

Research Article

# On the Heat Transfer in LENR Experiments

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## Abstract

Thermal conduction is considered in deuterated palladium. We assume that the energy released in a single LENR event thermalizes in a region that has a typical dimension on the order of a hundred nanometers. It is shown that thermal conductivity enables the heat transfer of the energy released in repeated events without causing the lattice to melt. Consequently, continuous power is possible. It is argued that power of close to one watt can arise from a single nuclear active environment, NAE. Both the experimental and theoretical consequences of the results are discussed.

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## 1. Introduction

In the LENR research there are two important – and unsolved – problems. Firstly, what is the mechanism which allows detectable energy production even though the interparticle distances between hydrogen nuclei remain much larger compared to the nuclear distances. Secondly, how is the nuclear energy, that is released, transferred to the heat without penetrating radiation. Several different approaches have been proposed to solve either or both of these questions [1–8]. However, the discussion on these problems have shown practically no convergence over the years. The total lack of agreement in these principal questions forces us to skip these principal questions here and to proceed further on more general grounds, in order to extract some useful information anyway. Therefore, in this article we will consider the issue, how the heat is transferred away after the thermalisation has taken place.

It is possible that initially the energy released in a fusion reaction is in the form of the energies of the initial energy carriers which can be phonons, photons, electrons or hydrogen nuclei (protons or deuterons), depending on the theoretical model. In the first stage these energy carriers may have a non-thermal energy distribution. Due to collisions the thermalisation of the hydrogen loaded metal will be reached. We shall assume here that this thermalisation takes place in a submicron region, of order hundred nanometers, large enough that no melting of the lattice will occur.

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## 2. Heat transfer

Considering how the heat is transferred, we need to use the thermal diffusion equation [9]

$$\frac{\partial T(\mathbf{x}, t)}{\partial t} = D \nabla^2 T(\mathbf{x}, t), \quad (1)$$

where the thermal diffusion constant  $D$  is defined by the thermal conductivity  $\kappa$  and the specific heat per unit volume

$$D = \frac{\kappa}{C/V} = \frac{\kappa}{c_p \rho}. \quad (2)$$

Here  $c_p$  is specific heat per unit mass and  $\rho$  is the mass density.

Assume here that the temperature is large compared to the Debye temperature, which is clearly true as we are dealing with temperatures corresponding to the room temperature or above it. The specific heat of the metal increases with hydrogen loading due to the additional vibration modes corresponding to the hydrogen atoms. However, the same is also true for the thermal conductivity. The thermal conductivity is proportional to the number of free electrons in metal [10]. This increases with loading due to the additional free electrons associated with the loaded hydrogen atoms. Therefore we can use, as the first approximation, the known values of the thermal conductivity and the specific heat of the host metal to calculate the thermal diffusion constant. To be specific, we shall here consider (fully loaded) palladium metal. For palladium we get the value of the thermal diffusion constant  $D = 2.5 \times 10^{-5} \text{ m}^2/\text{s}$ .

In the standard solution of the diffusion equation (1) the thermal distribution at some later time  $t_0$ ,  $T(\mathbf{r}, t_0)$ , can be expressed by the initial temperature distribution,  $T(\mathbf{x}, 0)$

$$T(\mathbf{r}, t_0) = \frac{1}{(4\pi Dt_0)^{3/2}} \int d^3\mathbf{x} e^{-(\mathbf{r}-\mathbf{x})^2/4Dt_0} T(\mathbf{x}, 0). \quad (3)$$

In the case of the spherical symmetry this reads

$$T(r, t_0) = \frac{1}{r\sqrt{4\pi Dt_0}} \int_0^\infty dx x \left[ e^{-(r-x)^2/4Dt_0} - e^{-(r+x)^2/4Dt_0} \right] T(x, 0). \quad (4)$$

Now, assume that the initial state is ambient temperature  $T_0$  plus (due to a fusion reaction) a temperature increase by an amount of  $\Delta T$  inside a sphere of radius  $r_0$ :

$$T(x, 0) = T_0 + \Delta T \theta(r_0 - x). \quad (5)$$

Then the temperature distribution becomes

$$T(r, t_0) = \frac{1}{r} \int_0^\infty dx x f(r, x) [T_0 + \Delta T \theta(r_0 - x)] = T_0 + \frac{\Delta T}{r} \int_0^\infty dx x f(r, x) \theta(r_0 - x), \quad (6)$$

where a shorthand notation has been used:

$$f(r, x) = \frac{1}{\sqrt{4\pi Dt_0}} \left[ e^{-(r-x)^2/4Dt_0} - e^{-(r+x)^2/4Dt_0} \right] \quad (7)$$

