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Research Article

Underlying Mechanism of the Nuclear of Implied by the Energy–momentum Conservation [I]

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Abstract

By studying the conservation of energy and momentum, it is found that in the nuclear cold fusion, existence of the localized external potential is necessary to absorb the large momentum transfer. We can narrow down the candidate of the required external field to the magnetic field produced by the magnetic monopole. The roll of the magnetic monopole in lowering the repulsive Coulomb barrier when two deuterons come close and fuse is considered.

Keywords: Charge–monopole system, Coulombic field, d–d Reaction, Momentum conservation, Momentum transfer

1. Introduction

The conservation laws of the energy and the momentum are the most important thing to obey when we study the nuclear reaction for example. As Noether's theorem taught us, the conservation law appears from the symmetry of the system. In particular, the translational invariance in time implies the energy conservation, whereas the momentum conservation arises from the homogeneity of the space.

The final states of the d–d reactions in vacuum at low energy are t +p and ^{3}He + n with 50% each in the branching ratio. In the c.m. system, the produced energy Q is shared by the two final state particles by $Q = \vec{p}'^2/2m_1 + (\vec{p}')^2/2m_2$. On the other hand, in the nuclear CF, the reaction changes to d + d $\rightarrow ^4$He. When we examine the conservation laws in the center of mass system, from the momentum conservation the momentum $\vec{q}$ of ^{4}He must be zero, whereas the energy conservation requires $q = \sqrt{2M_4 Q}$, where M_4 is the mass of ^{4}He. Therefore, two conservations are not compatible. We know that the homogeneity of the space is often destroyed when the external field exists. Since $Q = 23.9\,\mathrm{MeV}$, the momentum transfer becomes $q = 422\,\mathrm{MeV/c}$. In the potential scattering we know the scattering amplitude is the Fourier transformation of the external potential $V(r) : a(q) = -(2m/4\pi\hbar) \int \mathrm{d}^3 r' \exp[i\vec{q} \cdot \vec{r'}] V(r')$. Since $|a(q)|^2$ is the probability of the momentum transfer, the spread of the potential Δr must be very small in order to produce such a large momentum transfer. They are related by the uncertainty relation $q \cdot \Delta r \sim \hbar$, which means $\Delta r = 0.47$ fm. in our case.

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The candidates of the source X of the external field are severely restricted. The external fields, to which the nucleus can respond, must be either electric or magnetic. However, X cannot be the electric charge. This is because the electron cloud cannot become as small as $\Delta r = 0.47$ fm., since the size of the electron cloud must be larger than its Compton wave length, which is around 400 fm. Another candidate of X is the magnetically charged particle (magnetic monopole), since the nuclei with the non-zero magnetic moment can respond to the magnetic field. Because of the charge quantization condition of Dirac, the magnetic field of the magnetic monopole is super- strong, namely $^*e^2/\hbar c = 137/4$. The interaction potential between the nucleon and the magnetic monopole is $V(r) = -\kappa_{\text{tot}}(e/2m_p)^*e - \vec{\sigma} \cdot -\vec{r})/r^3$. This strong potential serves to form the bound states of the monopole and the nucleons or small nuclei. For example, the binding energy of the $^*e - $d system is around 2.3 MeV. Therefore, if the d–d reaction proceeds after two deuterons are trapped by the magnetic monopole, it is energetically impossible to become the (t+p) or (^{3}He + n) states. This is because the binding energy of t and ^{3}He is 8.5 and 7.7 MeV, respectively. The only allowed final state is ^{4}He, however it must leave the monopole, because the spin of ^{4}He is zero and there is no bound state of the monopole and ^{4}He.

In this way, when the reaction proceeds under the influence of the magnetic monopole, the channels (t + p) and (^{3}He + n) are energetically closed. On the other hand, the reac- ton of (two-body → one-body) type such as (d+ d→^{4} He) becomes possible. In Section 2, how the external potential, whose existence destroys the homogeneity of the space, can "absorb" the momentum transfer will be explained. Since the magnetic monopole plays the central roll in the nuclear cold fusion, the brief review of the theory of the magnetic monopole will be given in Section 3. In Section 4, the Schrödinger equation in the external magnetic Coulomb field is solved. Existence of the magnetic monopole can change the purely repulsive Coulomb potential between deuterons to a potential which enables for the two zero-incident energy deuterons to approach to the fm region. In Section 5, the mechanism of the change of the potential will be discussed along with the evaluation of the penetration factor. Other characteristic feature of the nuclear cold fusion is the sporadic property in starting the reaction. This phenomenon does not occur on demand, but some kind of probability comes into the scene. The magnetic monopole, which is the rare particle, plays the roll of the "one-particle catalyst" of the nuclear CF reaction. In Section 6, the meaning of the lack of the reproducibility is considered.

2. Energy–momentum Conservation

The most important difference of the very low energy (d + d) reaction in vacuum and that of the nuclear cold fusion is that although the former reaction is the (two-body → two-body) type, the latter reaction is the (two-body → one-body) type. In fact, the final states of reactions in vacuum are (p+t), (n +^{3} He) or (^{4}He + γ). On the other hand, in the nuclear CF the final state is ^{4}He. Let us examine the conservation laws in the center of mass system. The momenta of the final state particles, whose masses are m_1 and m_2, are $\vec{p}'$ and $(-\vec{p}')$, respectively. The energy conservation of the zero-incident energy becomes

$$Q = \frac{(\vec{p}')^2}{2m_1} + \frac{(-\vec{p}')^2}{2m_1}, \tag{1}$$

where Q is the energy produced in the exothermal reaction. The equation indicates that the energy Q is shared by the two particles with proportional to the inverse of the masses. For example, in the (d+d→ p + t) reaction 75% of the energy is taken by the proton and 25% goes to the triton.

For the case of (d +d→^{4} He + γ) reaction, small modification is necessary because the photon γ is the relativistic particle and whose energy is $p'c$, the energy conservation becomes

$$Q = \frac{(\vec{p}')^2}{2M_4} + p', \tag{2}$$

where M_4 is the mass of ^{4}He, and the natural units $c = \hbar = 1$ are adopted. Since numerically $Q = 23.9$MeV and $M_4 = 3732$MeV, by solving Eq. (2) we obtain $p'/Q = 0.99682$, which means that 99.682% of the energy is taken away by the γ ray and only 0.318% becomes the kinetic energy of ^{4}He. These sharings of the energy $Q$are confirmed by the experiments of d+d reactions at very low energy.

On the other hand, in the (two-body $\to$ one-body) reaction, the energy conservation and the momentum conservation can not be compatible. This is because in the c.m. system, the momentum of the final state particle p' must be zero from the momentum conservation. However, the energy conservation requires $p' = \sqrt{2M_4 Q}$. Therefore, for exothermal reaction: $Q > 0$, the two conservations are not consistent.

It is well-known from Noether's theorem that the translational invariance of the nuclear system in vacuum is responsible for the momentum conservation. Since the Lagrangian of the nuclear system has the form

$$L = \sum_i \big(-\frac{1}{2m_i}\big)\nabla_i^2 - \sum_{i>j} V_{ij(\vec{r}_i - \vec{r}_j)}, \tag{3}$$

L does not change when we shift all the coordinates $\vec{r}_i$ by $\vec{a}$, namely ($\vec{r}_i \to \vec{r'}_i = \vec{r}_i + \vec{a}$). In the new coordinate $\vec{r'}_i$, L remain the same. So the homogeneity of the system in space direction is responsible for the momentum conservation. Likewise the homogeneity in the direction of time is the cause of the energy conservation.

