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Research Article

Progress in Condensed Cluster Fusion Theory *

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Abstract

The theoretical models on Condensed Cluster Fusion in the dynamic ordering process of deuterons in condensed matter (especially PdDx lattice) have been elaborated in three steps in the period from 1989 to 2009. The present paper briefly reviews theoretical modeling, mathematical formulation and quantitative estimations of multi-body deuteron fusion rates, time-dependent screening effect by electron clouds, and time-dependent size of condensing clusters as 4D/TSC. TSC is the tetrahedral symmetric condensate and key idea for clustering and dynamic condensation. Final products of 4D fusion are mainly ^{4}He nuclei with 2–5 MeV main component and 23.8 MeV minor component.

Keywords: Condensed cluster fusion, Deuteron cluster fusion, 4D Fusion, Helium-4, Tetrahedral symmetric condensate, Time-dependent condensation

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1. Introduction

Major questions concerning theoretical modeling of the process of experimentally claimed "radiationless excess heat with ^{4}He ash" as condensed matter nuclear effects (CMNE) are as follows.

(A) How can the mutual Coulombic repulsion between deuterons be overcome at low deuteron energy, so as to attain significant levels of deuteron-related fusion?
(B) How can the ^{4}He generation channel be predominant?
(C) How can hard radiations be suppressed?
(D) What kind of environments in/on condensed matter are conductive to CMNE?

Many theoretical efforts in the past have concentrated on questions A and D, and controversial answers have been provided for questions B and C. We focus on question B "The major ash of ^{4}He due to a two-body deuteron fusion

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reaction cannot be expected" is undoubtedly true, as far as concerned the emission of particles in final state interactions by fusion reactions, commonly seen in nuclear physics. This means that we need to invent some new nuclear processes in condensed matter to explain ^{4}He production in correlation with excess heat. Takahashi has argued that the direct coupling of an excited nucleus in the final state interaction with a lattice (e.g., a metal–deuteride system) to transfer the nuclear excitation energy directly to the lattice (vibration) is impossibly difficult [1,6,16].

There are, in principle, several ways to get to certain lower excited states of ^{4}He* (probably the assumed levels of 20.21 MeV as noted by Swartz [17] or the Schwinger–Preparata P-wave state [18], as shown in Fig. 1. See Fig. 1 for ^{4}He level charts and related reactions. One simple way is excitation by incident gamma-rays with energy more than 20.21 MeV, which may show resonance photon absorption at $E_\gamma = 20.21$ MeV with some narrow energy width (maybe of the order of several tens of meV). Another way is by using the p + t reaction with $(20.21-19.814) = 0.396$ MeV relative kinetic (beam) energy. Conventional d + d reactions cannot go there since the ^{4}He* excited energy with zero relative kinetic energy is 23.8 MeV. So, Swartz [17] assumed that this 3.6 MeV difference — (23.8 minus 20.21) — was due to the hypothetical Phuson interaction (the feasibility is discussed below). In any case, the assumed 20.21 MeV state of 'special d–d fusion' should be reached through the d + d strong interaction, and therefore we need to consider the competition among n-, p- and gamma-emission channels with their own partial energy widths, because we have to start or compete with the $< \mathrm{d} - \mathrm{d} >$ admixture state by d + d to ^{4}He* (E_x =23.8 MeV) or by-passed routes. We therefore need to treat the competition among particle emission break-ups and gamma (or electromagnetic) transitions. To treat only the photon channel is consequently misleading. We stress that, in the p + t break-up channel, the [t] channel, is dominant due to its large partial energy width, defeating the gamma-ray emission transition, the [γ] channel, for the 20.21 MeV state of ^{4}He if this were attained in the d–d interaction in condensed matter at all (the author however regards it as very pessimistic to construct a feasible model considering this possibility).

We cannot change the branching ratios after the (virtual) intermediate compound state ^{4}He* ($E_x = 23.8\,\mathrm{MeV} + E_\mathrm{k}$) with very short life (on the order of 10^{-22} s), since no force-exchange bosons can transfer energy (or any information) from the intermediate compound nucleus to surrounding nuclei, atoms or lattices – all of which are located at distances larger than 0.1 nm. We need to consider a time length for the force-exchange boson (the photon for an electromagnetic interaction) of more than 3×10^{-19}s, which is very much longer than the life time 10^{-22} s of ^{4}He* ($E_x = 23.8\,\mathrm{MeV}+E_\mathrm{k}$). The branching ratios, [n]/[t]/[γ] of d–d fusion should therefore almost maintain at constant values, 0.5/0.5/10^{-7} for $E_\mathrm{k} = 0.025$ eV to about 100 keV. (See Fig. 2 for an illustration of this feature of the branching.) Here E_k is the relative kinetic energy of the d–d interaction.

To change out-going channels, namely branching ratios, we need the participation of a Third Interaction with the d–d strong interaction process during the initial interaction state. The third interaction should be effective enough to quantitatively change the virtual intermediate excited state of ^{4}He$^*(E_x)$, or other deuteron-related compound nuclei states.

As discussed in detail in the literature [16] on possible third interactions to change the out-going channels (branching ratios), we can conclude:

(1) The lowest excited energy of ^{4}He*, intermediate compound nucleus, by two body d + d fusion reaction is 23.8 MeV. Lower excited energy than 23.8 MeV is forbidden by kinematics. As a result, the [n]/[t]/[^{4}He] branching ratios become almost constant at: 0.5/0.5/10^{-7} for $E_k = 0$ eV to 100 keV (relative kinetic energy of reaction).

