J. Condensed Matter Nucl. Sci. 3 (2010) 31–34

Research Article

Simple Parameterizations of the Deuteron–Deuteron Fusion Cross Sections

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Abstract

Simple parameterizations of the deuteron–deuteron fusion cross sections are given in a form suitable for numerical calculations.

Keywords: dd-Fusion cross section, Fitting functions for cross section, Nuclear fusion, Numerical fits

1. Introduction

We have recently been pursuing calculations of neutron yields due to the presence of fast alphas and deuterons in PdD and D_2O, in order to better understand the relative absence of neutrons in excess heat experiments. In the course of our calculations, we needed a fit for the deuteron–deuteron fusion cross section for neutron production. The cross sections are available online in the Los Alamos ENDFB-VI nuclear data compilation. A table of them can be downloaded [1]. However, for numerical work it is useful to have a simple fitted version. After searching for a while in the literature, it seemed worthwhile to develop fits that were useful for our purposes. Perhaps these may also be useful for others as well.

We note that Li and coworkers have put forth a simple physics-based model for the deuteron–deuteron fusion cross section [2], which from our perspective represents a major step forward. The parameterizations that we present here are not intended to be physics-based fits; instead, we were interested in developing a more standard numerical type of fit.

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2. Parameterization of the cross section

At low energy, the asymptotic dependence of the fusion cross section in the absence of anomalous screening can be found in terms of a constant astrophysical S-factor S_0 according to

$$\sigma(E_\mathrm{r}) = \frac{S_0}{E_\mathrm{r}} \exp\left\{-\sqrt{\frac{E_\mathrm{G}}{E_\mathrm{r}}}\right\}, \tag{1}$$

where we have indicated the relative energy as E_r. The relative energy for the deuteron–deuteron reaction is half of the incident deuteron energy E in the lab frame. The Gamow energy is

$$E_\mathrm{G} = \left(\frac{\pi}{\alpha}\right)^2 M_D c^2 = 985.8 \text{ keV}. \tag{2}$$

To develop a parameterization, we follow the approach of Erba [3], which is based on the energy-dependent astrophysical S-factor $S(E)$

$$S(E_\mathrm{r}) = \sigma(E_\mathrm{r}) E_\mathrm{r} \exp\left\{\sqrt{\frac{E_\mathrm{G}}{E_\mathrm{r}}}\right\}. \tag{3}$$

It is much easier to fit this function than the cross section.

2.1. Fitting function

To proceed, we adopt a fitting function of the form

$$\ln S(E_\mathrm{r}) = \left\{\frac{a + bE_\mathrm{r} + cE_\mathrm{r}^2}{1 + \mathrm{d}E_\mathrm{r}}\right\}. \tag{4}$$

2.2. Fitting parameters for the ^{2}H(^{2}H,n)^{3}He reaction

In the case of the ^{2}H(^{2}H,n)^{3}He reaction, we have obtained fitting parameters given by

$$a = 10.8947, \qquad b = 34.5732 \text{ MeV}^{-1},$$
$$c = 0.254611 \text{ MeV}^{-2}, \qquad d = 2.6419 \text{ MeV}^{-1}. \tag{5}$$

The $S(E)$ function derived from the ENDFB-VI data set is shown along with the fitted version of the function in Fig. 1. The units of S associated with the fit are barns eV.

2.3. Fitting parameters for the ^{2}H(^{2}H,p)^{3}H reaction

For the ^{2}H(^{2}H,p)^{3}H reaction, the fitting parameters that we found are

$$a = 10.921, \qquad b = 22.5417 \text{ MeV}^{-1},$$
$$c = 0.150056 \text{ MeV}^{-2}, \qquad d = 1.71503 \text{ MeV}^{-1}. \tag{6}$$

The $S(E)$ function derived from the ENDFB-VI data set is shown along with the fitted version of the function in Fig. 2

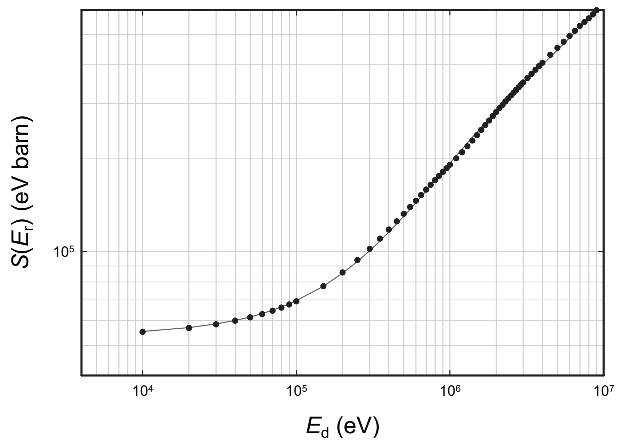

Figure 1. Astrophysical $S(E_{\mathrm{r}})$ factor for the n+^{3}He branch as a function of the deuteron energy in eV.

