz-invariant relativistic EOM for the electron. Remarkably the neutrino EOM is used in combination with the electron EOM to find a neutrino-electron or neutrino-positron temporarily bound state which has the mass and length scales of a nucleon and whose lifetime has the time scale of the neutron's lifetime.

2. Neutrino Equation of Motion (EOM)

Since the neutrino is ubiquitous in nuclear reactions it would be useful in the theory of LENR to have an EOM for the neutrino which could be then be combined with the EOM for the electron, which is Dirac's equation, in a theory which is general enough to make some progress in explaining LENR observations. In their classic text Morse and Feshbach discuss the theorem that the scalar product of 4-vectors is always Lorentz-invariant [5]. Two examples are the Lorentz gauge equation, which is the scalar product of the 4-gradient and the electromagnetic 4-potential,

$$\left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right) \cdot (\Phi, \vec{A}) = \frac{1}{c}\frac{\partial \Phi}{\partial t} + \vec{\nabla} \cdot \vec{A} = 0, \tag{6}$$

and the equation of continuity, which is the scalar product of the 4-gradient and the 4-current,

$$\left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right) \cdot (c\rho, \vec{j}) = \frac{1}{c}\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0. \tag{7}$$

Proceeding heuristically Dirac's equation for a free electron can be inferred from a form of Lorentz gauge equation, in which the electromagnetic 4-potential is replaced by a generalized 4-potential $(\Psi, \vec{X})$ posited for the electron,

$$\left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right) \cdot (\Psi, \vec{X}) = \frac{1}{c}\frac{\partial \Psi}{\partial t} + \vec{\nabla} \cdot \vec{X} = 0. \tag{8}$$

The scalar and vector potentials can be written in the form of carrier-wave expansions for an assumed dominant frequency component, thusly,

$$\Psi = \chi(\vec{r}, t)\, \mathrm{e}^{-i\omega t} + \psi(\vec{r}, t)\, \mathrm{e}^{i\omega t}, \tag{9a}$$

$$\vec{\mathrm{X}} = \vec{\mathrm{X}}_{-}(\vec{r}, t)\, \mathrm{e}^{-i\omega t} + \vec{\mathrm{X}}_{+}(\vec{r}, t)\, \mathrm{e}^{i\omega t}. \tag{9b}$$

On substituting Eqs. (9a) and (9b) into Eq. (8) and separately setting the coefficients of the exponential factors equal to zero, we obtain

$$\left(i\hbar\frac{\partial}{\partial t} - mc^2\right) \psi(\vec{r}, t) + i\hbar c\vec{\sigma} \cdot \vec{\nabla}\chi\,(\vec{r}, t) = 0, \tag{10a}$$

$$\left(i\hbar\frac{\partial}{\partial t} + mc^2\right) \chi\,(\vec{r}, t) + i\hbar c\vec{\sigma} \cdot \vec{\nabla}\psi\,(\vec{r}, t) = 0, \tag{10b}$$

which are identically Dirac's pair of first-order equations for a free electron on setting

$$\omega = \frac{mc^2}{\hbar},$$

$$\vec{\mathrm{X}}_{+}(\vec{r}, t) = \vec{\sigma}\chi\,(\vec{r}, t),$$

$$\vec{\mathrm{X}}_{-}(\vec{r}, t) = \vec{\sigma}\psi\,(\vec{r}, t),$$

where $\vec{\sigma}$ is Pauli's vector and the wave functions are the Dirac two-component spinors. Dirac's own derivation, which flows from the tradition of matter as opposed to radiant physics [6], follows from his demands that a correct relativistic equation of motion (EOM) for the electron should satisfy the relationship between energy and momentum of special relativity ($E = \gamma mc^2$, for Lorentz factor

$$\gamma = \sqrt{1 + \frac{p^2}{m^2 c^2}}$$

subject to the quantization rules

$$E \to i\hbar \frac{\partial}{\partial t}$$

and $\vec{p} \to -i\hbar\vec{\nabla}$) and further should satisfy the equation of continuity given by Eq. (7). The latter demand is satisfied by Dirac's equation, giving a current,