On the other hand, such a homogeneity in space is destroyed when the external potential exists at the specified point in space. As is well-known in the problem of the scattering by the potential, the momentum is transferred to the external potential $V(r)$ with the scattering amplitude $a(q)$ introduced by

$$a(q) = -(2m/4\pi\hbar)\int \mathrm{d}^3r' \exp[i\vec{q}\cdot\vec{r'}]V(r'), \tag{4}$$

and the probability of the occurrence of such a momentum transfer is $|a(q)|^2$. In our case of d +d $\to ^4$He), the momentum transfer is $q = \sqrt{2M_4 Q} = 422\,\mathrm{MeV/c}$, which corresponds to the wave length $\lambda = 2\pi\hbar/q = 3.0$ fm. Large value of the momentum transfer q means the very localized potential, whose spread Δr is related to q by the "uncertainty relation" $q \cdot \Delta r \sim \hbar$, when the functional form of $V(r)$ is Gaussian. In this way we can conclude that the nuclear CF can progress under the influence of the external potential whose locality is the order of fm.

We can narrow down the underlying mechanism of the nuclear CF, further. In general the nucleon can respond only to the two types of external fields, which are the electric field and the magnetic field. In fact the electric charges of protons interact with the electric field, whereas the magnetic dipole moments of nucleons interact with the magnetic field. Let us introduce the source particle X of the external field. Such a source particle X must attract fuel deuterons to fuse, so the candidate is the electron when the external field is electric. However, this does not work well, since the lower bound of the size of the wave packet of the electron is limited by its Compton wave length, which is around 400 fm, and so it cannot produce the sharply localized potential $V(r)$ with 1 fm in size. On the other hand if the source X is the magnetically charged particle (magnetic monopole), it attracts the nucleus when the tail of the magnetic moment orients to the direction of the magnetic monopole. Since the magnetic Coulomb field $^*e\vec{r}/r^3$ accompanies the magnetic monopole, the interaction potential of the magnetic moment of the nucleon is

$$V_\mathrm{m}(r) = -\kappa_{tot}\frac{^*ee}{2m_p}\frac{(\vec{\sigma}\cdot\vec{r})}{r^3}F(r), \quad \text{with} \quad F(r) = 1 - \mathrm{e}^{-ar}(1 + ar + a^2r^2/2), \tag{5}$$

where $F(r)$ is the form factor of the nucleon, and numerically $a = 6.04\mu_\pi$. If we substitute $V_\mathrm{m}(r)$ of Eq. (5) into Eq. (4), we shall find that the scattering amplitude $a(q)$ has appreciable value at the required value of q, and which means that the momentum transfer to the external field in the d+d $\to ^4$He) goes smoothly. Moreover the potential $V_\mathrm{m}(r)$ has the strength of the strong interaction. In the next section we shall see that the Coulomb field of Dirac's magnetic monopole is super-strong, namely $^*e^2/\hbar c = 137/4$, which is the magnetic counterpart of the ordinary fine structure constant $e^2/\hbar c = 1/137$. Therefore, the coefficient ($^*ee$) of Eq. (5) is 1/2.

In this way we found that the reaction (d+d $\rightarrow ^4$He, which is forbidden in the nuclear physics in vacuum, can occur if the reaction progress under the influence of the external field produced by the magnetic monopole. In Section 3, a brief review of the magnetic monopole will be given.

3. Brief Review of the Theory of the Magnetic Monopole

It is well-known that the Maxwell equations in vacuum are symmetric under the duality transformation, which interchange the electricity and the magnetism. On the other hand, the full Maxwell equations with the source and current terms do not have such a symmetry, and therefore the electricity and the magnetism are not treated on the same footing. For example, although $\vec{D}$ has source ρ, $\vec{B}$ does not have.

3.1. Maxwell equations with dual symmetry

In 1931, Dirac proposed to modify the Maxwell equations to restore the duality symmetry, which are:

$$\begin{aligned} \mathrm{div}\,\vec{D} &= 4\pi\rho, \\ \mathrm{div}\,\vec{B} &= 4\pi{}^*\rho \end{aligned} \tag{6}$$

and

$$\begin{aligned} \mathrm{rot}\,\vec{H} - \frac{1}{c}\frac{\partial\vec{D}}{\partial t} &= \frac{4\pi}{c}\vec{i}, \\ \mathrm{rot}\,\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} &= -\frac{4\pi}{c}{}^*\vec{i}. \end{aligned} \tag{7}$$

It is not difficult to examine the invariance of the equations under the following duality transformations.

$$\begin{aligned} \vec{D} &\to \vec{B}, \\ \vec{B} &\to -\vec{D}, \quad \text{etc.} \end{aligned} \tag{8}$$

and

$$\begin{aligned} \rho &\to \rho \\ {}^*\rho &\to -\rho, \quad \text{etc.} \end{aligned} \tag{9}$$

Once such a modification is done, we can consider a system in which the electric charge and the magnetic charge coexist. In the next subsection we shall see that, in such a system, an extra angular momentum emerges in addition to the ordinary orbital angular momentum.

3.2. Extra angular momentum characteristic to charge–monopole system

In order to see the existence of the angular momentum ${}^*QQ/c$, let us consider monopole with magnetic charge ${}^*Q$ is fixed at the origin, and a particle of mass M and electric charge Q is moving in the magnetic Coulomb field produced by the magnetic monopole. The equation is

$$M\ddot{\vec{r}} = (Q/c)\dot{\vec{r}} \times \vec{B}, \quad \vec{B} = {}^*Q\frac{\vec{r}}{r^3}, \tag{10}$$

and as in the case to prove the conservation of the angular momentum, we make the vector product $\vec{r}\times$ of the equation of motion

$$M\frac{\mathrm{d}}{\mathrm{d}t}(\vec{r}\times\dot{\vec{r}}) = \big(\frac{{}^{*}QQ}{c}\big)\frac{\vec{r}\times(\dot{\vec{r}}\times\vec{r})}{r^3}. \tag{11}$$

Because of $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}\big(-\vec{c}(\vec{a}\cdot\vec{b}\big)$, we obtain

$$M\frac{\mathrm{d}}{\mathrm{d}t}(\vec{r}\times\dot{\vec{r}}) = \big(\frac{{}^{*}QQ}{c}\big)\big(\frac{\dot{\vec{r}}}{r}-\frac{\vec{r}\vec{r}\cdot\dot{\vec{r}}}{r^3}\big) = \big(\frac{{}^{*}QQ}{c}\big)\frac{\mathrm{d}}{\mathrm{d}t}\big(\frac{\vec{r}}{r}\big). \tag{12}$$

The same extra angular momentum is also obtained if we integrate the angular momentum density $\vec{r'}\times\epsilon\mu\vec{S}(\vec{r'})$ stored in space, in which $\vec{S}$ is Poynting vector. Therefore, what is conserved in time is $M\vec{r}\times\dot{\vec{r}} - ({}^{*}QQ/c)\hat{r}$.

