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Erratum

Errata and Comments on a Recent Set of Papers in *Journal of Condensed Matter in Nuclear Science*

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Abstract

Recently we published a series of papers that focused on coherent energy exchange in the context of the lossy spin–boson model in this journal. Minor errors have been identified, and we provide corrections here. In addition, we give additional discussion of some of the issues.

Keywords: Excess heat, Fleischmann–Pons effect, Spin–boson model

1. Introduction

Over the years we have been developing the lossy spin–boson model as part of a proposed explanation for coherent energy exchange for the excess heat effect in the Fleischmann–Pons experiment. We recently documented some of our work on the model in a series of papers that appeared recently in this journal. After the papers were published, we noticed some errors while using them in the course of our research efforts. We found enough errors in both the text and in the equations that it seemed useful to go through them systematically and develop an errata.

However, in working with the papers we noticed an additional issue as well. In some cases results are given that probably should have been discussed further, and there are subtleties which are probably worth additional comment. In what follows, we list the errors we have found, and provide some additional thoughts on a paper by paper basis.

2. Energy Exchange in the Lossy Spin–Boson Model

In [1] there is a discussion on page 59, in which degenerate states are discussed. As published, the text explains (paragraph 1): "States that are resonant (such as Φ_1 and Φ_{12}) are assumed to be stable against zero-energy decay

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process.” The word “resonant” here should probably have been instead “degenerate”.

2.1. Loss model

The loss model discussed in the main text of the paper [1], and derived in Appendix A, probably deserves further discussion. Perhaps the biggest issue is that the approach will be most useful in the case where the decay process in one-directional, so that we do not need source terms in the sector Hamiltonian. Within this restriction, pretty much any loss process can be modeled using the infinite-order Brillouin–Wigner formalism.

Oscillator loss mechanisms near ω_0 do not work this way if $k_{\mathrm{B}}T$ is on the order of $\hbar\omega_0$ or larger. In this case, it would still be possible to develop a model based on the formalism described, as long as it were augmented to include source terms for the coupling from the different sectors. The coupling between the different sectors in such a problem would greatly increase the amount of work needed to develop solutions, since one would need to develop solutions for the different sectors self-consistently.

As a result, we have recently been coming around to a point of view in which loss generally impacts the coherent energy exchange rate in the multiphoton regime of the spin–boson model, for the reasons described in the paper. It is not the case that we require some very restricted loss model or process to see the effect; instead, it is likely that the loss mechanisms associated with a physical system may be good enough in principle to do the job qualitatively, as long as the associated decay rates are sufficiently fast.

3. Dynamics in the Case of Coupled Degenerate States

In models based on operators that form a finite closed Lie group, Ehrenfest’s theorem will lead to a finite set of evolution equations that involve expectation values of those operators. This occurs for linear and quadratic position and momentum operators in the simple harmonic oscillator model. Although we have presented two-coupled evolution equations for the model under discussion describing coupling between neighboring degenerate states [2], the associated operators do not form a closed Lie group. We stop with the evolution equation of Eq. (17) on p. 76, since this is sometimes done in potential well problems in order to make a connection between the quantum version of the problem and the classical version of the problem. In the classical limit of the problem we can determine the force if we know the position (whereas in the associated quantum problem we only have an estimate of the expectation value of the force given the expectation value of the position).

4. Second-order Formulation and Scaling in the Lossy Spin–Boson Model

In the lossless version of the spin–boson problem, coherent energy exchange in the multiphoton regime only occurs when the coupling is moderately strong. Such coupling causes the levels of the two-level system to shift substantially. As a result, the resonance condition for coherent energy exchange would include this shift. We would denote this in the case of a three-quantum exchange as

$$\Delta E(g) = 3\hbar\omega_0, \tag{1}$$

where $\Delta E(g)$ means the shifted transition energy, and where g is the dimensionless coupling constant, which in the lossless spin–boson model is