(2) If a ^{4}He$^*(E_x)$ state with $E_x < 19.8$ MeV occurs, the final product becomes ^{4}He in a ground state, after electromagnetic transition. To realize this process in view of d + d reactions, there should exist a third coupling field which must reach more than 4 MeV difference of energy $(23.8 - 19.8)$ of the d–d system in the initial state interaction.

(3) A many-body interaction process between the d + d pairing and the third field of photon–phonon coupling in the lattice of condensed matter may be considered. Due to the very short range force of the d + d strong interaction

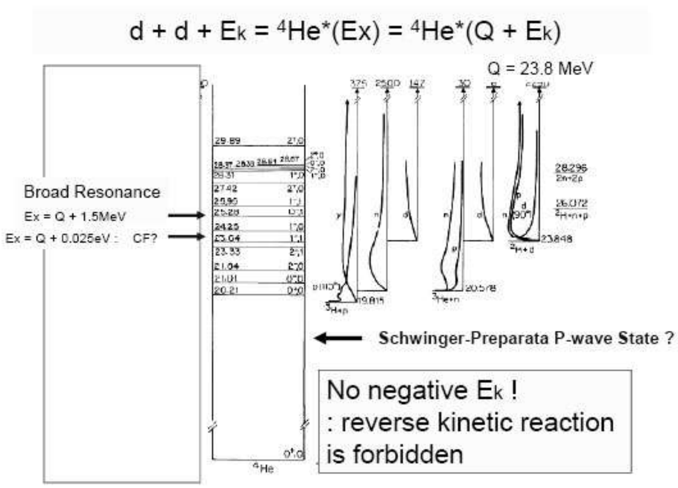

Figure 1. DD (d + d) reaction and energy levels of ^{4}He.

and its very short life time as a virtual intermediate compound state, no processes have ever been proved to remove the 4 MeV gap-energy [7−14]. Moreover, the field coupling constant of electromagnetic interaction looks too weak, of the order of 10^{-2} relative to the strong interaction, to drastically change the state of d + d strong interaction for fusion. Quantitative studies on transition probabilities for proposed models [7–15] will be further needed.

(4) Deuteron-cluster fusion, i.e., 4D fusion, may produce ^{4}He as a major final ash of the reaction. To realize the conditions for 4D fusion, the microscopic ordering/constraint process for the dynamic Platonic symmetry must be satisfied. The EQPET/TSC model is one of these theoretical models, although it requires further investigation to become established [1–6].

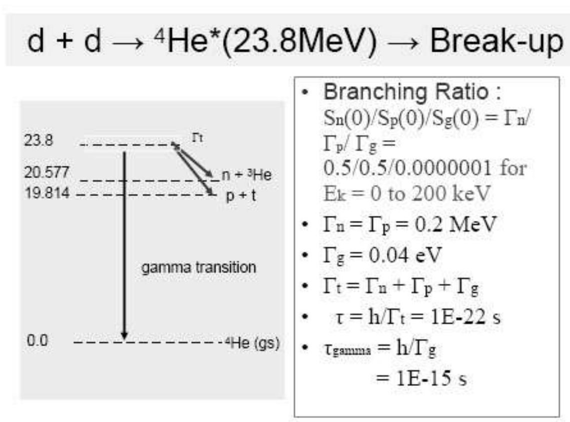

Figure 2. Break-up (out-going, decay) channels of d + d fusion.

This theoretical model treats a process of "condensed cluster fusion" of multi-body deuteron interactions in condensed matter.

2. Brief Explanation of Condensed Cluster Fusion Models

2.1. Step 1: Multi-body deuteron fusion models

Just two weeks after the announcement of the Fleischmann–Pons "cold fusion" experiment [25] claiming large excess heat evolution without corresponding intense neutron emission from the heavy-water electrolysis using Pd cathodes, the author submitted a short note to the *J. Nucl. Sci. Technol.* [19] proposing a three-body deuteron interaction, 3D fusion, considering the following D-catalyzed cascade reaction channel to produce the main ash of ^{4}He,

$$\mathrm{d} + \mathrm{d} \rightarrow {}^4\mathrm{He}^*(E_x = 23.8\ \mathrm{MeV}); \quad {}^4\mathrm{He}^* + \mathrm{d} \rightarrow {}^4\mathrm{He}(g.s) + \mathrm{d} + 23.8\ \mathrm{MeV}. \tag{1}$$

Ordinary d + d (2D) fusion should have two main outgoing channels with a 50%/50% branching ratio, the $\mathrm{n} + {}^3\mathrm{He} + 3.27$ MeV channel and the $\mathrm{p} + \mathrm{t} + 4.03$ MeV channel should have very small branching ratios (10^{-5}%) of the electromagnetic transition, $^4\mathrm{He}(g.s.) + \gamma + 23.8$ MeV, in the range of low deuteron kinetic energy. For this reason, it is reasonable to consider that a third hadronic interaction should participate in the d+d strong interaction to realize a main branch of the ^{4}He producing reaction.