3.3. Charge quantization condition

Since in quantum mechanics a component of the angular momentum can assume only the integer multiple of $\hbar/2$, we can derive the charge quantization condition of Dirac[?]

$$\frac{{}^{*}QQ}{2} = \frac{\hbar}{2}n, \quad n = 0, \pm1, \pm2, \cdots \quad \text{(Dirac)}. \tag{13}$$

On the other hand, Schwinger claims that the component of the extra angular momentum assumes not $\hbar/2$ but $\hbar n$ as in the case of the orbital angular momentum, because the term $-({}^{*}QQ/c)\hat{r}$ is obtained classically. Since we are going to consider both cases, let us introduce a number D and assign $D = 1$ for Dirac and $D = 2$ for Schwinger, and write the charge quantization as

$$\frac{{}^{*}QQ}{\hbar c} = \frac{D}{2}n, \quad n = 0, \pm1, \pm2, \cdots \tag{14}$$

If we write the non-zero smallest magnetic charge as ${}^{*}e$, Q becomes

$$Q = \frac{\hbar c}{{}^{*}e}n, \quad n = 0, \pm1, \pm2\cdots \tag{15}$$

This equation indicate that the electric charge Q is discrete and is integer multiple of $(\hbar c/{}^{*}e)$. Moreover we can understand the equality of the electric charges of the electron and proton up to the sign, on the other hand experimentally the equality has been known with extremely high relative accuracy 10^{-22}.

3.4. Magnetic counterpart of the fine structure constant

For the non-vanishing smallest charges e and ${}^{*}e$, the charge quantization condition becomes ($n = 1$)

$$\frac{{}^{*}ee}{\hbar c} = \frac{D}{2}. \tag{16}$$

If we combine it with the value $e^2/\hbar c = 1/137.036$, the "fine structure constant" of the magnetic charge becomes

$$\frac{{}^{*}e^2}{\hbar c} = 1/137.036\frac{D^2}{4}. \tag{17}$$

Therefore, the Coulomb force between the magnetic monopole is super-strong, and if we compare it with the Coulomb force between the electric charges it is stronger by 4,695 times for Dirac's case ($D = 1$) and 18,780 times for Schwinger's case ($D = 2$).

Since the magnetic Coulomb field is super-strong, we may expect the anomalous magnetic moment of a nucleus is attracted to the monopole and forms the bound states. This is in fact the case, since the potential of such an interaction is

$$V_{\mathrm{m}}(r) = -(\frac{\kappa_{\mathrm{tot}} e}{2m_{\mathrm{p}}})\vec{\sigma} \cdot \vec{B}(r) \quad \text{with} \quad \vec{B} =^{*} eD\frac{\hat{r}}{r^2}, \tag{18}$$

which is valid in the region of $r \gg r_A$, where r_A is the radius of the nucleus. From the charge quantization condition

$$V_{\mathrm{m}}(r) = -(\frac{\kappa_{\mathrm{tot}} D}{4m_{\mathrm{p}}})\frac{\vec{\sigma} \cdot \hat{r}}{r^2} \tag{19}$$

Therefore, when $\kappa_{\mathrm{tot}} > 0$, the spin state belongs to the eigenvalue $(\vec{\sigma} \cdot \hat{r}) = +1$ becomes attractive, and the strength V_{m} is nearly the same as that of the nuclear potential. For example, for proton the potentials V_{m} are -9.3 MeV and -2.33 MeV at $r = 1.41$fm. and at $r = 2.82$fm. respectively (for $D = 1$).

4. System of the Nucleus and the Magnetic Monopole

In order to understand the roll of the magnetic monopole as the catalyst of the nuclear fusion reaction, we must solve the Schrödinger equation in the external magnetic Coulombic field. We shall set the mass of the magnetic monopole to infinity, although Schwinger estimated its mass was 8 GeV. Such an approximation must be useful in understanding the effect of the magnetic monopole on the small nuclei ($A < 5$), qualitatively. To include the magnetic field into the Schrödinger equation, the standard procedure is to make the replace $-i\nabla j \to (-i\nabla j - (Ze/c)A_j)$, in which $\vec{A}$ is the vector potential whose rotation is the magnetic Coulomb field. However, such a procedure does not work well, because div(rot $\vec{A}$) = 0 for the regular function $\vec{A}$. In the next subsection a method to circumvent this difficulty will be explained.

4.1. The vector potential of magnetic Coulomb field

The magnetic Coulombic field $\vec{B}$ origin satisfies produced by a magnetic monopole $^{*}Q$ fixed at the

$$\mathrm{div}\vec{B} = 4\pi^{*}Q\delta^3(\vec{r}) \tag{20}$$

and if we substitute $\vec{B} = \mathrm{rot}\vec{A}$ we obtain a contradictory equation, because div(rot $\vec{A}$) is identically zero if $\vec{A}$ is regular. On the other hand, a vector potential

$$\vec{A}^{(\mathrm{a})} = {}^{*}Q\frac{(1-\cos\theta)}{r\sin\theta}\hat{\phi}, \text{ north} \tag{21}$$

or

$$\vec{A}^{(\mathrm{b})} = -{}^{*}Q\frac{(1+\cos\theta)}{r\sin\theta}\hat{\phi}, \text{ south} \tag{22}$$

becomes the magnetic Coulomb field when we compute its rotation. These $\vec{A}$'s have singularities on the negative and positive z-axis, respectively, and whose rotation become strings (Dirac string) on the half z-axis.

Wu and Yang proposed to use $\vec{A}^{(\mathrm{a})}$ in the north hemisphere and $\vec{A}^{(\mathrm{b})}$ in the south hemisphere as the vector potential. On the overlapping region around the equator, these potentials are related by the gauge transformation, because

$$\vec{A}^{(\mathrm{a})} - \vec{A}^{(\mathrm{b})} = -(\frac{2^{*}e}{r\sin\theta})\hat{\phi} = \nabla(-2^{*}e\phi). \tag{23}$$

It is well-known that the change of the gauge of $\vec{A}^{(\mathrm{a})}$ results in only the change of the phase of the wave function Ψ. In the next subsection, we shall see this property explicitly.

4.2. The monopole harmonics $Y_{q,l,m}(\theta, \phi)$

If $\vec{L} = M\vec{r} \times \dot{\vec{r}} - q\hat{r}$, where q is the magnitude of the extra angular momentum, the eigen-function of $\vec{L}^2$ and Lz is the monopole harmonics $Y_{q,l,m}(\theta, \phi)$, [?] and which reduces to the ordinary spherical harmonics $Y_{q,l,m}(\theta, \phi)$ when $q = 0$. If we write $Y_{q,l,m}(\theta, \phi) = \mathrm{e}^{\pm iq\phi}\mathrm{e}^{im\phi}\Theta(\theta)$, where $\pm$ of the exponential corresponds to the northern and the southern hemisphere, respectively, $\Theta(z)$ must satisfy

$$[l(l+1) - q^2]\Theta = -(1-z^2)\Theta'' + 2z\Theta' + \frac{(m+qz)^2}{1-z^2}\Theta. \tag{24}$$

The explicit form of $Y_{q,l,m}(\theta,\ \phi)$ is

$$Y^{(\mathrm{n})}_{q,l,m}(\theta,\ \phi) = \sqrt{\frac{2l+1}{4\pi}}\mathrm{e}^{+iq\phi}d^{(l)}_{-q,m}(\theta)\mathrm{e}^{+im\phi} \quad \text{in } R_{\mathrm{n}} \tag{25}$$

and

$$Y^{(\mathrm{s})}_{q,l,m}(\theta,\ \phi) = \mathrm{e}^{-2iq\phi}Y^{(n)}_{q,l,m}(\theta,\ \phi) \quad \text{in } R_{\mathrm{s}}, \tag{26}$$

where R_{n} and R_{s} is the northern and southern hemispheres of the sphere, respectively, $d^{(l)}_{m',m}(\theta)$ is Wigner's d-function of rotation[?] which is widely used in the nuclear physics.