$$g = \frac{V\sqrt{n}}{\Delta E} \qquad \text{(lossless problem)} \tag{2}$$

In the lossy spin–boson problem, we would also expect a shift to occur. As a result, the resonance condition should be written so as to include the shift. Equation (7) of [**?**] could reasonably be revised so as to take this into account:

$$\Delta E = 3\hbar\omega_0 \quad \rightarrow \quad \Delta E(g) = 3\hbar\omega_0,$$

where the relevant dimensionless coupling coefficient for the lossy spin–boson model is

$$g = \frac{V\sqrt{n}}{\Delta E}\sqrt{S^2 - m^2} \qquad \text{(lossy problem)}. \tag{3}$$

4.1. Difference between lossy and lossless spin–boson models

However, there is an important difference between the two models that is worth some discussion. In the lossless spin–boson model, the individual two-level systems are very nearly independent (and especially so in the large n limit), and the dimensionless coupling constant can be interpreted as the ratio of an (adiabatic) interaction matrix element to the transition energy. What this means is that the interaction for each two-level system has to be very strong in order for the coupling to be strong. The large level shifts that occur in the lossless model are on the order of the transition energy for g near unity. This is reflected in the associated experiments where a great deal of effort is put in to find transitions with small energy differences and large dipole moments.

In the lossy spin–boson model, the dimensionless coupling constant must be large for substantial coherent energy exchange to occur when many quanta are exchanged. But in this case, the associated phonon exchange matrix elements are individually very small, and a large dimensionless coupling constant occurs only because of the presence of the Dicke factor $\sqrt{S^2 - m^2}$ in the definition of g. This is not just a matter of different definitions, in the numerical calculations we find that the lossless model respects the smaller g definition, and the lossy model respects the larger g definition. The presence of loss results in cooperation between the two-level systems that is not present in the lossless version of the model.

4.2. Level shifts in the lossy model

As a result, the level shifts that are included when we write $\Delta E(g)$ for the lossy problem are very small, especially when $|m| \ll S$. The transition energy in the local approximation can be estimated from

$$\Delta E(g) \approx \Delta E + \Sigma\left(\frac{V\sqrt{n}}{\Delta E}\sqrt{S^2 - (m+1/2)^2}\right) - \Sigma\left(\frac{V\sqrt{n}}{\Delta E}\sqrt{S^2 - (m-1/2)^2}\right). \tag{4}$$

In the strong coupling limit

$$\Sigma(g) \rightarrow -4g \tag{5}$$

so that

$$\Delta E(g) \approx \Delta E + 4(V\sqrt{n})\frac{m}{\sqrt{S^2 - m^2}}. \tag{6}$$

Away from the boundaries at $\pm S$, the shift in the transition energy is small.

We have recently analyzed the coupling for an E1 transition between the ground state and first excited state of ^{181}Tm, with the result that the zero-phonon exchange matrix element is expected to be on the order of 10^{-7} eV, and the single phonon exchange matrix element $(V\sqrt{n})$ is near 10^{-9} eV. These are very much less than the associated transition energy (6.24 keV). As a result the shifts expected for the lossy spin–boson model for $|m| \ll S$ in Eq. (**??**) are quite small.

4.3. Level shift near $m = -S$

Near the boundary at $m = -S$, the dimensionless coupling coefficient g becomes smaller since $\sqrt{S^2 - m^2}$ approaches zero. In this case, we require an expression for the self-energy in the weak coupling approximation

$$\Sigma(g) \rightarrow -2g^2\Delta E \tag{7}$$

We can use this to estimate

$$\Delta E(g) \approx \Delta E + 4\frac{(V\sqrt{n})^2}{\Delta E}m \tag{8}$$

The transition between the two self-energy expressions occurs near $g = 1$, so the maximum shift has a magnitude on the order of $2\sqrt{2}|V|\sqrt{n}\sqrt{S}$, which may be significant.