The quantitative model of Eq. (1) meets, however, a difficulty in predicting a high level reaction rate, due to the very short life time (about 10^{-22} s) of ^{4}He* (E_x =23.8 MeV), from proven nuclear physics knowledge. The model was therefore elaborated [20,21] to the "simultaneous" 3D and 4D fusion models in the dynamic environment of the PdD lattice with excited D-harmonic oscillators (phonons) to be able to predict more than several watts/cc-PdD excess heat with ^{4}He ash (23.8 MeV/^{4}He). It also predicted much less (of the order of 10^{-6}–10^{-12} of helium yield) of tritium and neutron production. Fusion reactions by transient deuteron clusters were modeled to take place as the following competition processes of 2D, 3D and 4D fusions.

$$\mathrm{d} + \mathrm{d} \rightarrow \mathrm{n} + {}^3\mathrm{He} + 3.27\ \mathrm{MeV}(50\%); \quad \mathrm{p} + \mathrm{t} + 4.03\ \mathrm{MeV}(50\%), \tag{2}$$

$$\mathrm{d} + \mathrm{d} + \mathrm{d} \rightarrow {}^3\mathrm{He} + \mathrm{t} + 9.5\ \mathrm{MeV}(\text{about } 50\%); \quad {}^4\mathrm{He} + \mathrm{d} + 23.8\ \mathrm{MeV}(\text{about } 50\%), \tag{3}$$

$$\mathrm{d} + \mathrm{d} + \mathrm{d} + \mathrm{d} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He} + 47.6\ \mathrm{MeV}. \tag{4}$$

The formation of transient clusters of 2D, 3D and 4D was modeled to approximately quantify the concept shown in Fig. 3. Deuterons sit at O-sites as Einstein oscillators (harmonic oscillators), and have Gaussian wave function for the ground state (energy eigenvalue is 32 meV) [20]. At higher phonon-excited states, D-wave function changes to form a "U" shape distribution to enhance the probability of multiple deuterons meeting around the T-site. Fusion rates for 3D and 4D formation can overtake 2D fusion rates at high phonon excited states [20].

A time-window of about 50 fs was conceived for a transient 4D cluster formation with about 10 GHz lattice plasma-oscillation under D-phonon excitation [21]. We have also roughly estimated D-cluster formation probabilities, as shown in Fig. 4, as a function of D-phonon energy (one phonon = 64 meV, namely 0.064 eV). Competing fusion rates were then estimated, as shown in Fig. 5.

Later, we considered electron spin combinations (in Step 2 [1]) and the Platonic symmetries (in Step 3 [4,5]) for D-cluster formation. We also later found by using Langevin equation analyses [4–6] that the initial time window for 4D/TSC formation with a much shorter time interval of 1.0×10^{-18} s was enough for further 4D/TSC condensation. Also, a very small 4D cluster formation probability of 10^{-11} can already be realized for 4D fusion rates to meet a

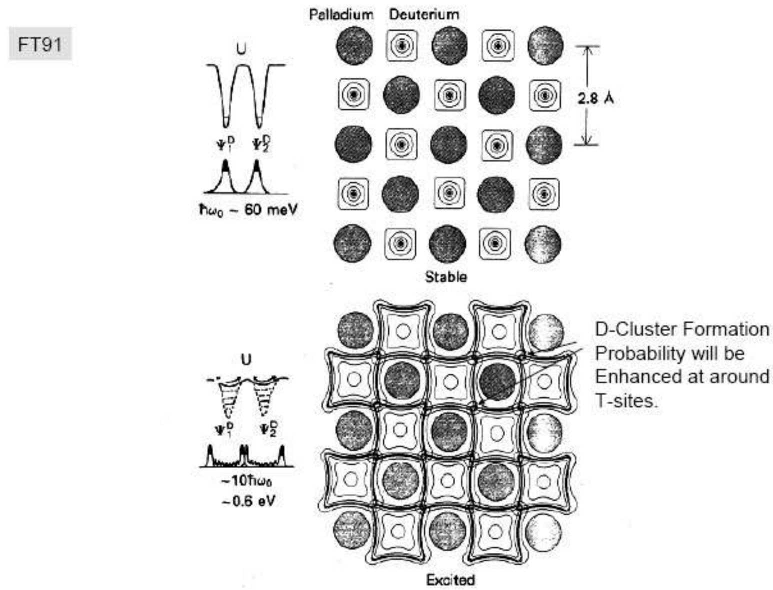

Figure 3. Modeling of transient D-cluster formation under a D-harmonic oscillator in the PdD lattice [21].

1 W/cm^3 level of nuclear heat-power level. Henceforth, we need to re-quantify cluster formation probabilities with more sophisticated solid state physics (or surface physics) modeling.

However, our Step-1 theory can already explain why deuteron-related nuclear reactions with observable excess heat level and ^{4}He main ash are possible with apparently radiation-less nuclear products. To prove the super-screening scenario of mutual Coulomb barriers among deuterons, we need further elaboration of Steps 2 and 3.

2.2. Step 2: EQPET/TSC models

Elaboration of EQPET/TSC (electronic quasi-particle expansion theory/tetrahedral symmetric condensate) models were reviewed more in detail in our recent papers [1,2]. There, we proposed a multi-body deuteron fusion process by formation of Tetrahedral Symmetric Condensates (TSC) and Octahedral Symmetric Condensates (OSC). Some numerical results were given by EQPET analyses, which could explain the 3–78 W/cm^3 heat–power level with 1.0×10^{11} $f/s/cm^3$ to 1.0×10^{13} $f/s/cm^3$ of ^{4}He-atoms production by 4D and 8D fusion reactions, with neutron production rates smaller than 10 $neutrons/s/cm^3$. There remain, however, open questions as to where TSC is formed. We have proposed two mechanisms, as transient motion forming deuteron-clusters with short lifetimes (60 fs). In the near surface region of the PdDx cathode, deuterium full loading ($x = 1$; PdD) may be attained by electrolysis, gas discharge or gas-permeation, at least locally. No experimental techniques have been developed to measure local distribution of the x-value, although we know that it would be vital information. At very small densities (namely 1ppm, as assumed in our papers [20,21]), PdD_2 states may exist.