$$\vec{j}(\vec{r}, t) = c[\psi^+(\vec{r}, t)\vec{\sigma}\chi(\vec{r}, t) + \chi^+(\vec{r}, t)\vec{\sigma}\psi(\vec{r}, t)], \quad (11)$$

where the superscripts denote Hermitian conjugates. Notice that the only identification of Eqs. (10) with the electron is in the mass term since Pauli's vector, originally identified with the electron by Pauli, can be generally identified with any spin-1/2 particle or fermion. This means that the Dirac equation with $m = 0$ can sensibly be considered to be the EOM for a neutrino, although again the free-particle EOM tells us nothing about the property of charge either for the electron or the neutrino.

The neutrino shares its zero-mass and charge neutrality with electro- magnetic radiation. It is therefore assumed that a 4-potential exists for the neutrino such that its EOM can be inferred from the Lorentz invariant found from the scalar product of an electromagnetic 4-momentum and the neutrino's posited 4-potential thusly,

$$\left(\frac{\hbar}{c}\frac{\partial}{\partial t}, \hbar\vec{\nabla} + \frac{e\hbar}{mc^2}\vec{E}, \vec{H}\right) \cdot (\Phi_\nu, \vec{A}_\nu) = \frac{\hbar}{c}\frac{\partial}{\partial t}\Phi_e + \left(\hbar\vec{\nabla} + \frac{e\hbar}{mc^2}\vec{E}, \vec{H}\right) \cdot \vec{A}_\nu = 0 \quad (12)$$

for either electric or magnetic fields $\vec{E}, \vec{H}$. The electromagnetic 4-momentum is found from $\hbar$ times a 4-gradient,

$$\left(\frac{\partial}{c\partial t}, \vec{\nabla} + \frac{e}{mc^2}\vec{E}, \vec{H}\right),$$

whose scalar product with the electromagnetic 4-current,

$$\left(c\left(u + \int_0^t \mathrm{d}t'\vec{j} \cdot \vec{E}\right), \vec{S}\right),$$

where

$$u = \frac{1}{8\pi}(\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B})$$

is the electromagnetic energy density and $\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{H}$ is the electromagnetic 3-current, gives the Lorentz-invariant electromagnetic continuity equation,

$$\frac{\partial u}{\partial t} + \vec{\nabla} \cdot \vec{S} + \vec{j} \cdot \vec{E} = 0. \quad (13)$$

This is simply the electromagnetic analog of writing the Lorentz-invariant material continuity equation given by Eq. (7) as the scalar product of the known 4-gradient,

$$(\frac{\partial}{c\partial t}, \vec{\nabla})$$

,
and the known material 4-current, $(c\rho, \vec{j})$. Notice that in the theory developed above the known 4-gradient is simply renormalized by the replacement

$$\vec{\nabla} \to \vec{\nabla} + \frac{e}{mc^2}\vec{E}, \vec{H},$$

which gives a Lorentz- invariant electromagnetic continuity equation since the scalar product of $\vec{E}$ or $\vec{H}$ with the electromagnetic 3-current, $\vec{S}$, vanishes. As with the electron the neutrino scalar and vector potentials can be written in the form of carrier-wave expansions for an assumed dominant frequency component, thusly,

$$\Phi_\nu = \Phi_{\nu_-}\mathrm{e}^{-i\omega_\nu t} + \Phi_{\nu_+}\mathrm{e}^{i\omega_\nu t}, \tag{14a}$$

$$\vec{A}_\nu = \vec{A}_{\nu-}\mathrm{e}^{-i\omega_\nu t} + \vec{A}_{\nu+}\mathrm{e}^{i\omega_\nu t}, \tag{14b}$$

from which on substituting Eqs. (14a) and (14b) into Eq. (12) and separately setting the coefficients of the exponential factors equal to zero, we obtain,

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + i\frac{\omega_\nu}{c}\right)\Phi_{\nu+} + \left(\vec{\nabla} + \frac{e}{mc^2}\vec{E}, \vec{H}\right)\cdot\vec{A}_{\nu+} = 0, \tag{15a}$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t} - i\frac{\omega_\nu}{c}\right)\Phi_{\nu-} + \left(\vec{\nabla} + \frac{e}{mc^2}\vec{E}, \vec{H}\right)\cdot\vec{A}_{\nu-} = 0. \tag{15b}$$