Assume now that the next fusion reaction takes place at the same sphere at the time  $t_0$ , leading to another temperature increase  $\Delta T$  in the sphere. The temperature distribution at the time  $2t_0$  can be written then

$$\begin{aligned} T(r, 2t_0) &= \frac{1}{r} \int_0^\infty dx_2 x_2 f(r, x_2) [T(x_2, t_0)] \\ &= \frac{1}{r} \int_0^\infty dx_2 x_2 f(r, x_2) \left[ T_0 + \frac{\Delta T}{x_2} \int_0^\infty dx_1 x_1 f(x_2, x_1) \theta(r_0 - x_1) + \Delta T \theta(r_0 - x_2) \right]. \end{aligned} \quad (8)$$

By renaming the integration variable  $x_2$  to  $x_1$  in the last term this can be written

$$T(r, 2t_0) = T_0 + \frac{\Delta T}{r} \left\{ \int_0^\infty dx_1 dx_2 f(r, x_2) f(x_2, x_1) x_1 \theta(r_0 - x_1) + \int_0^\infty dx_1 f(r, x_1) x_1 \theta(r_0 - x_1) \right\}. \quad (9)$$

Assuming several consequent fusion reactions, so that the temperature inside the sphere of the radius  $r_0$  increases by the amount  $\Delta T$  after every time interval  $t_0$ . Then the above solution can be iterated, leading to the thermal distribution

$$\begin{aligned} T(r, nt_0) &= T_0 + \Delta T \left\{ \frac{1}{r} \left[ \int_0^\infty dx_1 \cdots dx_n f(r, x_n) f(x_n, x_{n-1}) \cdots f(x_2, x_1) x_1 \theta(r_0 - x_1) \right. \right. \\ &\quad \left. \left. + \cdots + \int_0^\infty dx_1 f(r, x_1) x_1 \theta(r_0 - x_1) \right] + \theta(r_0 - r) \right\} \end{aligned} \quad (10)$$

where the temperature increase at the time  $t = nt_0$  has been also included. Every term in this sum (10) represents a separate fusion reaction; the first term corresponding to the reaction occurred at zero time, and so on, the last one corresponding to the last reaction at the time  $t = nt_0$ .

The temperature has its largest value in the center. Noting that

$$\lim_{r \rightarrow 0} \frac{f(r, x)}{r} = \frac{x}{Dt_0} \frac{e^{-\frac{x^2}{4Dt_0}}}{\sqrt{4\pi Dt_0}}, \quad (11)$$

we get

$$\begin{aligned} T(0, nt_0) &= T_0 + \Delta T \left\{ \int_0^\infty dx_1 \cdots dx_n \frac{x_n}{Dt_0} \frac{e^{-\frac{x_n^2}{4Dt_0}}}{\sqrt{4\pi Dt_0}} f(x_n, x_{n-1}) \cdots f(x_2, x_1) x_1 \theta(r_0 - x_1) \right. \\ &\quad \left. + \cdots + \int_0^\infty dx_1 \frac{x_1}{Dt_0} \frac{e^{-\frac{x_1^2}{4Dt_0}}}{\sqrt{4\pi Dt_0}} x_1 \theta(r_0 - x_1) + 1 \right\}. \end{aligned} \quad (12)$$

By using the dimensionless variables

$$z_i = \frac{x_i}{\sqrt{4\pi Dt_0}}, \quad (13)$$

this can be written

$$T(0, nt_0) = T_0 + \Delta T \left\{ 4 \int_0^\infty \frac{dz_1 \cdots dz_n}{\pi^{n/2}} z_n e^{-z_n^2} g(z_n, z_{n-1}) \cdots g(z_2, z_1) z_1 \theta \left( \frac{r_0}{\sqrt{4\pi Dt_0}} - z_1 \right) + \cdots + 4 \int_0^\infty \frac{dz_1}{\pi^{1/2}} z_1 e^{-z_1^2} z_1 \theta \left( \frac{r_0}{\sqrt{4\pi Dt_0}} - z_1 \right) + 1 \right\} \quad (14)$$

with

$$g(z_n, z_{n-1}) = e^{-(z_n - z_{n-1})^2} - e^{-(z_n + z_{n-1})^2}. \quad (15)$$