4.3. The equation of the monopole-nucleus of spin-0

It is instructive to start from the simplest problem of the motion of the single charged particle such as ^{4}He in the magnetic Coulomb field, whose Hamiltonian is

$$H_0 = \frac{1}{2m_A}\big(\vec{p} - \frac{Ze}{c}\vec{A}\big)^2. \tag{27}$$

The radial function $R(r)$, introduced by

$$\psi = R(r)Y_{q,l,m}, \tag{28}$$

satisfies the following equation:

$$\Big[-\frac{1}{2m_A r^2}\frac{\mathrm{d}}{\mathrm{d}r}\big(r^2\frac{\mathrm{d}}{\mathrm{d}r}\big) + \frac{l(1+1)-q^2}{2m_A r^2} - E\Big]R(r) = 0. \tag{29}$$

The solution, which does not blow up at $r = 0$, is the J-Bessel:

$$R(r) = \frac{1}{\sqrt{kr}}J_\mu kr, \tag{30}$$

where

$$\mu = \sqrt{l(l+1) - q^2 + 1/4} = \sqrt{(l+1/2)^2 - q^2} > 0 \quad \text{and} \quad l = |q|, |q|+1, |q|+2\cdots \tag{31}$$

For $E < 0$, there is no meaningful solution, and therefore there is no bound state.

4.4. Angular function of spin 1/2 particle in the monopole field

For the spin-0 particle, we know the monopole harmonics $Y_{q,l,m}$ is the angular function. For the spin-1/2 particle, we can construct the state of the total angular momentum (j, m) by combining spin-up and spin-down states with the monopole harmonics $Y_{q,l,m}$, in which the Clebsch–Gordan coefficients appear. In general, for given j there are two states

$$\Phi^1_{j,m} = \sqrt{\frac{j+m}{2j}}Y_{qj-1/2,m-1/2}\begin{bmatrix}1\\0\end{bmatrix} + \sqrt{\frac{j-m}{2j}}Y_{qj-1/2,m+1/2}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\sqrt{\frac{j+m}{2j}}Y_{qj-1/2,m-1/2}\\ \sqrt{\frac{j-m}{2j}}Y_{qj-1/2,m+1/2}\end{bmatrix}$$

and

$$\Phi^2_{j,m} = -\sqrt{\frac{j-m+1}{2j+2}}Y_{qj+1/2,m-1/2}\begin{bmatrix}1\\0\end{bmatrix} + \sqrt{\frac{j+m+1}{2j+2}}Y_{qj+1/2,m+1/2}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sqrt{\frac{j-m+1}{2j+2}}Y_{qj+1/2,m-1/2}\\ \sqrt{\frac{j+m+1}{2j+2}}Y_{qj+1/2,m+1/2}\end{bmatrix}$$

except for the smallest j, namely for $j = |q| - 1/2$. After Wu and Yang we shall call the (j, m) state with $j \geq |q| + 1/2$ type-A, to which two states $\Phi^1_{j,m}$ and $\Phi^2_{j,m}$ belong.

On the other hand, the state of $j = |q| - 1/2$. is the type-B and to which only one state $_{j,m}$ belongs. Since the ground state appears in the type-B state, we shall write it explicitly:

$$\eta_m \begin{bmatrix}-\sqrt{\frac{|q|-m+1/2}{2|q|+1}}Y_{q,|q|,m-1/2}\\ \sqrt{\frac{|q|+m+1/2}{2|q|+1}}Y_{q,|q|,m+1/2}\end{bmatrix}, \tag{32}$$

in which $l = |q|$ and $j = |q| - 1/2$.

4.5. The eigenvalue problem of the proton–monopole system

The equation to be solved is

$$\Big[\frac{1}{2M}(-i\vec{\nabla} - e\vec{A})^2 + V(r) - \frac{1}{r^2}\frac{b(r)}{2M}(\vec{\sigma}\cdot\hat{r})\Big]\psi = E\psi, \tag{33}$$

where

$$b(r) = \bar{b}\Big(1 - \mathrm{e}^{ear}\big(1 + ar + \frac{a^2r^2}{2}\big)\Big) \tag{34}$$

with

$$\bar{b} = \kappa_{\mathrm{tot}}q = 2.7928, \quad q = D/2 \quad \text{and} \quad a = 6.04\mu\pi. \tag{35}$$

If we remember

$$\vec{L} = \vec{r}\times(\vec{p} - e\vec{A}) - q\hat{r} \quad \text{with} \quad q = D/2 \tag{36}$$

and

$$(\vec{p} - e\vec{A})^2 = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\big(r^2\frac{\mathrm{d}}{\mathrm{d}r}\big) + \frac{1}{r^2}(\vec{L}^2\vec{q}^2) \tag{37}$$

for the type-B amplitude

$$\psi = \frac{1}{r}g(r)\eta m \tag{38}$$

the Schrödinger equation of the radial function becomes

$$\Big[\frac{\mathrm{d}^2}{dr^2} + 2M(E - V(r)) - \frac{R}{r^2}\Big]g(r) = 0 \tag{39}$$

with

$$R = (j + 1/2)(j + 3/2) - q^2 - b(r)q/|q| = |q|(1 - b(r)/q). \tag{40}$$

When $V(r) = 0$, the equation is

$$\Big[\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \kappa - \frac{1}{r^2}(|q| - b(r))\Big]g(r) = 0 \quad \text{with } \kappa^2 = -2ME \tag{41}$$

For $ar \gg 1$, the solution, which damps for large ar, is the K-Bessel:

$$g(r) = \sqrt{\kappa r}K_{i\mu}(\kappa r) \quad \text{with} \quad i\mu = \sqrt{|q|}(1 - \kappa_{tot}) + 1/4. \tag{42}$$

Since in small ar region the regular solution is determined by the index equation:

$$g(r) = c_1(ar)^{\alpha} \quad \text{with} \quad \alpha = 1/2 + \sqrt{|q|} + 1/4, \tag{43}$$

we can solve the equation numerically and match to the logarithmic derivative of the Bessel function at $ar = x_m \gg 1$.

We can see that there are infinitely many energy levels which shrink exponentially to $\kappa = 0$. Since for pure imaginary index $i\mu$, the asymptotic form of the K-Bessel is

$$K_{i\mu}(x) = \frac{\pi}{\sin h\pi\mu}\sin(\mu \log x/2)[1 + o(x)], \tag{44}$$

the logarithmic derivative reduces to

$$r\frac{\mathrm{d}}{\mathrm{d}r}log(\sqrt{\kappa r}K_{i\mu}(\kappa r)) = \frac{1}{2} + \mu\cot(\mu\log\frac{\kappa r}{2}). \tag{45}$$

The periodicity of cot implies the eigenvalues appear in the way that $\mu\log(\kappa/a) = -\pi n' + \text{const.}$ with the integer n'. In terms of the binding energy $-E$,

$$-E_n = \frac{\kappa_n^2}{2M} = C_\infty \exp\Big[-\frac{2\pi}{\mu}(n-1)\Big], \tag{46}$$

where n is the principal quantum number.