On setting $\Phi_{\nu+} = \xi_{E,H}$, $\vec{A}_{\nu+} = \vec{\sigma}_\nu\zeta_{E,H}$, $\Phi_{\nu-} = \zeta_{E,H}$, $\vec{A}_{\nu-} = \vec{\sigma}_\nu\xi_{E,H}$, where $\vec{\sigma}_\nu$ is Pauli's vector for the neutrino's spin, we obtain a Dirac form for the neutrino EOM,

$$\frac{\partial\xi_{E,H}}{c\partial t} + i\frac{\omega_\nu}{c}\xi_{E,H} + \vec{\sigma}_\nu\cdot\left(\vec{\nabla} + \frac{e}{mc^2}\vec{E}, \vec{H}\right)\zeta_{E,H} = 0, \tag{16a}$$

$$\frac{\partial\zeta_{E,H}}{c\partial t} - i\frac{\omega_\nu}{c}\zeta_{E,H} + \vec{\sigma}_\nu\cdot\left(\vec{\nabla} + \frac{e}{mc^2}\vec{E}, \vec{H}\right)\xi_{E,H} = 0 \tag{16b}$$

including the interaction of the neutrino with the electron. Writing $\xi_{E,H} = \mathrm{e}^{-i\omega t}\psi_{E,H}$ and $\zeta_{E,H} = \mathrm{e}^{-i\omega t}\chi_{E,H}$ in Eqs. (16a) and (16b) we derive stationary equations for$\psi_{E,H}$ and $\chi_{E,H}$; then we eliminate the equation for $\chi_{E,H}$ in favor of a second-order equation for $\psi_{E,H}$,obtaining equations for the neutrino wave functions which have the Helmholtz form

$$\left\{\nabla^2 + \frac{\omega^2 - \omega_\nu^2}{c^2} + \frac{e}{mc^2}\left[\vec{\nabla}\cdot\vec{E} + 2\vec{E}\cdot\vec{\nabla} + i\vec{\sigma}_\nu\cdot(\vec{\nabla}\times\vec{E}) + \frac{e}{mc^2}E^2\right]\right\}\psi_E = 0, \tag{17a}$$

$$\left\{\nabla^2 + \frac{\omega^2 - \omega_\nu^2}{c^2} + \frac{e}{mc^2}\left[\vec{\nabla}\cdot\vec{H} + 2\vec{H}\cdot\vec{\nabla} + i\vec{\sigma}_\nu\cdot(\vec{\nabla}\times\vec{H}) + \frac{e}{mc^2}H^2\right]\right\}\psi_H = 0, \tag{17b}$$

where we have used the identity, $(\vec{\sigma}_\nu \cdot \vec{A})\,(\vec{\sigma}_\nu \cdot \vec{B}) = \vec{A} \cdot \vec{B} + i\vec{\sigma}_\nu \cdot (\vec{A} \times \vec{B})$. On setting $\omega_\nu = 0$ the 0-mass, charge-neutral neutrino EOM is obtained. On setting $\vec{E} = \vec{H} = 0$ Dirac's 0-mass equation is recovered. The neutrino EOM given by Eq. (17b) has also been interpreted as the EOM for the radiant aspect of the electron, for which it has been used to calculate a divergence-free Lamb shift [11] and the electron's anomalous magnetic moment [12]. The material aspect of the electron is of course described by Dirac's equation, whose physical interpretation regarding the phenomena of matter versus antimatter states and of Zitterbewegung are discussed at length in previous work [13–15].

Notice that the proposed neutrino EOM agrees with observation that (a) the neutrino is a spin-1/2 particle, (b) the neutrino has zero mass (or for

$$\omega_\nu = \frac{m_\nu c^2}{\hbar} > 0$$

a finite mass m_ν to be determined from experiment), and (c) the neutrino interacts with the electron in a scaled electromagnetic interaction which is weaker than the electromagnetic interaction between charged particles such that it is sensible to investigate if the neutrino-matter interaction terms in Eqs. (17a) and (17b) arise from fundamental electroweak forces.