3. More complicated fitting function

It is possible to develop a better fit using a similar fitting function with more fitting parameters. In this case, we adopt a fitting function of the form

$$S(E_{\mathrm{r}}) \; = \; \exp\left\{\frac{a + bE_{\mathrm{r}} + cE_{\mathrm{r}}^2 + \mathrm{d}E_{\mathrm{r}}^3}{1 + eE_{\mathrm{r}} + fE_{\mathrm{r}}^2}\right\}. \tag{7}$$

In the case of the ^{2}H(^{2}H,n)^{3}H reaction, the fitting parameters we found are

$$a \; = \; 10.8757, \qquad b \; = \; 51.579 \; \mathrm{MeV}^{-1},$$

$$c \; = \; 24.822 \; \mathrm{MeV}^{-2}, \qquad d \; = \; 0.070247 \; \mathrm{MeV}^{-3},$$

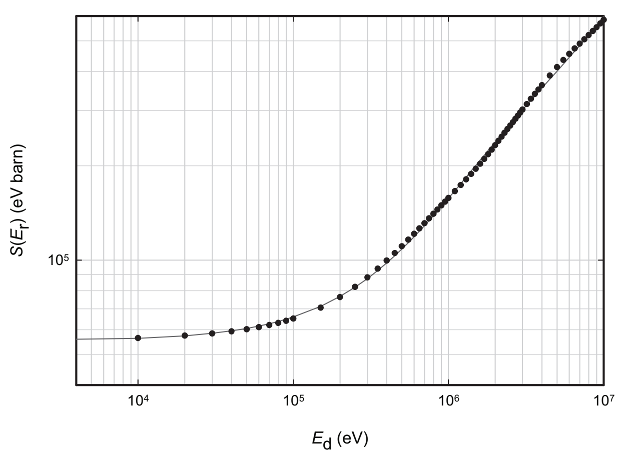

Figure 2. Astrophysical $S(E_{\mathrm{r}})$ factor for the p+^{3}H branch as a function of the deuteron energy in eV.

Table 1. Comparison of data with fits; E_r is the relative deuteron-deuteron energy; $S(n)$ is the astrophysical S-factor $S(E_r)$ for the $^2H(^2H,n)^3H$ reaction from the ENDFB-VI library; S_4(n) is the 4-parameter fitted result and S_6(n) is the 6-parameter fitted result; S(p) is the astrophysical S-factor $S(E_r)$ for the $^2H(^2H,p)^3H$ reaction from the ENDFB-VI library; S_4(p) is the 4-parameter fitted result and S_6(p) is the 6-parameter fitted result.

E_r(eV)	S(n)	S_4(n)	S_6(n)	S(p)	S_4(p)	S_6(p)
10000	56876	57019	56333	57492	57440	56911
30000	63014	63315	63205	61191	61691	61596
50000	69270	69634	69996	65132	65965	66245
100000	85718	85370	86491	76320	76700	77665
300000	145915	143856	144386	121356	118775	119873
500000	190496	192830	191405	157860	157512	156953
1000000	281353	283389	281088	232613	238054	234996
3000000	494163	487800	494855	454451	443165	454228
5000000	643531	647065	632824	583799	599158	576813

$$e = 4.1407 \text{ MeV}^{-1}, \qquad f = 1.8212 \text{ MeV}^{-2}. \tag{8}$$

For the $^2H(^2H,p)^3H$ reaction, we obtained

$$a = 10.9070, \qquad b = 32.995 \text{ MeV}^{-1},$$

$$c = 5.2903 \text{ MeV}^{-2}, \qquad d = -0.050120 \text{ MeV}^{-3},$$

$$e = 2.6291 \text{ MeV}^{-1}, \qquad f = 0.34445 \text{ MeV}^{-2}. \tag{9}$$

4. Discussion

We have developed simple fits to the deuteron–deuteron fusion cross sections which provide a reasonable match to the ENDFB-VI data set. Numerical values for the two cross sections and the different fits at selected relative energy points are given in Table 1. One can see that the fits match the data within a couple of percent. The standard error for the six parameter fits in both cases are about 60% of the standard error for the four parameter fits.

References

[1] The LANL website is: http://t2.lanl.gov/data/deuteron.html.
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