5. Local Approximation for the Lossy Spin–boson Model

On p. 103 (paragraph 3) of [?] one finds written in connection with the two-laser experiment: “The excess power in these experiments is seen in many cases to persist when the lasers are turned off.” Unstated (but it should have been stated) is that this is in contrast to the single laser experiments, where the excess power turns off when the laser turns off.

A more significant error appears on p. 107 in Eq. (21), where the existing incorrect equation should be fixed to read

$$\Psi = \sum_m \sum_n d_m v_{n+m\Delta n} |S, m\rangle |n\rangle$$

The $|n\rangle$ term here appears as $|n + m\Delta n\rangle$ in [?]. As we shall see, this problem occurred more than once in the papers.

Equation (35) needs to be fixed as well; it should read

$$\frac{E(0) - E(\phi)}{E(0) - E(\pi)} \rightarrow \frac{1 - \cos\phi}{2}.$$

6. Coherent Energy Exchange in the Strong Coupling Limit of the Lossy Spin-Boson Model

On p. 120, we find in Eq. (18) the notation $E(g)$, which seems odd since previously the energy eigenvalue has been considered a function of ϕ. In the local approximation, the energy eigenvalue is a function of ϕ and also g, so that we might have used a notation $E(\phi, g)$ throughout the papers. However, our focus was on the determination of the indirect coupling matrix element, so that the ϕ-dependence was more important to us. So, in Eq. (18), we are interested in the dependence of the eigenvalue on the coupling strength g, where the spread of energies with different ϕ is too small to be seen in a plot of the energy eigenvalues as a function of g.

We read on page 121 in Section 4 of [?]: “From the results of the previous section, we see that the solutions do not change suddenly between different values of n, which suggests that we might be able to develop a continuum model.” Sadly, the previous section discussed something else. The results which this refers to appear later on, and can be seen in Figs. 3 and 4.

On p. 129 Eq. (47) reads

$$v(z) = \mathrm{Ai}\left[\left(\frac{\Delta n^2}{2g}\right)^{1/3}(z - z_0)\right].$$

Now, as defined z only ranges between 0 and 1, so this solution should be assumed approximately good only in this range.

On p. 129, Eq. (51) is missing an a_m; it should be corrected to

$$\epsilon_0\langle u_n|u_n\rangle a_m = \langle u_n|\frac{n}{\Delta n}|u_n\rangle a_m - g\langle u_n|u_{n+1}\rangle a_{m+1} - g\langle u_n|u_{n-1}\rangle a_{m+1}$$
$$-g\langle u_n|u_{n+1}\rangle a_{m-1} - g\langle u_n|u_{n-1}\rangle a_{m-1}.$$

In this equation, we should interpret the notation $\langle u_n|u_n\rangle$ and $\langle u_n|u_{n\pm1}\rangle$ as

$$\langle u_n|u_n\rangle = \sum_n u_n^2, \qquad \langle u_n|u_{n\pm1}\rangle = \sum_n u_n u_{n\pm1}, \tag{9}$$

where the summation includes the n values within one pulse over a range of Δn.

On p. 130, Eq. (53) is

$$\left\langle \frac{n}{\Delta n} \right\rangle = m + \delta n \left(\frac{g}{\Delta n^2} \right).$$

This should be interpreted as m plus the function δn, which is a function of the parameter $g/\Delta n^2$. The problem is that the same notation could indicate the some parameter δn should be multiplied by $(g/\Delta n^2)$, which was not intended. The existing text does not clarify this.