Finally, in order to highlight the structural similarity of the neutrino and electron EOM's, Dirac's equation for an electron in the presence of electro-magnetic fields follows if the 4-gradient in Eq. (8) for $(\Psi, \vec{\mathrm{X}})$ is replaced by the electron's 4-momentum as follows

$$(\gamma mc, -\gamma m\vec{\upsilon}) = \left(i\hbar\frac{\partial}{c\partial t} - \frac{e}{c}\Phi,\, i\hbar\vec{\nabla} + \frac{e}{c}\vec{A} \right), \tag{18}$$

where γ is the Lorentz factor, and the scalar and vector components of the 4-momentum on the left-hand side of Eq. (18) have been replaced by substitution using the classical relativistic expressions for the energy and canonical momentum, $\vec{P}$,

$$E = \gamma mc^2 + e\Phi, \tag{19a}$$

$$\gamma m\vec{\upsilon} = \vec{p} = \vec{P} - \frac{e}{c}\vec{A}, \tag{19b}$$

where the quantized forms of energy and the kinetic momentum have been used,

$$E \to i\hbar\frac{\partial}{\partial t}, \tag{20a}$$

$$\vec{P} \to -i\hbar\vec{\nabla}. \tag{20b}$$

The scalar product of the electron's 4-momentum and the 4-potential posited for the electron is,

$$\left(i\hbar\frac{\partial}{c\partial t} - \frac{e}{c}\Phi,\, i\hbar\vec{\nabla} + \frac{e}{c}\vec{A} \right) \cdot (\Psi, \vec{\mathrm{X}}) = \left(i\hbar\frac{\partial}{c\partial t} - \frac{e}{c}\Phi \right)\Psi + \left(i\hbar\vec{\nabla} + \frac{e}{c}\vec{A} \right) \cdot \vec{\mathrm{X}} = 0. \tag{21}$$

Using the same carrier-wave expansions for $(\Psi, \vec{\mathrm{X}})$ as we used in Eqs. (9) for a free electron we obtain,

$$\left(i\hbar\frac{\partial}{\partial t} - e\Phi - mc^2 \right) \psi(\vec{r}, t) + \vec{\sigma} \cdot (i\hbar c\vec{\nabla} + e\vec{A})\chi\,(\vec{r}, t) = 0, \tag{22a}$$