4.6. Binding energy of magnetic monopole and nuclei of spin 1/2

The binding energies of the spin 1/2 nuclei can be estimated in the same way as the proton, by simply changing the mass, charge, $\kappa_{\rm tot}$ and the parameter a of the form factor, as long as the deformations of the nuclei in the magnetic Coulomb field are negligible. Following table is the first few binding energies $-E_n$ of the Dirac's magnetic monopole ($D = 1$) with p, t and ^{3}He along the the parameters μ and C_∞ of Eq. (46) along with the orbital radius of the ground state $\bar{r}_1$.

	Proton	Triton	^{3}He
$D = 1$	$-E_1 = 0.1882\,\text{MeV}$	$-E_1 = 1.516\,\text{MeV}$	$-E_1 = 0.2434\,\text{MeV}$
	$-E_2 = 76.046\,\text{eV}$	$-E_2 = 58.085\,\text{keV}$	$-E_2 = 2.7413\,\text{keV}$
	$-E_3 = 0.0307\,\text{eV}$	$-E_3 = 2.226\,\text{keV}$	$-E_3 = 30.047\,\text{eV}$
	$\vdots$	$\vdots$	$\vdots$
	$C_\infty = 0.1884\,\text{MeV}$	$C_\infty = 1.516\,\text{MeV}$	$C_\infty = 0.2502\,\text{MeV}$
	$\mu = 0.8040$	$\mu = 1.9263$	$\mu = 1.3921$
	$\bar{r}_1 = 11.00\text{fm}$	$\bar{r}_1 = 3.820\text{fm}$	$\bar{r}_1 = 7.371\text{fm}$

When the magnetic monopole is the Schwinger type $D = 2$, the binding energies are:

	Proton	Triton	^{3}He
$D = 2$	$-E_1 = 2.4065\,\text{MeV}$	$-E_1 = 4.366\,\text{MeV}$	$-E_1 = 1.063\,\text{MeV}$
	$-E_2 = 15.457\,\text{keV}$	$-E_2 = 0.5479\,\text{MeV}$	$-E_2 = 51.115\,\text{keV}$
	$-E_3 = 98.231\,\text{eV}$	$-E_3 = 57.766\,\text{keV}$	$-E_3 = 2.3239\,\text{keV}$
	$\vdots$	$\vdots$	$\vdots$
	$C_\infty = 2.4322\,\text{MeV}$	$C_\infty = 5.4085\,\text{MeV}$	$C_\infty = 1.1259\,\text{MeV}$
	$\mu = 1.2421$	$\mu = 5.7697$	$\mu = 2.0312$
	$\bar{r}_1 = 3.666\text{fm}$	$\bar{r}_1 = 2.779\text{fm}$	$\bar{r}_1 = 4.596\text{fm}$

4.7. Binding energy of the magnetic monopole and the deuteron

The hamiltonian of the deuteron and the magnetic monopole system $H_{^*e-\rm d}$ is

$$H_{^*e-\rm d} = H_{^*e-\rm p} + H_{^*e-\rm n} + V_{\rm p,n}, \tag{47}$$

where $V_{\rm p,n}$is the nuclear potential of p and n. The hamiltonians of the magnetic monopole and the nucleons are

$$H_{^*e-p} = \frac{1}{2m_{\rm p}}(-i\vec{\nabla} - e\vec{A})^2 - \kappa_{\rm tot}^{(\rm p)} D \frac{^*ee}{2m_{\rm p}} \frac{(\vec{\sigma}\cdot\hat{r})}{r^2} F(r) \tag{48}$$

and

$$H_{^*e-\rm n} = \frac{1}{2m_{\rm n}}(-i\vec{\nabla})^2 - \kappa_{\rm tot}^{(\rm n)} D \frac{^*ee}{2m_{\rm p}} \frac{(\vec{\sigma}\cdot\hat{r})}{r^2} F(r), \tag{49}$$

where the form factor function $F(r)$ is common to the proton and the neutron, and

$$F(r) = (1 - \exp[-ar](1 + ar + \frac{a^2}{2}r^2)) \qquad \text{with} \qquad a = 6.04\mu_\pi, \tag{50}$$

in which a is related to the radius of the nucleon $\bar{r}$ by $a = \sqrt{12}/\bar{r}$, and numerically $\bar{r} = 0.81\text{fm}$.

We searched for the energy level of the ground state of the $^*e - \mathrm{d}$ system by the variational calculation, namely minimizing $\langle\Psi|H_{^*e-\mathrm{d}}|\Psi\rangle/\langle\Psi|\Psi\rangle$ by changing the trial function Ψ, which is the function of $\vec{r}_\mathrm{p}$ and $\vec{r}_\mathrm{n}$ including spin. We restrict the trial function to the type-B spin-angular function of Eq. (38) for proton, and for $(\vec{r}_\mathrm{n} - \vec{r}_\mathrm{p})$ the two S-waves are retained, namely both the spin-singlet and the spin-triplet are included. Concerning the radial part, the prescription of the Gauss expansion method (GEM) is applied, namely the radial trial function is the sum of the Gauss functions whose range parameters are the "geometric series". The numerical result obtained is $E = -4.5$MeV in which the energy level of the state, where the three particles separate infinitely, is chosen as zero. Therefore, since the binding energy of the deuteron is $B_\mathrm{d} = 2.2$MeV, we may say that the binding energy of ($^*e - \mathrm{d}$) is $B_{\mathrm{d}+^*e} = 2.3$ MeV.

This value of the binding energy leads to an important consequence that the two deuterons trapped by the same magnetic monopole is energetically impossible to decay to (t+p) or (^{3}He + n) state since the binding energies are 8.5 and 7.7 MeV, respectively for t and ^{3}He, while the binding energy of $\mathrm{d} -^* e - \mathrm{d}$ state is 9.0 MeV for $D = 1$ and 14.0 MeV for $D = 2$ magnetic monopoles. Since our calculation is variational, there remains some possibility for the binding energy of the deuterons with $^*e$ to in- crease slightly. In the next Subsection, we shall consider this important phenomenon characteristic to the nuclear cold fusion that the main channels (t+p) or (^{3}He + n) are energetically closed in some detail.