There is an unfortunate prefactor error in Eqs. (80) and (81) that badly needs to be fixed. Equation (80) should read

$$\Gamma_0 = \frac{1}{S} \left| \frac{\mathrm{d}m}{\mathrm{d}t} \right|_{\max} = \frac{8}{S} \left(\frac{\Delta E}{\hbar} \right) \left(\frac{g_{\max}}{\Delta n^2} \right) \Phi \left(\frac{g_{\max}}{\Delta n^2} \right) = 8\Omega_0 \Phi \left(\frac{g_{\max}}{\Delta n^2} \right)$$

and Eq. (81) should read

$$\Gamma_0 = 8 \frac{1}{(\Delta n)^2} \frac{V\sqrt{n}}{\hbar} \Phi \left(\frac{g_{\max}}{(\Delta n)^2} \right).$$

7. Generalization of the Lossy Spin-Boson Model to Donor and Receiver Systems

On p. 141 (paragraph 4) of [**?**], there are some minor typos. If we translate the resulting garbled text into English, we would read: "Unfortunately, in the experiments that have been done so far, we generally lack the experimental clarity that would result in unambiguous choices. Consequently, there is no agreement within the community of scientists working on the problem as to what states should be focused on, and at this point some discussion of the problem is worth while."

On p. 146, Eq. (15) has an error and should be corrected to read

$$\Psi = \sum_{m_1} \sum_{m_2} \sum_n d_{m_1,m_2} u_{n+m_1\Delta n_1+m_2\Delta n_2} |S_1, m_1\rangle |S_2, m_2\rangle |n\rangle.$$

The $|n\rangle$ appeared instead as $|n + m_1\Delta n_1 + m_2\Delta n_2\rangle$ in the published paper. A similar error shows up later on p. 148 in Eq. (29), which should read

$$\Psi = \sum_{m_1} \sum_{m_2} \sum_n d_{m_1}(t) \mathrm{e}^{im_2\phi_2} u_{n+m_1\Delta n_1+m_2\Delta n_2} |S_1, m_1\rangle |S_2, m_2\rangle |n\rangle.$$

Once again, the oscillator state $|n\rangle$ appeared incorrectly as $|n + m_1\Delta n_1 + m_2\Delta n_2\rangle$.

A minor error appears in Equation (30), which should read as

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t} d_{m_1}(t) \mathrm{e}^{im_2\phi_2} u_{n+m_1\Delta n_1+m_2\Delta n_2} =$$

$$\left[\Delta E_1 m_1 + \Delta E_2 m_2 + n\hbar\omega_0 - i\frac{\hbar}{2}\hat{\Gamma}(E)\right] d_{m_1}(t)\mathrm{e}^{im_2\phi_2} u_{n+m_1\Delta n_1+m_2\Delta n_2}$$

$$+g_1\mathrm{e}^{-G}\Delta E_1\Big[d_{m_1+1}(t)\mathrm{e}^{im_2\phi_2}\big(u_{n+1+(m_1+1)\Delta n_1+m_2\Delta n_2} + u_{n-1+(m_1+1)\Delta n_1+m_2\Delta n_2}\big)$$

$$+d_{m-1}(t)\mathrm{e}^{im_2\phi_2}\big(u_{n+1+(m_1-1)\Delta n_1+m_2\Delta n_2} + u_{n-1+(m_1-1)\Delta n_1+m_2\Delta n_2}\big)\Big]$$

$$+g_2\Delta E_2\Big[d_{m_1}(t)\mathrm{e}^{i(m_2+1)\phi_2}\big(u_{n+1+m_1\Delta n_1+(m_2+1)\Delta n_2} + u_{n-1+m_1\Delta n_1+(m_2+1)\Delta n_2}\big)$$

$$+d_{m_1}(t)\mathrm{e}^{i(m_2-1)\phi_2}\big(u_{n+1+m_1\Delta n_1+(m_2-1)\Delta n_2} + u_{n-1+m_1\Delta n_1+(m_2-1)\Delta n_2}\big)\Big].$$

This error is that $\mathrm{e}^{im_2\phi_2}$ in the second line was instead e^{im_2}.

7.1. Limit of slow receiver coherent energy exchange

In Section 5, we obtain results for the donor dynamics in the case that the receiver is strongly coupled to the oscillator. However, if we consider the donor dynamics under conditions where coherent energy exchange between the receiver and the oscillator is very slow (conditions under which the model would be expected to break down in some way), the overlap matrix elements may still be near unity for Δn_1 values near Δn_2, $2\Delta n_2$, $3\Delta n_2$, and so forth (as shown in Fig. 2 of [**?**]). In this case, Eq. (34) seems to predict that donor transitions still occur under conditions where we would not expect them to. This issue seems worthy of further discussion.