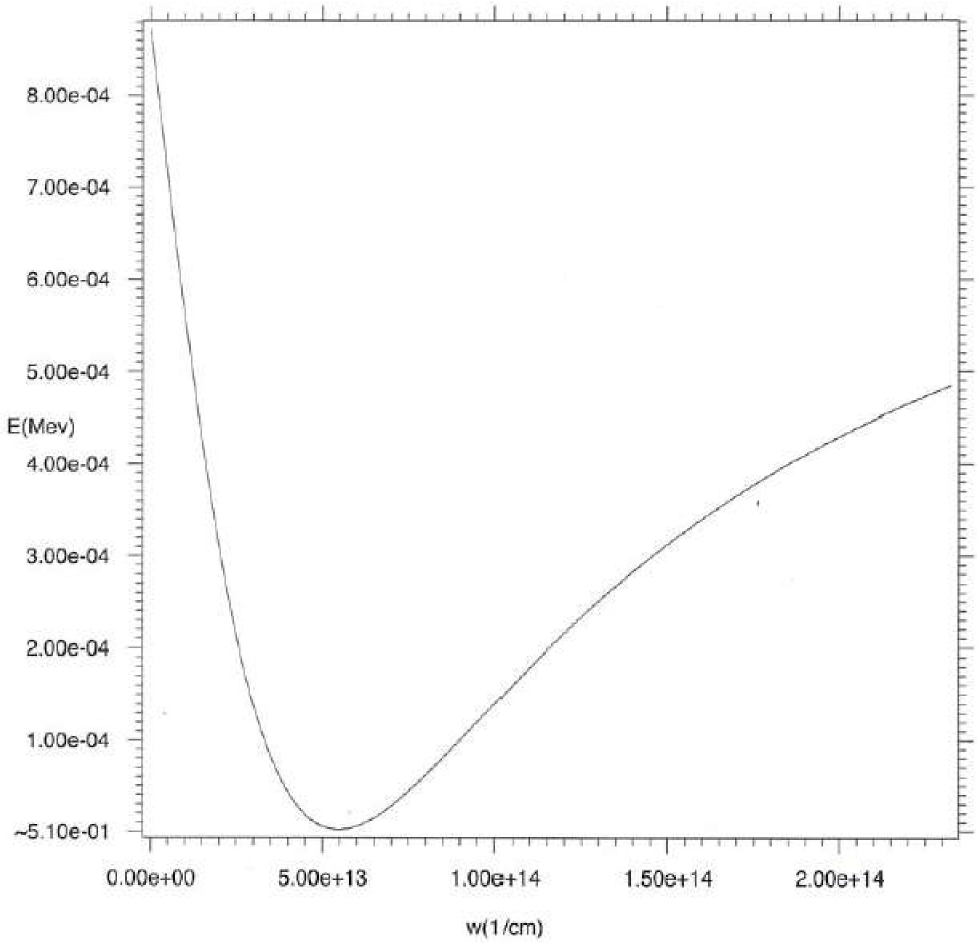

Figure 1. Electron energy versus variational parameter. The ordinate numbers above the origin are the energies in MeV to be added to 0.510 MeV at the origin. The binding energy is the energy difference from 0.511 MeV (positive-energy threshold) to the minimum of the well at 0.510 MeV or about 1 keV.

$$\left(i\hbar\frac{\partial}{\partial t} - e\Phi + mc^2\right)\chi(\vec{r},t) + \vec{\sigma}\cdot(i\hbar c\vec{\nabla} + e\vec{A})\,\psi(\vec{r},t) = 0, \tag{22b}$$

which is identically Dirac's equation on setting $\hbar\omega = mc^2$, $\vec{\mathrm{X}}_+(\vec{r},t) = \vec{\sigma}\chi(\vec{r},t)$, and $\vec{\mathrm{X}}_-(\vec{r},t) = \vec{\sigma}\psi(\vec{r},t)$. Notice that Eqs. (22a) and (22b) follow from Eq. (8) on renormalizing the 4-gradient as follows

$$\frac{1}{c}\frac{\partial}{\partial t} \to \frac{1}{c}\frac{\partial}{\partial t} - \frac{e}{i\hbar c}\Phi \qquad \text{and} \qquad \vec{\nabla} \to \vec{\nabla} + \frac{e}{i\hbar c}\vec{A}.$$