4.8. Mechanism to close the d + d → (t + p) and (n +^{3} He) channels in CF

Since the most important feature of the nuclear CF reaction is the change of the open channels of the final states of the d+d reaction, it is interesting to see whether a "theory" can successfully explain such a change. It is experimentally well-known that the final states of the low energy d+d reaction in vacuum are (t + p) and (^{3}He + n) with momentum conservation and the isotopic invariance. On the other hand, in the nuclear cold fusion (CF), the process becomes d + d ^{4}He, namely (two-body) → (one-body) type. As explained in Section 2, the momentum conservation of the such a system is not satisfied, and so in order to restore the momentum conservation, we need the external (namely, outside of the nuclear system) potential to absorb the momentum transfer $\vec{q}$. Numerically in our case of the ^{4}He production, the the magnitude of the transferred momentum is $q = \sqrt{2M_4Q} = 422\,\mathrm{MeV/c}$, where M_4 is the mass of α-particle and $Q = 23.9$MeV. However, a soft and spread external potential $V(r)$ cannot do the job to absorb such a large momentum transfer q, instead we can estimate spacial size Δr of the external potential by using the relation $q\Delta r \sim \hbar$. We obtain $\Delta r \approx 0.45$ fm. We can narrow down the type of the external field further. In order to obtain the external potential, the external field must interact with the nucleons. We know that the nucleon has two attributes, namely the electric charge and the magnetic dipole moment. Therefore, the candidate of the external field is either the electric field or the magnetic field. However, if we remember that the electron cloud spreads too widely and cannot produce the localized electric field, the only candidate of the external field becomes the magnetic field whose source particle is the magnetic monopole. Because of the charge quantization condition of Dirac, $^*ee/\hbar c = 1/2$, the magnetic monopole is accompanied by the super-strong magnetic Coulomb field, which attracts surrounding nucleons or small nuclei with non-zero magnetic moment, and forms the bound state. In Section 4.6, the energy levels of the bound states are shown along with the orbital radius $\bar{r}_1$ of the ground state. Since the orbital radius is several fm, when two deuterons are trapped by the same magnetic monopole, it is expected that the nuclear reaction starts to occur and become the more stable particle ^{4}He. When the binding of the deuterons to the magnetic monopole are too deep, we can expect such a state cannot energetically transit to the continuous states of (t+p) and (n +^{3} He). Let us consider such closing of the t or n channels in more detail.

It is important to understand how the main channels (t + p) and (n+^{3}He) of the d + d reaction are closed when the nuclear reaction starts after the two deuterons are trapped by the same magnetic monopole $^*e$. The energy level of the starting state E_{ini} is

$$E_\mathrm{ini} = -2(B_{\mathrm{d}+^*e} + B_\mathrm{d}), \tag{51}$$

in which the zero of the energy level is chosen where all the particles, namely the nucleons and the magnetic monopole, separate infinitely. In Eq. (51), $B_{\rm A}$ is the binding energy of the nucleus A, and numerically they are $B_{\rm d} = 2.2$, $B_{\rm t} = 8.5$, $B_{^3{\rm He}} = 7.7$ and $B_{^4{\rm He}} = 28.3$MeV, respectively. $B_{^*e-{\rm d}}$ is the binding energy of the $d -^* e$. If we remember that the continuous spectrum of the (t + p) system appears in $E \geq -B_{\rm t}$, from the energy conservation initial state cannot become (t + p) as long as $E_{\rm ini} < -B_{\rm t}$. Therefore, from Eq. (51) the condition of the closing of the (t + p) channel is:

$$B_{{\rm d}+^*e} > \left(\frac{B_{\rm t}}{2} - B_{\rm d}\right) = 2.05\,{\rm MeV} \equiv B^{\rm critical}({\rm t+p}). \quad (52)$$

Likewise the continuous (n $+^3$ He) channel is closed when

$$B_{{\rm d}+^*e} > \left(\frac{B_{^3{\rm He}}}{2} - B_{\rm d}\right) = 1.65\,{\rm MeV} \equiv B^{\rm critical}({\rm n}+^3{\rm He}). \quad (53)$$

is satisfied. However, from the variational calculation given in Section 4.7, we know $B^*e - {\rm d} = 2.5$ MeV, and which means both of (t + p) and (n $+^3$ He) channels are closed which agrees with the observations of the CF experiments.

Concerning the one particle ^{4}He state, since the binding energy of ^{4}He is very large, namely $B_{^4{\rm He}} = 28.3$MeV, the ^{4}He channel stays open as long as

$$B_{{\rm d}+^*e} > \left(\frac{B_{^4{\rm He}}}{2} - B_{\rm d}\right) = 11.95\,{\rm MeV} \equiv B^{\rm critical}({\rm n}+^4{\rm He}). \quad (54)$$

is satisfied. Therefore, the only open channel is ^{4}He. Since α-particle is spin-0, it is not attracted by the magnetic monopole, so it must be emitted by the monopole.

5. Penetration factor

5.1. Penetration factor in vacuum

The most disturbing mystery of the nuclear cold fusion is why the two positively charged nuclei of zero-incident energy can come close to the fm region. The standard argument against such a phenomenon is to calculate the penetration factor T by using the WKB approximation:

$$T = {\rm e}^{-2\tau} \quad \text{with} \quad \tau = {\rm int}_{\rm a}^{\rm b}\sqrt{2m_{\rm red}(V(x) - E)}{\rm d}x/\hbar, \quad (55)$$

where the domain of penetration is $[a, b]$, and numerically we shall put $a = 1$fm and $b = 1$Å, since nucleus is shielded by the atomic electron. $m_{\rm red}$ is the reduced mass of the system. We shall consider the penetration of the (electric) Coulomb barrier of t+p and d+d system in the low energy limit $E \to 0$. Since $V_{\rm C}(x) = +e^2/x$, τ becomes

$$T = {\rm int}_{\rm a}^{\rm b}\sqrt{2m_{\rm red}e^2/x}{\rm d}x = \sqrt{4m_{\rm red}e^2}(\sqrt{b} - \sqrt{a}). \quad (56)$$

Therefore, numerically $T = 6.78 \times 10^{-92}$ for t +p system, whereas for the system of d+ d, $T = 5.33 \times 10^{-106}$. These extremely small values of T mean that such phenomena "never" occur.

5.2. Change of the penetration potential

Let us consider the case of t + p reaction. In the last section, we saw that a proton can form the bound state with the magnetic monopole $^*e$, whose orbital radius is several fm. If the second nucleus, in our case it is the triton t, approaches to the bound state ($^*e$ − p), the potential felt by the incoming second particle is the sum of the repulsive Coulomb potential and the interaction between the magnetic monopole and the magnetic dipole of the triton. When the direction of the magnetic moment of the trition orients to the opposite to the location of the magnetic monopole, the interaction becomes attractive, namely the sum of the potentials becomes

$$V_1(x) = +e^2/x - \kappa_{\rm tot}\frac{D^* e\, e}{2m_{\rm p}}\frac{1}{x^2}. \qquad (57)$$

It becomes easier for the triton to penetrate to the region of the composite system $^*e - \mathrm{p}$. In this way two nuclei p and t are trapped by the same magnetic monopole $^\star e$.

Since t and p are confined in the few fm region, it is unstable and fuses to become more stable particle ^{4}He. Since the spin of ^{4}He is zero, it cannot form a bound state with $^*e$, and the energetic ^{4}He is emitted from the magnetic monopole. There remains a fresh magnetic monopole, and it starts to attract the surrounding nuclei anew. In this way, the magnetic monopole plays the roll of the catalyst of the nuclear cold fusion reaction t + p→^{4}He. Likewise d + d→^{4}He of zero-incident energy must also occur.

If we compute the penetration factor T by using the new potential $V_1(x)$ of Eq. (53), although the value of T is improved, it is not sufficient to cause the expected nuclear cold fusion. However, in the electron rich environment, the magnetic monopole is shielded by an electron cloud whose radius is around the electron Compton wave length $\hbar/m_{\rm e}c = 386.2$ fm. In the next subsection, we shall show that the Dirac equation of the electron in the external magnetic Coulomb field has such a solution which was found by Kazama and Yang.[?] Therefore, the pure repulsive (electric) Coulomb term $+e^2/x$ of Eq. (53) is modified to $+(e^2/x)\exp[-2m_{\rm e}x]$.

