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Research Article

Low-energy Subbarrier Correlated Nuclear Fusion in Dynamical Systems

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Abstract

In the work the method of formation of a correlated coherent states of nuclei in the nuclear-synthesis systems and application of the method for essential optimization of low-energy nuclear interaction are considered. The relation of the correlation coefficient of these nuclei to the probability of their passage through a Coulomb barrier in order to realize a nuclear reaction is studied. We have determined the form of such an optimum dependence of the correlation coefficient on time, at which the formation of a maximally correlated states of particles and the attainment of the maximum variance of their coordinates under a parametric pumping of a harmonic oscillator are possible. The conditions allowing the choice of those possible laws of variations of the frequency of a harmonic oscillator, which cause the formation of a correlated state, are established. The possible type of a parametric pumping which induces the fast asymptotic formation of the completely correlated state of a particle with giant increasing variance of the coordinate under the parabolic barrier and similar increasing of nuclear reactions probability are determined.

Keywords: Coherent correlated states, Schrodinger–Robertson uncertainty relation, Subbarrier low-energy reactions

1. Introduction

It is well known that the total probability for nuclear reactions to run with the participation of charged particles at a low or middle energy (for $E \ll Ze^2/R$) is defined, in the first turn, by the action of a Coulomb barrier Ze^2/R and, as a result, is bounded by a very small probability of the tunnel effect. This fundamental limitation complicates sharply the solution of the problem of nuclear synthesis and stimulates the use of fast particles in the composition of a hot thermonuclear plasma, which leads at once to the necessity to solve the extremely complicated technological problems related to the formation and confinement of such a plasma. The experience of studies performed in various countries for 50 years showed that the perspectives to realize the large-scale power-releasing thermonuclear synthesis

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remain to be unclear and unfavorable for the time being even for lightest particles (d and t). It is also obvious that the choice of the "thermonuclear" way makes any attempt to use, under the terrestrial conditions, the reactions of synthesis on the base of isotopes heavier than deuterium or tritium (they, in their turn, are not optimum candidates) to be absolutely unreal. For example, much more optimum is the synthesis on the basis of the ecologically safe reaction $^{11}B + p = 3\,^{4}He$ with the participation of heavier nuclei ^{11}B. This reaction is not accompanied by both the appearance of neutrons and an additional radioactivity, but none of the promising thermonuclear-energy projects considers this reaction due to a significantly less probability of the tunnel effect.

Together with these "classical" and extremely expensive thermonuclear studies, whose efficiency is very small now and is not proportional to the undertaken efforts and the financial expenditures, many different disconnected experiments were carried out, where the nuclear synthesis ran at a low energy and under clearly "nonthermonuclear" conditions with the probability incomparably greater than those limitedly small values which must be revealed on the basis of the tunnel effect.

Convincing results were obtained at the Electrodynamics Laboratory "Proton-21" in Kiev for last 12 years in the experiments, where a state and a composition of needle-like targets were changed under the spatially symmetric action of nanosecond pulses of the current of an electron beam with the amplitude $J_0 \approx$ 50–70 kA at the energy of accelerated electrons of about 300–400 keV. In all the experiments, the current pulse energy of a beam did not exceed 200–300 J. In these experiments (their total number in 12 years is at least 18 000, we observe the various effects related to the fundamental transformations of elements and isotopes of the initial chemically pure targets. In particular, after each of the experiments, we registered the synthesis products with the extremely wide spectrum of isotopes (from hydrogen to transuranium elements). As a distinctive peculiarity of the experiments, we mention the very high efficiency of nucleosynthesis. It corresponded to the transformation of about $N_n \approx 10^{19}$–10^{21} nucleons per one cycle of the pulse action with the total duration of at most 50 ns. In addition, we registered a great number of protons with their energy in the interval from several keV to several MeV and greater, many fast deuterons, very intense X-rays, and gamma emission in each of the experiments. The scales of nuclear transformations corresponded to a change of the total binding energy $\Delta Q_{\rm bind} \approx 30 - 50$ MJ or about $\Delta Q_{\rm bind}/N_n \approx 1$ Mev/nucleon, i.e., it was by many orders higher that the mean specific energy of an electron driver (at most 1 eV/ nucleon). The more complete data on the results of these studies are presented in Refs. 1–4, à their detailed description is given on the Web-page of the Electrodynamics Laboratory "Proton-21" [5].

Various isotopic anomalies under clearly "nonthermonuclear" conditions were observed (though on considerably smaller scales) also in experiments performed by other groups.

The essential change of the isotope composition of the structural materials (mainly, Fe and Ti, for which the variations were at the level of 3–5%) of switching elements which realized the rapid commutation of hard currents (1–50 kA) and high voltages (up to 5 kV under idling conditions) was observed in industrial hard-current nets after the long-term operation [6]. Of importance is the circumstance that an electric arc (plasma) appeared at the time moment of the breaking of a hard current in all the studied facilities, where the isotope anomalies were registered.

Other experiments [7] have demonstrated the considerable variations of the isotope composition (about 5–7%) on the explosion of wires and foils immersed in a liquid under the action of millisecond hard-current pulses with the total energy of 20–30 kJ.

The transformations of nuclei, whose efficiency was close to that of the above-mentioned experiments, were observed in Ref. 8. On the passage of powerful submicrosecond electric pulses through an aqueous solution of $ZnSO_4$, a significant decrease in the zinc concentration was registered.