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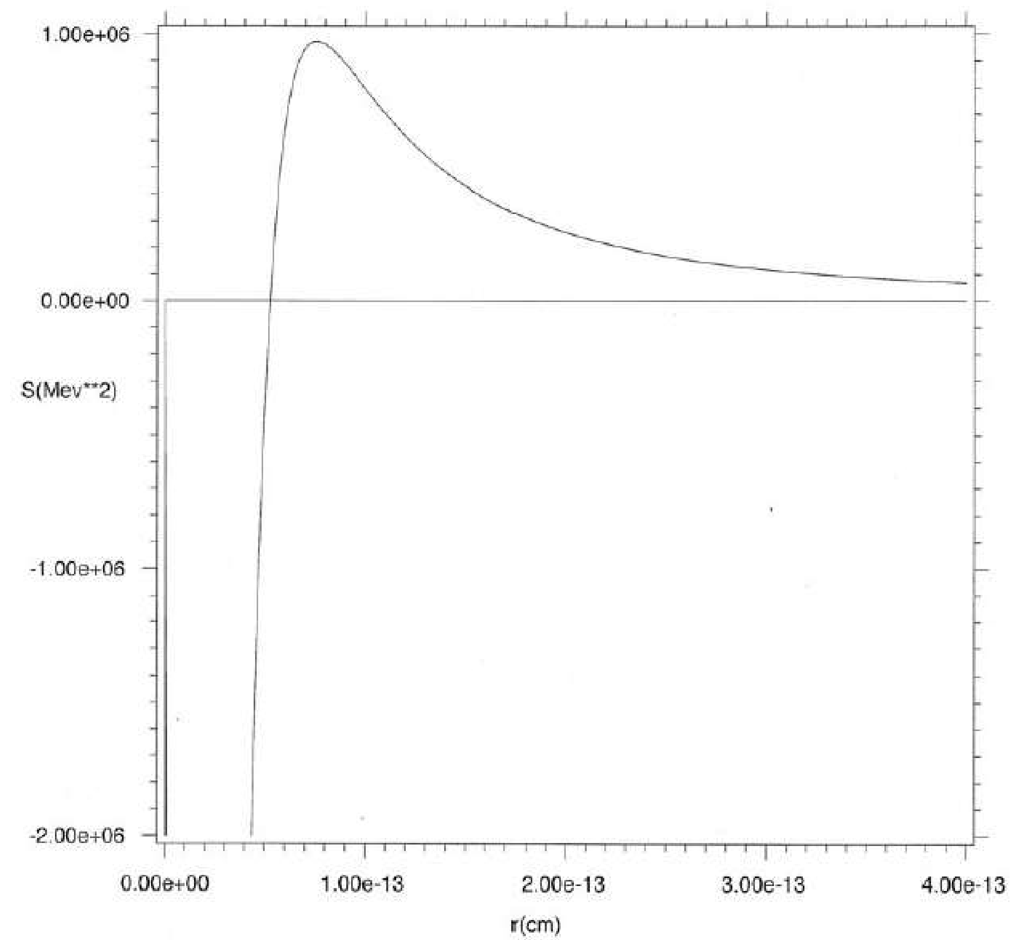

Figure 2. The "potential" function $S(r)$ in Eq. (28a) versus r. The rate of tunneling through the barrier is roughly equal to the reciprocal of the lifetime of the neutron ($\approx$ 15 min) for an energy behind the barrier $E_\nu = 1.7$ MeV.

3. Mutual Binding of the Neutrino and Electron

The EOM's for the neutrino and electron, as discussed in the previous section, are given respectively by Eq. (17a) and by the second-order form of Dirac's equation (Eqs. (22a) and (22b)) for $\Phi = 0$ as follows,

$$\text{neutrino}: \quad E_\nu^2 \psi_E + \hbar^2 c^2 \left\{ \nabla^2 + \frac{e}{mc^2} \left[\vec{\nabla} \cdot \vec{E} + 2\vec{E} \cdot \vec{\nabla} + \frac{e}{mc^2} E^2 \right] \right\} \psi_E = 0; \tag{23a}$$

$$\text{electron}: \quad (E_e^2 - m^2 c^4)\psi + \hbar^2 c^2 \left\{ \nabla^2 - \frac{e}{\hbar c} \left[i\vec{\nabla} \cdot \vec{A} + 2i\vec{A} \cdot \vec{\nabla} + \frac{e}{\hbar c} A^2 - \vec{\sigma} \cdot \vec{H} \right] \right\} \psi = 0, \tag{23b}$$

where in Eq. (17a) $\omega_\nu = m_\nu c^2/\hbar = 0$ (neutrino assumed to have zero mass), $\omega = E_\nu/\hbar$, and only electrostatic interaction terms have been retained in both equations. Eq. (23b) is essentially Pauli's equation, which is Dirac's equation for $\Phi = 0$ and for $E_\mathrm{e}^2 \cong m^2c^4 + 2mc^2E_\mathrm{nr}$, where E_nr is an energy in the nonrelativistic regime. Notice that the interaction terms in the neutrino EOM are weaker by a factor of c^{-1} than the electromagnetic interactions of the electron EOM, such that they may possibly be associated with the electroweak interaction. This difference occurs due to the scaling of $e\vec{E}$ as a force (energy divided by length) in the neutrino EOM rather than to the scaling of $e\vec{A}$ as an energy in the electron EOM. The interaction of the neutrino with the electron is given by the electric field arising from the charge density of the electron (or positron), $e\vec{E} = -\vec{\nabla}V$, where