Trapped D in a Bloch potential has discrete energies with 32 meV ground state and 64 meV of one phonon energy for excited states. Over 0.22 eV, all D-ions in the lattice diffuse out of the solid if excitation happens at every O-site. Following the classical Drude model, transient clusters of TSC can be formed with certain probabilities, by excitation with an external UV or an EUV laser, in limited places, such as near the surface region. An illustration of a possible

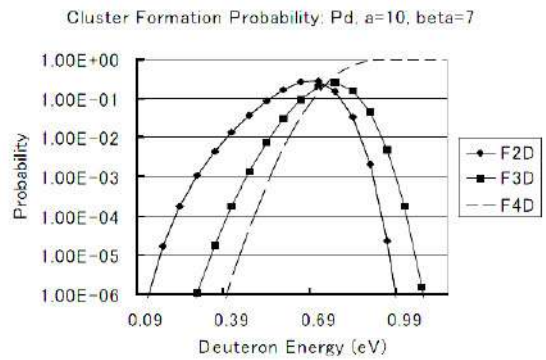

Figure 4. Estimation of 2D, 3D and 4D cluster formation probabilities around T-sites in the PdD lattice as a function of D-phonon excitation energy.

state of 4D/TSC ($t = 0$) formation at a focal point (T-site in this case) is shown in Fig. 6.

In EQPET models, we assume that the total 4D wave function can be expanded by a linear combination of partial wave functions of dde* type molecules with regular electron states e(1,1) and electronic quasi-particle states as e*(2,2) Cooper pair, e*(4,4) quadruplet, and so on.

$$|\Psi_N\rangle = a1|\Psi_{(1,1)}\rangle + a2|\Psi_{(2,2)}\rangle + a4|\Psi_{(4,4)}\rangle + a6|\Psi_{(6,6)}\rangle + a8|\Psi_{(8,8)}\rangle \quad (5)$$

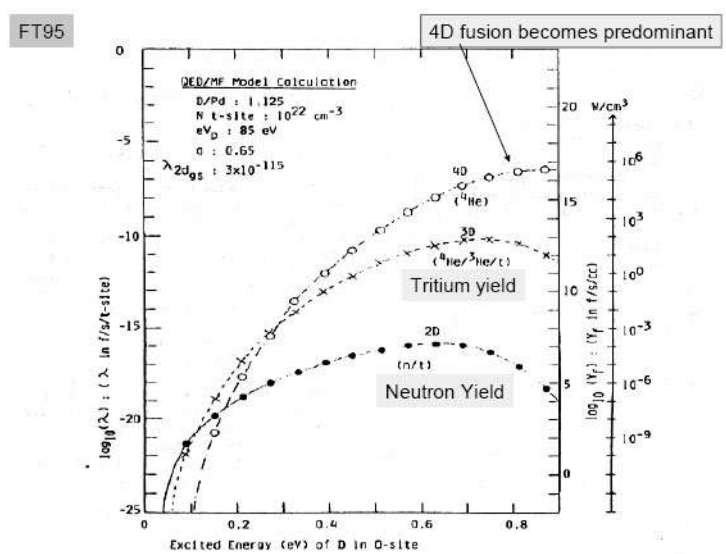

Figure 5. Comparison of logarithmic fusion rates between 2D, 3D and 4D fusions in PdD as a function of D-phonon excitation energy (from Fig. 4 [20]).

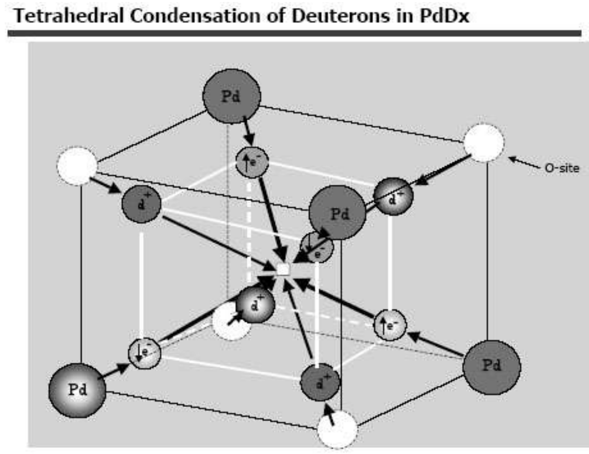

Figure 6. Illustration of initially formed 4D/TSC ($t = 0$) around some T-sites in a PdD lattice under D-phonon excitation; 4 centers of electron waves are drawn with "e^-" and will form tetrahedra by an exchange interaction of a 1s electron from d^+ and a 5s (or 5f) electron from Pd^- states.