5.3. Dirac equation in the external magnetic Coulomb field

The equation to solve is obtained by making the standard gauge substitution $\vec{p} \to \vec{p} - (Ze/c)\vec{A}$ of the free Dirac equation, where $\vec{A}$ is the vector potential whose rotation is the magnetic Coulomb field of strength $^*eD$ and which is given in Eqs.(21) and (22). If we choose the representation of the 4×4 Dirac matrices $\vec{\alpha}$ and β as,

$$\vec{\alpha} = \begin{bmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{bmatrix} \quad \text{and} \quad \beta = \begin{bmatrix} I_2 & 0 \\ 0 & -I_2 \end{bmatrix}, \qquad (58)$$

the Hamiltonian becomes

$$H = \vec{\alpha}\cdot(-i\vec{\nabla} - e\vec{A}) + \beta M - \frac{\kappa_\alpha q}{2MR^2}\begin{bmatrix} \vec{\sigma}\cdot\hat{r} & 0 \\ 0 & -\vec{\sigma}\cdot\hat{r} \end{bmatrix}, \qquad (59)$$

where the last term is the Pauli term, and in which κ_a is the anomalous magnetic moment of the electron in the unit of the Bohr magneton $e^2/2M$, whose value is 0.00116, or 2.13 in the unit of the nuclear magneton $e^2/2m_{\rm p}$.

Since t and p are confined in the few fm region, it is unstable and fuses to become more stable particle ^{4}He. Since the spin of ^{4}He is zero, it cannot form a bound state with $^*e$, and the energetic ^{4}He is emitted from the magnetic monopole. There remains a fresh magnetic monopole, and it starts to attract the surrounding nuclei anew. In this way, the magnetic monopole plays the roll of the catalyst of the nuclear cold fusion reaction t + p→^{4}He. Likewise d + d →^{4}He of zero-incident energy must also occur.

If we compute the penetration factor T by using the new potential $V_1(x)$ of Eq. (53), although the value of T is improved, it is not sufficient to cause the expected nuclear cold fusion. However, in the electron rich environment, the magnetic monopole is shielded by an electron cloud whose radius is around the electron Compton wave length $\hbar/m_{\rm e}c = 386.2$ fm. In the next subsection, we shall show that the Dirac equation of the electron in the external magnetic Coulomb field has such a solution which was found by Kazama and Yang.[?] Therefore, the pure repulsive (electric) Coulomb term $+e^2/x$ of Eq. (53) is modified to $+(e^2/x)\exp[-2m_{\rm e}x]$.

Let us consider an eigen-value problem $H\psi = E\psi$. In particular, the type-(B) solution has the form:

$$\psi_m^{(B)} = \begin{bmatrix} f(r)\eta_m \\ g(r)\eta_m \end{bmatrix}, \tag{60}$$

where η_m is the hedgehog solution of the spin 1/2 particle given in Eq. (32), and which is the eigen-function of $(\vec{\sigma} \cdot \hat{r})$ with eigen-value $+1$. By substituting this form into the eigen-value equation, we obtain

$$[M - E - \kappa_a|q|(2Mr^2)^{-1}]f(r) - iq|q|^{-1}(\partial_r + r^{-1})g(r) = 0 \tag{61}$$

and

$$-iq|q|^{-1}(\partial_r + r^{-1})f(r) - [M + E - \kappa_a|q|(2Mr^2)^{-1}]g(r) = 0. \tag{62}$$

If we introduce $F(r)$, $G(r)$ by

$$f(r) = \frac{\kappa_a q}{|\kappa_a q|}\frac{F(r)}{r} \quad \text{and} \quad g(r) = -i\frac{G(r)}{r}. \tag{63}$$

Equations (61) and (62) become

$$\frac{\mathrm{d}G}{\mathrm{d}r} = \left[-\frac{(E - M)\kappa_a}{|\kappa_a|} - \frac{|\kappa_a q|}{2Mr^2}\right]F(r), \tag{64}$$

$$\frac{\mathrm{d}F}{\mathrm{d}r} = \left[\frac{(E + M)\kappa_a}{|\kappa_a|} - \frac{|\kappa_a q|}{2Mr^2}\right]G(r). \tag{65}$$

Furthermore, if we introduce a new independent variable ρ by $r = |\kappa_a q|\rho(2M)^{-1}$, the equations change to the simpler form:

$$\frac{\mathrm{d}G}{\mathrm{d}\rho} = \left(A - B - \frac{1}{\rho^2}\right)F \tag{66}$$

$$\frac{\mathrm{d}F}{\mathrm{d}\rho} = \left(A + B - \frac{1}{\rho^2}\right)G, \tag{67}$$

where

$$A = \frac{\kappa_a|q|}{2} \text{ and } B = \frac{\kappa_a|q|}{2M}E \tag{68}$$

By changing the energy variable B, we must find the eigen-functions, which satisfy the boundary conditions that for $\rho \to \infty$ they must damp, and at $\rho = 0$ they must vanish: $F(0) = 0 \quad G(0) = 0$. The solution which satisfies the requirements is

$$B = 0, \quad F(\rho) = -G(\rho) = \exp[-A\rho - 1/\rho]. \tag{69}$$

In terms of the former variable r, the eigen-value and the wave function is

$$E = 0, \quad F = -G = \exp[-Mr - \kappa_a|q|/(2Mr)]. \tag{70}$$

The requirement $\kappa_a > 0$ is necessary for the wave function to vanish at $r = 0$. However, in QED we know that the first term of the perturbation of κ_a is the Schwinger term $\kappa_a = e^2/2\pi = 0.00116$ and so it is positive.

The form of Eq. (70) of the eigen-function indicates that the radius of the orbit of the wave function is $r = 1/M$, the electron Compton wave length. Therefore, a magnetic monopole or the ($^*e$ – p) system is shielded by the electron cloud whose size is the Compton length of the electron It /Mc. This fact makes it much easier for the second nucleus t to penetrate to the region of the composite particle ($^*e$ − p). In the next subsection, we shall see the drastic change of the numerical value the penetration factor T of ($^*e$ − p) − t in the electron-rich environment. Since the (electric) Coulomb potential is shielded, the penetration potential becomes

$$V_2(x) = +(e^2/x)\exp[-2Mx] - \kappa_{\rm tot}\frac{D^*e\,e}{sm_{\rm p}}\frac{1}{x^2}, \tag{71}$$

where M is the mass of the electron.