Now, consider the integral

$$I = \frac{1}{k^{3/2} \sqrt{\pi}} \int_0^\infty dz z e^{-z^2/k} \left( e^{-(z-w)^2} - e^{-(z+w)^2} \right). \quad (16)$$

This integral can be written

$$I = \frac{1}{k^{3/2} \sqrt{\pi}} \int_0^\infty dz z \left( e^{-\frac{k+1}{k}(z - \frac{k}{k+1}w)^2} - e^{-\frac{k+1}{k}(z + \frac{k}{k+1}w)^2} \right) e^{-w^2/(k+1)}. \quad (17)$$

Substituting in the latter integral  $z \rightarrow -z$ , we obtain

$$I = \frac{e^{-w^2/(k+1)}}{k^{3/2} \sqrt{\pi}} \int_{-\infty}^\infty dz z e^{-\frac{k+1}{k}(z - \frac{k}{k+1}w)^2}. \quad (18)$$

This can be evaluated analytically [11] giving

$$I = \frac{w e^{-w^2/(k+1)}}{(k+1)^{3/2}}. \quad (19)$$

Comparing this to Eq. (16) we find that in Eq. (14) all the multiple integrals can be evaluated analytically one by one (except the last one) the integer  $k$  just increasing by one in every integration, leading to the sum

$$T(0, \infty) = T_0 + \Delta T \left\{ 1 + \frac{4}{\sqrt{\pi}} \sum_{k=1}^\infty \frac{1}{k^{3/2}} \int_0^\infty dx x^2 e^{-\frac{x^2}{k}} \theta \left( \frac{r_0}{\sqrt{4\pi Dt_0}} - x \right) \right\} \quad (20)$$

or

$$T(0, \infty) = T_0 + \Delta T \left\{ 1 + \frac{4}{\sqrt{\pi}} \sum_{k=1}^\infty \int_0^{\frac{r_0}{\sqrt{4\pi Dt_0 k}}} dx x^2 e^{-x^2} \right\}. \quad (21)$$

In these equations a continuous power is assumed by letting  $n \rightarrow \infty$ . For numerical calculations this sum converges relatively fast. Even faster convergence can be obtained if the exponential function is expanded as a Taylor expansion and the integrations are performed. We get then

$$T(0, \infty) = T_0 + \Delta T \left\{ 1 + \frac{4}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n \xi\left(\frac{2n+3}{2}\right)}{(2n+3) n!} \left(\frac{r_0}{\sqrt{4\pi D t_0}}\right)^{2n+3} \right\}, \quad (22)$$

where the Riemann zeta function is

$$\xi(z) = \sum_{k=1}^{\infty} \frac{1}{k^z}. \quad (23)$$

The temperature increase  $\Delta T$  depends on energy released  $Q$  and the number of the host metal atoms in the volume of the sphere inside which the energy thermalizes.

$$\Delta T = \frac{Q}{C} = \frac{Q}{6Nk} = \frac{Q}{8\pi r_0^3 \rho_n k}, \quad (24)$$

where  $\rho_n$  is the particle density of the host metal atoms. In Eq. (24) we have used the Dulong–Petit equation to express the heat capacity:

$$C = 6 N k, \quad (25)$$

where  $k$  is the Boltzmann constant and  $N$  is the number of Pd atoms in the  $r_0$ -sphere