The modal fusion rate is defined [21] as,

$$\lambda_N = a1^2\lambda_{(1,1)} + a2^2\lambda_{(2,2)} + a4^2\lambda_{(4,4)} + a6^2\lambda_{(6,6)} + a8^2\lambda_{(8,8)}, \quad (6)$$

$$\lambda_{nd(i,j)} = \nu(S_{nd}/E_d)\exp(-n\Gamma_{(i,j)}). \quad (7)$$

The modal fusion rate given by Eq. (6) for 4D fusion is attributed almost 100% to the quadruplet EQPET molecule dde*(4,4) state. Therefore, the accuracy of this model is closely related to what the minimum size state of 4D/TSC is. Screening energies for d–d reactions are compared in Table 1. The estimated fusion rates are shown in Table 2.

Subsequently, we have considered that the squeezing motion of TSC can be more simply treated by a semi-classical model, because of the three-dimensional constrained motion of 4d and 4e particles in the TSC into the central focal point. Every QM particle-center in the TSC can undergo a central squeezing motion with the same velocity, to keep charge neutrality of the total TSC system – in other words, to satisfy the minimum system energy state (as calculated by the variational principle of quantum mechanics, QM). Therefore, this squeezing motion can be treated approximately by Newtonian mechanics until the point at which four deuterons get into the range (about 5 fm) of the strong nuclear interaction. When four electrons start to separate at the minimum TSC state, four deuterons suddenly start to feel the mutual Coulomb repulsion. The nuclear interaction at this stage can be approximately treated by the STTBA (Sudden Tall Thin Barrier Approximation) [1]. We obtained: $\lambda_{4d} = 2.3 \times 10^{-4}$f/s/cl at the TSC-minimum state. This microscopic fusion rate is 10^7 times larger than that given in Table 2. We consider therefore that the EQPET model may have given significant underestimation for the 4D fusion rate when the rigid constraint of motion in the three-dimensional TSC motion in condensed matter is attained, as shown in Fig. 7.

TSC squeezes from about 100 pm size to its minimum-size of about 10–20 fm diameter and behaves as a charge-neutral pseudo-particle. The life time of TSC is estimated as the time difference from the 100 pm size state to a minimum size with a velocity of the order of 10^5 cm/s; we obtain about 60 fs. (This was later found to be much shorter (1.4 fs) in Step 3 [4–6].)

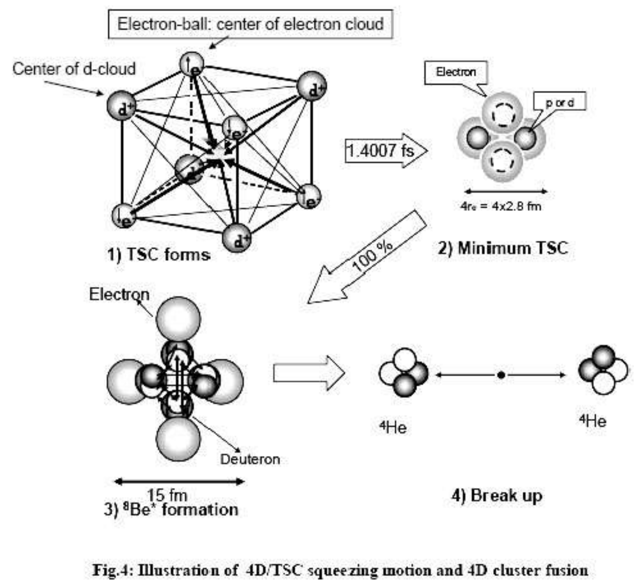

Figure 7. Schematic steps of 4D/TSC condensation motion; (1) TSC ($T = 0$) is just formed, (2) TSC gets to the minimum size state after about 1.4 fs condensation time [4–6], (3) strong interaction among 4D forms a ^{8}Be* intermediate excited nucleus, (4) ^{8}Be* undergoes a final state interaction to break up (from [4], Fig. 4).

2.3. Step 3: D-Cluster dynamics and fusion rate by Langevin equation

To explain the apparent hard-radiation-free excess heat with ^{4}He ash in CMNS experiments, especially in dynamic PdDx systems, we have done a series of studies to model D-cluster (or multi-body deuteron) fusion reaction mechanisms in 1989–2009, thus arriving at our latest theory in Step 3 studies based on quantum-mechanical Langevin equations [4–6] (stochastic differential equations).

The basics of the methods with Langevin equations for D-cluster dynamics, especially for the D-atom, D_2 molecule, D_2^+ ion, D_3^+ ion, 4D/TSC (tetrahedral symmetric condensates) and 6D^2-/OSC (octahedral symmetric condensates) are given in our latest papers [5,6] which are included in the LENR Source book Vols .1 and 2.

First, one-dimensional Langevin equations for D-clusters with the R_{dd} (d–d distance) are formulated under the Platonic symmetry [6] of multi-particle D-cluster systems with deuterons and quantum-mechanical electron centers. Under the orthogonally coupled Platonic symmetry for a Platonic deuteron-system and a Platonic electron system,

Table 1. Screened energies for various EQPET molecules.