$$V = e^2\left[\frac{1}{r}\int_0^r \mathrm{d}r' r'^2(G_{-1}^2 + F_{-1}^2) + \int_r^\infty \mathrm{d}r' r'(G_{-1}^2 + F_{-1}^2)\right], \tag{24}$$

where the electronic density is that inferred from Dirac's equation using the large and small components of Dirac's wave function for the electron $\psi = G_\kappa(r)\chi_{\kappa\mu}(\theta,\phi)$ and $\chi = iF_\kappa(r)\chi_{-\kappa\mu}(\theta,\phi)$. The interaction of the electron with the neutrino is given solely by the A^2 interaction and the Pauli interaction $\vec{\sigma}\cdot\vec{H}$ since $\vec{\nabla}\cdot\vec{A} = 0$, which is demonstrated below, which means that the $\vec{A}\cdot\vec{\nabla}$ term also gives a zero contribution by parts integration of its expectation value. The A^2contribution is found to be negligible compared to the Pauli contribution. The magnetic field interaction of the electron with the neutrino is calculated from Maxwell's equation,

$$\nabla^2\vec{A} = -\frac{4\pi e}{c}\vec{j}_\nu, , \tag{25}$$

where the current arises from the permanent magnetic moment due to the neutrino's spin, $\vec{j}_\nu = c(\xi_E^+\sigma_\nu\varsigma_E + \varsigma_E^+\sigma_\nu\xi_E)$, which follows from Eqs. (16a) and (16b) due to the Dirac form of the neutrino EOM, such that the spinor analysis for the neutrino is the same as that for the electron, for which $\xi_E = g_\kappa(r)\chi_{\kappa\mu}(\theta,\phi)$ and $\varsigma_E = if_\kappa(r)\chi_{-\kappa\mu}(\theta,\phi)$. The radial parts for the electron are of course different and are given by $G_{-1}(r)$ and $F_{-1}(r)$ for the large and small components, respectively. The cartesian components of the current for $\kappa = -1$ are

$$j_{\nu x} = \frac{c}{2\pi}\hat{y}R, \quad j_{\nu y} = -\frac{c}{2\pi}\hat{x}R, \quad \text{and} \quad j_{\nu z} = 0,$$

where $R = g_{-1}(r)f_{-1}(r)$. Finding the divergence of both sides of Eq. (25) the reader may easily verify that $\vec{\nabla}\cdot\vec{A} = 0$ due to the transverse nature of the current. The magnetic field is found by taking the curl of both sides of Eq. (25),

$$\nabla^2\vec{H} = -\frac{4\pi e}{c}\vec{\nabla}\times\vec{j}_\nu. \tag{26}$$

Only the diagonal or z-component of $\vec{H}$ is considered here; the z-component of the curl of the current is given by

$$\left(\vec{\nabla}\times\vec{j}_\nu\right)_z = \frac{c}{2\pi}\left[2\frac{R}{r} + \left(R' - \frac{R}{r}\right)\sin^2\theta\right],$$

where the prime denotes derivative with respect to r. Solving Eq. (26),

$$H_z = \frac{4}{3}e\left[\frac{1}{r}\int_0^r \mathrm{d}r' r'^2\left(R' + 2\frac{R}{r'}\right) + \int_r^\infty \mathrm{d}r' r'\left(R' + 2\frac{R}{r'}\right)\right]. \tag{27}$$

Proceeding heuristically the radial equations (for $\kappa = -1$) are solved variationally using the unnormalized trial forms $G_{-1}(r) = \mathrm{e}^{-wr}$ and $g_{-1}(r) = \mathrm{e}^{-w'r}$ for the "large" components, where $w = 1.15w'$. The "small" components are calculated using the trial forms,

$$f_{-1}(r) = \frac{\hbar c}{m_p c^2} g_{-1}^{'} \quad \text{and} \quad F_{-1}(r) = \frac{\hbar c}{mc^2} G_{-1}^{'}$$

for the neutrino and electron respectively, where m_p is the proton mass, which is the only empirical parameter in the calculation. The electron energy versus the variational parameter w is shown in Fig. 1, in which the minimum energy lies below 0.511 MeV indicating binding to the neutrino with binding energy of about 1 keV. Notice that the minimum energy occurs for w approximately equal to the reciprocal of the proton Compton wavelength, $w \cong m_p c/\hbar$, which is consistent with our choice in the denominator of the variational form for $f_{-1}(r)$ given above. Indeed the scaling of