After some consideration, it is possible to conclude that the origin of this effect is in the use of periodic solutions of the form

$$\Psi = \sum_{m_1}\sum_{m_2}\sum_{n} \mathrm{e}^{i(m_1\phi_1+m_2\phi_2)} u_{n+m_1\Delta n_1+m_2\Delta n_2}|S_1,m_1\rangle|S_2,m_2\rangle|n\rangle.$$

Peaks in the overlap matrix element arise because the solution contains basis states of the form

$$\cdots, |S_1,m_1\rangle|S_2,m_2-1\rangle|n+\Delta n_2\rangle\ ,\ |S_1,m_1\rangle|S_2,m_2\rangle|n\rangle\ ,\ |S_1,m_1\rangle|S_2,m_2+1\rangle|n-\Delta n_2\rangle\cdots$$

Now, if coherent energy exchange occurs between the receiver and the oscillator at a rate faster than the relevant dephasing rate, then one would expect these states to be present and contribute to produce peaks. On the other hand, if this coherent energy exchange is very slow, then we would expect dephasing to dominate, and there would be no coupling between these distant nearly degenerate states. In this slow energy exchange limit, we would not expect a significant superposition of states to develop. In this case, a formulation in terms of periodic states is probably not appropriate, and the overlap matrix element associated with localized basis states would not show peaks in the overlap matrix element.

To include this effect, we require the development of a more sophisticated version of the model that includes dephasing effects when the coherent energy exchange is slow.

7.2. Lack of receiver transitions described by two-level systems

In the Introduction, we discussed briefly our efforts to find transitions that correspond to two-level models relevant to the receiver, with the conclusion that there were no transitions which could couple strongly to the lattice and also have a long lifetime. Strictly speaking, the analysis that led to this conclusion focussed on transitions in the different Pd, Pt, and Li isotopes since these are present near the cathode surface.

Subsequent work has identified an E1 transition in ^{181}Ta at 6.24 keV which should be considered as a possible receiver for excess heat production in tantalum deuteride (or for the HD/^{3}He donor in tantalum hydride with some deuteration). In this case, the two-level lossy spin–boson model described in [**?**] could be relevant.

8. Discussion

There are many issues, discussions, and equations in these papers [1–6], so perhaps it should be expected that some of the equations might end up with errors, and also that some clarification might be needed. Although some of the errors that we found in the equations are unfortunate, in general they are either minor or annoying; fortunately, none of them as yet have resulted in a more significant problem with the basic theory under discussion.

In subsequent work we have found that if the strongly coupled two-level system is very lossy, then it does not get excited, yet still can act to produce a spread in the overlap matrix element corresponding to the lowest order peak in Fig. 2 of Ref. [6]. In this case there is no subdivision effect since there would be no real excitation of the receiver. The strongest such transitions appear to be due to nuclear configuration mixing under conditions where the oscillator is sufficiently strong to exchange phonons in association with the mass shift of the excited configurations. In this case, modeling the receiver using oscillator creation and annihilation operators is an approximation to what should be Duschinsky (phonon mode rearrangement) operators. In this case, when the oscillator is highly excited and the receiver is strongly coupled (with phonon exchange), donor transitions can occur if the spread of the equivalent lowest peak of the above-mentioned Fig. 2 is sufficiently broad.

References

[1] P.L. Hagelstein and I.U. Chaudhary, *Cond. Mat. Nucl. Sci.* **5** (2011) 52.
[2] P.L. Hagelstein and I.U. Chaudhary, *Cond. Mat. Nucl. Sci.* **5** (2011) 72.
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[6] P.L. Hagelstein and I.U. Chaudhary, *Cond. Mat. Nucl. Sci.* **5** (2011) 140.