$e^*(m^*/me, e^*/e)$	Screening Energy U_s (eV)		b_0 (pm)	
	dde*	dde*e^*	dde*	dde*e^*
(1, 1); Normal electron	36	72	40	20
(2, 2); Cooper pair	360	411	4	2
(4, 4); Quadruplet	4000	1108	0.36	1.3
(8, 8); Octal coupling	22154	960	0.065	1.5
(208, 1); muon	7579	7200	0.19	0.20

dynamic equations should be treated for many-body systems of deuterons and electrons with metal atoms. Systems with more than four deuterons plus four 1s electrons of deuterium atoms plus 40 4d-shell electrons of four Pd atoms in an fcc lattice plus surrounding lattice atoms under D-phonon excited states should be considered in the model. A simple one-dimensional Langevin equation for the internuclear d–d distance $R_{\rm dd}$ can be formulated, as shown in [4–6]. Considering the mean values taken from the Langevin equation with the weight on quantum mechanical wave-functions for electrons and deuterons, we could further derive a time-dependent one-dimensional Langevin equation for the expectation value $\langle R_{\rm dd}\rangle$, which is nonlinear, but could be solved by the Verlet's time step method [4,5]. We showed [5,6] that only 4D(or H)/TSC can condense ultimately to a final, very small charge neutral entity with about 10–20 fm radius. At the final stage of 4D/TSC condensation in about 2×10^{-20}s, 4D fusion with two ^{4}He products takes place with almost 100% probability, according to our HMEQPET calculation [4,5] for barrier factors and fusion rate formula using Fermi's first golden rule.

Basic Langevin equations for a Platonic symmetric D-cluster having $N_{\rm e}$ d–d edges and $N_{\rm f}$ faces of d–d–e (D_2^+) or d–e–d–e (D_2) type are written as in Eq. (8). Here, R is the d–d distance and $m_{\rm d}$ is the deuteron mass, $V_{\rm s}$ is the d–d pair trapping potential of either a d–e–d–e or a d–d–e type molecule. The first term on the right-hand side of Eq. (8) is the total Coulomb force of the D-cluster system, and $f(t)$ is the fluctuation of force for which we introduce a quantum mechanical fluctuation of deuteron positions under condensation motion. The quantum mechanical effect of electron clouds is incorporated as the second term on the right-hand side as "friction" in the Langevin equation for the D_2 molecule, $N_{\rm e} = N_{\rm f} = 1$. For the D_3^+ ion which is known to be stable in a vacuum, $N_{\rm e} = 3$ and $N_{\rm f} = 6$ are given. For 4D/TSC, $N_{\rm e} = 6$ and $N_{\rm f} = 6$ are given. For $6D^{2-}$, $N_{\rm e} = 12$ and $N_{\rm f} = 24$ are given. By taking a QM ensemble average with d–d pair wave functions, assumed to have Gaussian distributions, we derived the Langevin equation for 4D/TSC as shown in Eq. (9). By taking QM ensemble average of Eq. (10), we obtained Eq. (13). And we obtained the time-dependent TSC-cluster trapping potential as Eq. (14).

A similar Langevin equation and trapping potential were also derived for the $6D^{2-}$ molecule. We compare the central potential curve (at $R' = R_{\rm dd}$) in Fig. 8. We find that 4D(H)/TSC can condensate ultimately to a very small charge neutral entity and has no stable or ground state. This may be the reason that we do not observe D_4 molecules in nature. On contrary, the $3D^+$ molecule and the $6D^{2-}$ molecule have stable ground states. Equation (13) was numerically solved by the Verlet method [4], and shown in Fig. 9.

Time-dependent barrier penetration probabilities are given as a function of $R_{\rm dd}$, since we have a one-to-one relation between the elapsed time and $R_{\rm dd}(t)$.

$$N_{\rm e} m_{\rm d} \frac{{\rm d}^2 R}{{\rm d}t^2} = -\frac{k}{R^2} - N_{\rm f}\frac{\partial V_s}{\partial R} + f(t), \tag{8}$$

$$6m_{\rm d}\frac{{\rm d}^2 R_{\rm dd}(t)}{{\rm d}t^2} = -\frac{11.85}{[R_{\rm dd}(t)]^2} - 6\frac{\partial V_{s2}(R_{\rm dd}(t);1,1)}{\partial R_{\rm dd}(t)} + \langle f(t)\rangle + f'(t), \tag{9}$$

$$f'(t) = f(t) - \langle f(t)\rangle, \tag{10}$$

$$f(t) = \left[-\frac{\partial \Delta E_{\rm c}(R_{\rm dd})}{\partial R_{\rm dd}}\right] \bmod [X^2(R'_{\rm dd}; R_{\rm dd}(t))], \tag{11}$$

$$X^2(R'_{\rm dd}; R_{\rm dd}(t)) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp[-(R'_{\rm dd} - R_{\rm dd}(t))^2/(2\sigma^2)], \tag{12}$$

$$6m_{\rm d}\frac{{\rm d}^2\langle R_{\rm dd}\rangle}{{\rm d}t^2} = -\frac{11.85}{\langle R_{\rm dd}\rangle^2} - 6\frac{\partial V_{\rm s}(\langle R_{\rm dd}\rangle; m, Z)}{\partial\langle R_{\rm dd}\rangle} + 6.6\left\langle\frac{(R' - R_{\rm dd})^2}{R_{\rm dd}^4}\right\rangle, \tag{13}$$

$$V_{\rm tsc}(R' : R_{\rm dd}(t)) = -\frac{11.85}{R_{\rm dd}(t)} + 6V_s(R_{\rm dd}(t); m, Z) + 2.2\frac{|R' - R_{\rm dd}(t)|}{[R_{\rm dd}(t)]^4}. \tag{14}$$

Table 2. Typical results of EQPET/TSC for fusion rates, power levels and products, for TSC in PdDx, assuming $N_{4D} = 10^{22}$ (1/cm^3).