$$f_{-1}(r) \quad \text{as} \quad f_{-1}(r) = -\frac{\hbar c w'}{m_p c^2} g_{-1}(r) \cong -g_{-1}(r),$$

in which the large denominator $m_p c^2$ is cancelled by the numerator $\hbar c w$ near the minimum of the electron energy versus w (Fig. 1), is consistent with the inverse relationship of particle range and particle mass in particle theory. Once the derivative operation has been carried out on the trial function Eq. (23a) has the standard Schroedinger form,

$$\left[E_\nu^2 + \hbar^2 c^2 \nabla^2 - S(r)\right] \psi_E = 0, \tag{28a}$$

$$S(r) = -\frac{e^2 \hbar^2}{m} \left(\rho_r + \frac{V'}{e^2} w' + \frac{1}{mc^2 e^2} V'^2\right), \tag{28b}$$

where $S(r)$ is plotted versus r in Fig. 2. The first and third terms on the right-hand side of Eq. (28b) are attractive while the second term is repulsive such that a neutrino can bind to an electron or positron behind the potential barrier shown in Fig. 2. Standard WKB theory [16] can be used to estimate the energy levels behind the barrier and the tunneling rate through the barrier, whence

$$I = \int_{r_1}^{r_2} \mathrm{d}r \sqrt{\kappa_\nu^2 - \frac{S(r)}{\hbar^2 c^2}} = \left(n + \frac{1}{2}\right)\pi, \tag{29}$$

where $\kappa_\nu = E_\nu/\hbar c$ and the limits of integration are over the classical region in which the argument under the square-root sign is positive, and

$$R = \frac{\hbar}{m} \frac{\mathrm{e}^{-2\int_{r_2}^{r_3} \mathrm{d}r \sqrt{\frac{S(r)}{\hbar^2 c^2} - \kappa_\nu^2}}}{4\int_{r_1}^{r_2} \mathrm{d}r \left[\kappa_\nu^2 - \frac{S(r)}{\hbar^2 c^2}\right]^{-1/2}} \ \mathrm{s}^{-1}, \tag{30}$$

where the integration limits in the exponential term are over the barrier width For the value of w giving the minimum electron energy in Fig. 1 ($w \cong m_p c/\hbar$) an energy E_ν is chosen for which the reciprocal of the tunneling rate is nearly equal to the lifetime of the neutron. The calculated rate is 8.41×10^{-4} s^{-1}, whose reciprocal gives a lifetime of 1.19×10^3, which is close to the observed lifetime of nearly fifteen minutes. In Eq. (29) this rate corresponds to 1.7 MeV and a principal quantum number close to $n = 3$. The rate would of course be slower for smaller n and faster for larger n. In atomic units the numerator in Eq. (30) is 1.52×10^{-30} and the integral in the denominator is 1.87×10^{-11}, such that the quotient is 2.34×10^{-20}, which, converted to CGS units by division by the atomic unit of time, 2.42×10^{-17}, gives the the rate cited above.

4. Conclusions

An EOM has been presented for the neutrino which contains an interaction with charged matter. For zero interaction with matter the EOM reduces to the zero-mass Dirac equation, which is the generally accepted EOM for the neutrino. I believe that the postulated bosons, which we denote by the symbol $W_s^{\pm}$ for binding of a neutrino and a positron (+) or of a neutrino and an electron ($-$) and for spin s equal to 0 or 1, since they discover the mass, length, and lifetime scales of a nucleon, should be useful in future work as building blocks to construct a nucleon with constituents physically equivalent to quarks and intermediate force-carrier bosons.

Acknowledgements

The author is grateful to T. Scott Carman for supporting this work. He is grateful to Professor John Knoblock of the University of Miami and to Dr. Ray Garrett of the University of Tennessee for seminal discussion. This work was performed under the auspices of the Lawrence Livermore National Security, LLC, (LLNS) under Contract No. DE-AC52-07NA27344.

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