$$N = \frac{4\pi}{3} r_0^3 \rho_n. \quad (26)$$

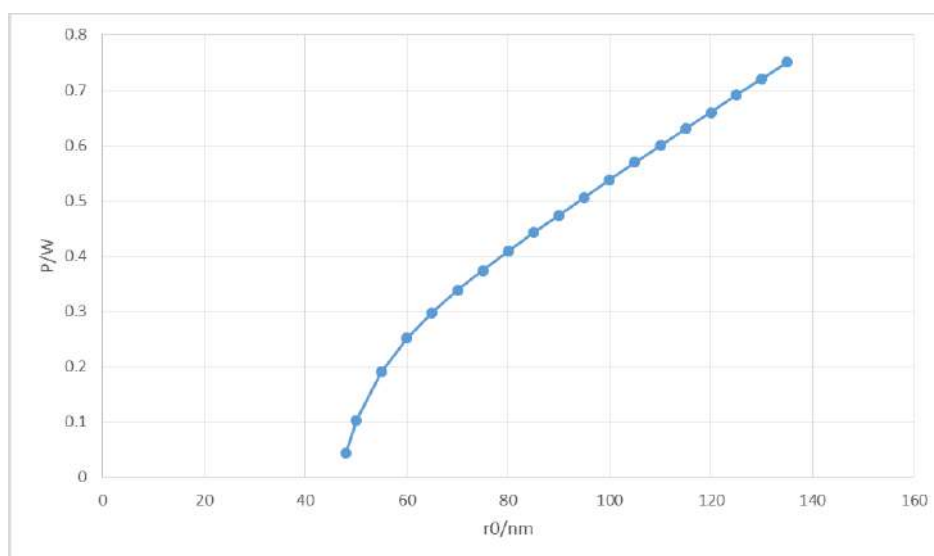
The extra factor 2 in Eq. (25) arises from the doubled vibration degrees of freedom in the fully loaded palladium metal, D/Pd  $\approx 1$ . In Eq. (25) the contribution from the degenerated electron gas is neglected as a small change, at most on the order of a few per cent. The time interval between the consequent fusion reactions can be written by the excess power  $P$

$$t_0 = \frac{Q}{P}. \quad (27)$$

Now, demanding that the central temperature will remain below the melting temperature, we get an equation relating the maximum excess power  $P_{\max}$  to the thermalisation sphere radius  $r_0$ :

$$T_m > T(0, \infty) = T_0 + \frac{Q}{8\pi r_0^3 \rho_n k} \left\{ 1 + \frac{4}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n \xi\left(\frac{2n+3}{2}\right)}{(2n+3) n!} \left(\frac{r_0}{\sqrt{4\pi D Q/P_{\max}}}\right)^{2n+3} \right\}. \quad (28)$$

We assume here the deuterium-deuterium to He-4 fusion, so that  $Q = 23.85$  MeV.  $T_0$  is assumed to be room temperature,  $+20^\circ\text{C}$ . Inserting also the values of the thermal diffusion constant and the particle density of palladium, we can calculate numerically the maximum power that can arise from a single spot while still leaving the central temperature below the melting point. This maximum power is shown in Fig. 1. as a function of the radius  $r_0$ .



**Figure 1.** Maximum power as a function of the radius.

### 3. Discussion

As we can see in Fig. 1, the thermal conductivity of loaded Pd-metal is large enough to allow continuous excess power to exist up to the order of, or close to, one watt, provided that the thermalization takes place in submicron range (on the order 100 nm). Or, to put it other way around, it is possible to have continuous excess power on the order of one watt originating from a single spot of about 100 nm. It should be noted, however, that the exact relation between  $P_{\text{max}}$  and  $r_0$  depends on the specific assumptions about the power production (i.e. does it originate as discrete events, as assumed here, or for example as constant power).

In roughly half of the electrochemical experiments in LENR research the excess power reported has been on the order of one watt. So, it is possible that in those experiments the excess power has originated from a single spot (or a single NAE [12]). Or if not just from a single spot - then from a very small number of spots. This notion can be regarded as compatible with the fact that LENR is a very rare phenomenon. It is possible that in the evolution of the surface region during electrolysis, the critical properties of NAE do not appear anywhere, leading to a null result. Sometimes, the requirements of NAE are met in only a single place (or in very few places) leading to excess power of 1 W or less. And only occasionally will several NAE emerge, leading to larger values of excess power.

The results of the above calculation also suggests that surface studies may have rather limited value. In surface studies, typically only the average values are considered. Now, if the critical requirements of NAE are met only in a single spot on the order 100 nm, they will remain hidden, because we cannot have control over the whole cathode surface within 100 nm accuracy.

As a final remark, we can note that the results shown here may be used to restrict possible theoretical models by their ability to reproduce the behavior found here. Especially the results may be used to restrict the possible energy carriers in the first stage.

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