Cluster	Microscopic fusion rate (f/cl/s)	Macroscopic yield (f/s/cm^3), Power (W/cm^3)	Ash (fusion products)
2D	1.9×10^{-21}	19 (f/s/cm^3), 1.9×10^{-11} (W/cm^3)	Neutron; 10 n/s/cm^3
3D	1.6×10^{-13}	1.6×10^9(f/s/cm^3), 1.6×10^{-3} (W/cm^3)	Tritium; 8×10^8 t/s/cm^3
4D	3.1×10^{-11}	3.1×10^{11}(f/s/cm^3), 3.1 (W/cm^3)	Helium-4; 3×10^{11} h/s/cm^3

The fusion rate is calculated by the following Fermi's golden rule [21],

$$\lambda_{\mathrm{nd}} = \frac{2}{\hbar}\langle W\rangle P_{\mathrm{nd}}(r_0) = 3.04 \times 10^{21} P_{\mathrm{nd}}(r_0)\langle W\rangle. \tag{15}$$

Here P_{nd} is the barrier factor for a nD-cluster and $\langle W\rangle$ is the averaged value of the imaginary part of the nuclear optical potential [2]. The extrapolation of the $\langle W\rangle$ value to 4d fusion was made by using the scaling law $\langle W\rangle \propto (\mathrm{PEF})^5$ with PEF value which is given in units of the derivative of a one pion exchange potential (OPEP) (the simple case of the Hamada–Johnston potential [6] for the pion exchange model for the nuclear strong interaction). We obtained the next value of 4D fusion yield per TSC generation, as:

$$\eta_{4\mathrm{d}} = 1 - \exp\left(-\int_0^{t_c} \lambda_{4\mathrm{d}}(t)\,\mathrm{d}t\right). \tag{16}$$

Using time-dependent barrier factors as given in [3–5], we obtained $\eta_{4\mathrm{d}} \cong 1.0$. This result means that: *We have found that 4D fusion may take place with almost 100% yield per a TSC generation, so that the macroscopic 4d fusion yield is given simply by the TSC generation rate Q_{tsc} in the experimental conditions of CMNS.*

However, when we consider that the deuteron has spin-parity 1^+ and combinations of 4d have total spin state 4, 3, 2, 1 and 0, the 4d fusion with outgoing channel to two ^{4}He (0^+: gs) particles is forbidden, by spin-parity conservation (for S-wave in/out channels), except for the 0^+ spin-parity state ($T = 0$). This result will be explained elsewhere in detail, including P-wave and D-wave states with isospins.

The ultimate condensation is possible only when the double Platonic symmetry of 4D/TSC is kept in its dynamic motion. A sufficient increase (super screening) of the barrier factor is also only possible as far as the Platonic symmetric 4D/TSC system is kept. Therefore, there should be always four deuterons in the barrier penetration and the fusion process, so that the 4d simultaneous fusion should take place predominantly. The portion of 2D (usual) fusion rate is considered to be negligible [4,6]. Typical nuclear products of 4D fusion are naively predicted to be two 23.8 MeV α-particles. But the final state interaction of ^{8}Be* is complex and yet to be studied. There may be dominant outgoing channels via excited states of fragmented composite particles, such as ^{4}He* and ^{6}Li*, which would produce ^{4}He-particles (α-particles) mostly in the 2–5 MeV region. These α-particles are difficult to detect in liquid-phase D-loading cells, and also somewhat difficult even in gas-loaded cells, due to attenuation of particles in liquid, gas and solid phases. Fragmentation may occur "symmetrically" as $^4\mathrm{He}^*(E_x) + ^4\mathrm{He}^*(E_x)$ + (47.6 MeV $- 2E_x$), or "asymmetrically" as ^{4}He(g.s.)+$^4\mathrm{He}^*(E_x)$+(47.6 MeV$-E_x$). If E_x is the first excited state 20.21 MeV (see Fig. 2), $^4\mathrm{He}^*(E_x = 20.21$ MeV) breaks up to t(1.8$-$3.4 MeV) + p(0.6$-$2.2 MeV) only. This triton may cause secondary DT (d + t) reactions in its slowing down in PdDx matter and the emission of energetic neutrons in the 10$-$17 MeV region, which may explain the SPAWAR triplet tracks [22–24]. There is possible minor channel emission of 1–5 MeV deuterons and protons. If ^{8}Be* has odd spin-party (3-), 46 keV α-particles may be final products after the electromagnetic transition by many QED photons to lattice [26].

We consider lastly the principle of dynamic condensation motion of TSC in view of the Heisenberg uncertainty principle. At the starting condition of 4D/TSC (t =0), the d–d distance R_{dd} was estimated to be the same value

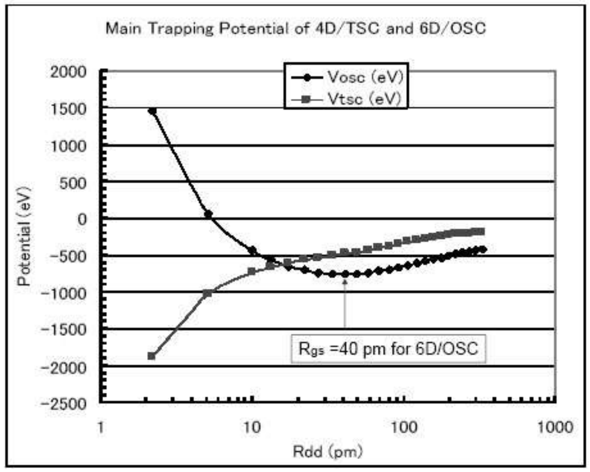

Figure 8. Comparison of cluster trapping potentials for 4D/TSC and $6D^{2-}$/OSC. TSC condenses ultimately to a very small R_{dd} value ($R_{dd} - min = 20 fm$), while OSC converges at $R_{dd} = 40 pm$ (corresponding to the ground state).