5.4. Penetration factor T of the (t+p) and (d +d) systems

In this subsection, we shall give the table of the penetration factor T for the d +d and p+t process when the penetration potential of the incoming second nucleus $V_2(x)$ of Eq. (67) is used in the WKB calculation of Eq. (51). We shall use the notation such as ($^*e$ − p) − t, which means that firstly the magnetic monopole $^*e$ traps the proton p and a composite particle ($^*e$ − p) is formed, and then the second nucleus t approaches to the composite particle ($^*e$ − p). In the process ($^*e$ − t) − p, the roll of p and t is interchanged. Numerical values of τ and T of Eqs. (52) and (51) are

	($^*e$ − t) − p	($^*e$ − p) − t	($^*e$ − d) − d
$D = 1$	$\tau = 7.52$ $T = 2.93 \times 10^{-7}$	$\tau = 3.76$ $T{=}5.39{\times}10^{-4}$	$\tau = 9.45$ $T{=}6.22{\times}10^{-9}$
$D = 2$	$\tau = 4.51$ $T{=}1.22{\times}10^{-4}$	$\tau = 2.41$ $T{=}8.01{\times}10^{-3}$	$\tau = 7.88$ $T{=}1.42{\times}10^{-7}$

These values of the penetration factor T should be compared with T in the vacuum

t+p	D+D
$\tau = 104.96$	$\tau = 121.20$
$T = 6.78 \times 10^{-92}$	$T = 5.33 \times 10^{-106}$

Therefore, in the framework of the ordinary nuclear physics, in which the constituent particles are proton and neutron, the nuclear reaction of the zero incident energy can- not occur. This is what almost all the nuclear physicists claim. On the other hand, if we extend the nuclear physics to include the magnetic monopole as an additional ingredient, the nuclear cold fusion reaction occurs rather freely. Since the Hamiltonian of such a system is known as given in Eq. (47), it is not difficult to construct the world in which $^*e$, p and n are the ingredients, because from 1990s we have developed the techniques to treat the few-body system by solving honestly the simultaneous Schr¨odinger equations.

6. Problem of the Reproducibility of Nuclear Cold Fusion

The last mystery of the nuclear cold fusion is that such a reaction does not start on demand. It is said that we have to wait at least a few weeks before the heat generation starts anew. This sporadic nature of the nuclear cold fusion is the main obstacle in understanding the phenomenon as the scientific fact, or in applying it to the practical use. It is believed that when we reuse the successful cathode, the probability of starting of the heat generation increases. It is interesting to consider how to understand the sporadic nature, when the magnetic monopole plays the roll of the catalyst of the nuclear fusion reaction.

6.1. "One-particle catalyst"

We saw that if a magnetic monopole $^*e$ exists in the domain of the high density deuteron, it forms the bound state (*e–d) and sometimes forms the doubly bound state (d–*e–d). However, since the size of the orbits of the deuterons at $^*e$ is around a few fm, (d–*e–d) is unstable and two deuterons become a more stable ^{4}He. Since the spin of ^{4}He is zero, there is no attractive potential between ^{4}He and $^*e$, ^{4}He must be emitted with energy 23.8 MeV. Since we rely on the magnetic monopole floating in nature, which is the rare particle, nothing occurs until a magnetic monopole moves into and stops in the region where the density of the deuteron is high. In this way the probability comes into the scene. We need not worry about the apparent sporadic nature of the phenomenon when the physical law of the underlying mechanism is understood well.

In order to understand the non-reproducibility, it is useful to introduce the concept of the "one-particle catalyst". In the ordinary catalyst, although the number of the catalyst particle $\langle N_c \rangle$ is much small compare to the particle number of the reactant $\langle N_r \rangle$, the value of $\langle N_c \rangle$ is still macroscopic number. Let us consider the case where $\langle N_c \rangle$ decreases to a finite number. Number $\langle N_c \rangle$ must be different from experiment to experiment and fluctuate around the average value $\langle N_c \rangle$ with the spread $\sqrt{\langle N_c \rangle}$. If the average $\langle N_c \rangle$ decreases further and becomes much smaller than 1, we must encounter the non-reproducibility. In decreasing $\langle N_c \rangle$, we must increase the strength of each catalyst particle properly.

The length of the track of the emitted α is around 0.1mm, the energy of the α is transferred to the electron and then to the lattice in the neighborhood of the trapped magnetic monopole, which is the one-particle catalyst. Since, as we shall see in the next subsection, the trapping potential of the magnetic monopole is proportional to $1/T$, where T is the temperature of the of the place of $^*e$, the magnetic monopole must hop to the neighboring cooler place. In fact Szpak and Mosier-Boss [?] observed by using the infrared camera, the discrete hot-spots are spreading in the cathode.

6.2. Energy of the magnetic monopole in the lattice

Let us consider a medium lattice, which consists of atoms with the magnetic dipole moment κ in the unit of the Bohr magneton $e/2m_e$. Suppose that at first the directions of the magnetic dipoles fixed on the lattice points are random, and consider what happen when a magnetic monopole of the strength $^*eD$ moves into and stops in the lattice. The dipoles of the atoms must align and the energy of each dipole is $\Delta V = -\kappa(e/2m_e)(^*eD)/r^2$, where r is the distance of the magnetic dipole from the magnetic monopole $^*e$. Because of the thermal agitation, whose energy is $k_B T/2$, there is a sphere of radius R determined by the temperature T, and outside of which the directions of the dipoles are random. R is determined by

$$\kappa(e/2m_e)^* \frac{eD}{R^2} = \frac{k_B T}{2}. \tag{72}$$

The trapping energy of the magnetic monopole is the sum of the alignment energy in the sphere R, namely

$$V = -\frac{4\pi}{a^3} \int_0^R r^2 \mathrm{d}r\, \kappa(e/2m_e)^* \frac{eD}{r^2}, \tag{73}$$

where a is the lattice constant.

Because of the charge quantization condition $^*ee = 1/2$, the numerical values of V and R become for the lattice constant 1Å

$$V = -\left(\frac{\kappa D}{2}\right)^{3/2} \frac{14.24\,\mathrm{keV}}{\sqrt{T}} \text{ and } R = \sqrt{\frac{\kappa D}{2}} \frac{2.97 \times 10^{-6}\,\mathrm{cm}}{\sqrt{T}}. \tag{74}$$

Therefore, for $T = 300°$, $|V|$ is several keV and R is few nm in size.

6.3. Energy loss of the magnetic monopole in matter

If the magnetic monopole plays the roll of the catalyst of the nuclear fusion reaction, we must understand its behavior in the medium. The stopping power of a charged particle ze is given by the Bethe–Bloch's equation:

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{KZ_{\mathrm{med}\rho\mathrm{med}}}{A_{\mathrm{med}}}\left(\frac{z}{\beta}\right)\left[\log\left(\frac{2m_e\gamma^2\beta^2c^2}{I}\right) - \beta^2 - \delta - \frac{C}{z_{\mathrm{med}}}\right], \tag{75}$$

where $K = 4\pi N_A r^2 m_e c^2 = 0.3070\mathrm{MeVcm}^2/g$. Here Z_{med} and A_{med} are the charge and mass numbers of the medium and ρ_{med} is the mass density of the medium. δ and C are frequently small function and I is the order of the binding energy of the electrons in the medium.

Because the force between the incoming magnetic charge $^*eD$ and the electron is the Lorentz force, we must change to $ze \rightarrow^* eD\beta$ in Eq. (71) to get the stopping power of the magnetic monopole in a medium. There is another source to lose the energy of the magnetic monopole. The loss of the energy by the radiation is given by the Larmor's formula $\mathrm{d}E/\mathrm{d}t = -(2/3)e2|a|^2$ where $|a|$ is the acceleration. The radiation loss of the magnetic monopole is

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{2}{3}(^*eD)^2|\overrightarrow{a}|^2. \tag{76}$$

Since on the surface of the medium the trapping potential V changes, the acceleration becomes non-zero. Because of the large value of the magnetic "fine structure constant" $^*e^2$, the radiation loss of the magnetic charge must be very large. In particular, when the medium is the collection of the nano-particle, the magnetic monopole passes through the surfaces many times and it looses energy effectively and trapped by the medium.

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