(74.1 pm) as that of the D_2 molecule. At this starting point, the mean electron kinetic energy of one "d–e–d–e" face EQPET molecule of the TSC six faces was 17.6 eV. During the non-linear condensation of TSC, the size of "d–e–d–e" EQPET molecule decreases from $R_{\rm dd} = 74.1$ pm at $t = 0$ to $R_{\rm dd} = 20.6$ fm at $t = 1.4007$ fs. From the uncertainty principle, the electron wave length should decrease accordingly to the decrement of $R_{\rm dd}$. At $t = 1.4007$ fs, the mean kinetic energy of electron for "d–e–d–e" EQPET molecule was estimated [4] to be 57.6 keV. Considering the relations, $\lambda = \hbar/(mv)$ of the de Broglie wave length and (kinetic-energy)$= \frac{1}{2}mv^2$, we understand that the effective quantum mechanical wave length of trapped electrons in TSC has decreased dramatically during the 1.4007 fs condensation time. The estimated trapping potential depth of TSC at $t = 1.4007$ fs was -130.4 keV. This is understood as an adiabatic state in very short time interval (about 10^{-20}s) to trap such high kinetic energy (57.6 keV) electrons in a very deep (-130.4 keV) trapping potential, in order to satisfy the uncertainty relation. By the way, the mean kinetic energy of relative d–d motion was estimated to be 13.68 keV at this adiabatic state, which is also diminished relative to the deuteron wave length trapped in the adiabatic TSC potential. In this way, a very short $R_{\rm dd}$ (in other words, super screening of the mutual Coulomb repulsion) was realized in the dynamic TSC condensation to give a very large 4D simultaneous fusion rate. It is also worthwhile to point out that the simultaneous 4D fusion in the final stage interval, about 2×10^{-20}s, of the TSC-minimum state should take place with a relative kinetic energy about 10 keV, by chance, similar to the target plasma temperature of the DT plasma-fusion device (ITER, for instance). In this sense, the 4D condensed cluster fusion is not "cold fusion".

3. Concluding Remarks

How to super-screen the Coulomb barrier, how to obtain ^{4}He ash and why there are no apparent hard radiations, these are questions that are fundamentally resolved by the 4D/TSC dynamic condensation motion. The Langevin equation-based analysis can be extended for neutral clusters, such as 6D/OSC and 8D/HSC, with further elaboration of the modeling. We can name Condensed Cluster Fusion Models for these processes as proposed and quantitatively studied in the last 20 years.

Future works should include:

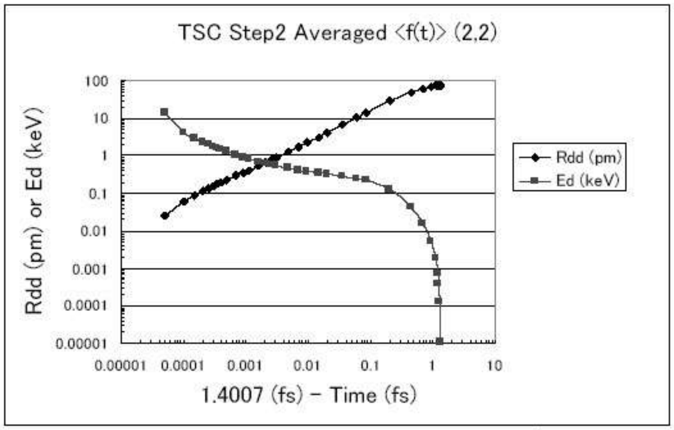

Figure 9. Numerical solution of Eq.(13) by the Verlet method[4]. Time is reversed starting from the condensation time at 1.4007 fs.

(1) How to enhance the 4D/TSC (t =0) rate in the nano-structure of metal–deuterium systems? This should be investigated, since this gives key information on the stimulation-conditions in experiments. The TSC formation process in regular PdD lattices under external stimulation, on the surface sub-nano-holes (we will show in separate paper elsewhere) of nano-particles or nano-structure samples or interfaces with incoming deuteron (or D_2) flux should be modelled or experimentally tested.

(2) Details of the ^{8}Be* final state interaction and out-going channels should be studied. This compound excited state may have a complex final state interaction to various out-going channels as symmetric and asymmetric fragmentations and $\alpha +^4$ He*(E_x < 47.6 MeV) + (47.6 MeV − E_x), gamma-transition of ^{8}Be*(47.6 MeV − E_γ) + $E\gamma$ and minor channels of n, p, and t emission, etc. Lower excited states of ^{8}Be* have many out-going channels to ^{4}He(g.s) $+^4$ He(g.s). Combinations of spin-parities and isospins are complex. Charged particle spectroscopy and experimental particle-identification should be cross-checked with such theoretical predictions.

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