Of great interest are the experiments on the registration of neutrons escaping from a volume of cold gaseous deuterium positioned in a strong magnetic field with variable amplitude [5]. In these experiments, the emission of intense bunches of neutrons was registered only under very specific conditions, namely on the cooling (!) of gaseous deuterium down to the temperature of liquid nitrogen (-196°C) and for a very short time interval at once after the change of the strong magnetic field with intensity of at least 8–10 kOe! These experimental results contradict directly the basic postulates and the very ideology of thermonuclear synthesis on the basis of tunnel effects, according to which the probability of the nuclear synthesis must exponentially decrease on a decrease in the temperature and increase on its growth.

The essential change in the isotope and element compositions was observed in studies [10,11] of the isotope composition of microbiological cultures rapidly growing in specially prepared nutrient media. The duration of the growth stage under study was several days. These media were impoverished by one of the vitally necessary chemical elements (in particular, by natural iron), but they contained the additional isotopes of other elements (in particular, concentrated heavy water D_2O and ^{55}Mn as an admixture or, respectively, light water H_2O and admixtures of ^{23}Na and ^{31}P). The elementary analysis shows that the simple addition of these isotopes allows one (in principle) to ensure the synthesis of the absent element, because the reactions $^{55}\mathrm{Mn} + \mathrm{d} = {}^{57}\mathrm{Fe}$ and $^{23}\mathrm{Na} + {}^{31}\mathrm{P} = {}^{54}\mathrm{Fe}$ are characterized by the positive energy of the reaction and satisfy all the conservation laws. The detailed combined mass-spectrometric and Mössbauer analyses of the composition of the grown microbiological cultures showed that they turned out, indeed, to be enriched by these isotopes. In this case, their increment was by many orders more, than this could be explained on the basis of the probability of the tunnel effect. We also mention the experiments on the synthesis of ^{57}Fe, where a synthrophic association of various types of microbiological cultures, which was stable under the action of depressing toxins and products (wastes) of the own metabolism, was used, rather than "pure" strains. In these experiments, not only the increase in the relative and absolute concentrations of isotope ^{57}Fe, but approximately the same decrease in the absolute concentration of the initial isotope ^{55}Mn were surely registered!

All the above-mentioned experiments executed by the independent scientific groups at the nuclear centers of various countries have demonstrated the large-scale nuclear transformations under such clearly "nonthermonuclear" subthreshold conditions, when their efficiency calculated within the standard model of the tunnel effect must be by many orders less than that determined in the experiments. This circumstance becomes else more obvious if we take into account that the majority of experiments involved the isotopes with the charge of nuclei $Z \ll 1$ which participated in the process of nuclear transformations. For such isotopes, the probability of the tunnel effect at a low temperature is extremely small, and the number of events with nuclear transformations must be by many orders lower than the registration threshold.

Of course, the certain attempts to substantiate a specific mechanism, which would promote specific reactions with the participation of nuclei with $Z \gg 1$, were made on the interpretation of the mentioned experiments. But such substantiations involve, most frequently, only the qualitative arguments [6–8] or are generally absent [9]. A sufficiently grounded quantitative theory of the observed global nuclear transformations on the basis of the conception of self-controlled collapse in the volume of a target was proposed only in the works devoted to the analysis of the experiments executed at the Electrodynamics Laboratory "Proton-21" [1–4].

In addition, we note that all the above-considered experiments, despite the basically different ideologies, equipments, methods, and scales of observed processes, are joined by the fact that the success was associated only with those experiments, in which the perturbation stimulating the nuclear transformations was nonstationary or corresponded to a transient mode. In this case, the following empiric rule was fulfilled in the successful experiments: the optimum duration of the relaxation of a perturbation is inversely proportional to the amplitude of this perturbation.

Below, we consider a general and sufficiently universal mechanism of the stimulation and optimization of nuclear reactions running at a low energy. This mechanism ensures the large probability of the nuclear reactions under conditions, where the ordinary tunneling effects (including resonant tunneling effect) obviously "does not work", and can be applied with the same efficiency to very different experiments (both the executed and planned ones).

2. Correlated Coherent States of Particles and Schrödinger–Robertson Uncertainty Relation

The presence of wave properties and the possibility of the tunnel effect for microparticles are ones of the basic distinctive peculiarities of the quantum-mechanical description of the Nature. In the concentrated form, these properties are expressed in the form of the uncertainty relations which determine, in fact, the limit of the applicability of the classical and quantum descriptions of the same object.

Atomic and nuclear physics use widely the well-known Heisenberg uncertainty relation,

$$\sigma_q \sigma_p \geqslant \hbar^2/4, \tag{1}$$

which connects the variances and mean square errors,

$$\sigma_q \equiv (\delta q)^2 = \langle (q - \langle q \rangle)^2 \rangle, \; \sigma_p \equiv (\delta p)^2 = \langle (p - \langle p \rangle)^2 \rangle, \tag{2}$$

of the coordinate q and the corresponding component of the momentum of a particle p.

The connection of this relation and similar ones to the commutativity of the operators of corresponding quantities in the form of the generalized uncertainty relation

$$\sigma_A \sigma_B \;\geqslant\; |\langle \widehat{A}\widehat{B} \rangle|^2/4, \tag{3}$$

was discovered by Robertson in 1929 for dynamical variables A and B, whose commutator $\widehat{A}\widehat{B}$ is nonzero.

Relation (1) can be used for quantitative estimation of the tunnel transparency of a potential Coulomb barrier of a nucleus $V(q)$ with width $L(E)$ on the identification of the quantity δp with the mean square effective radial momentum of a particle with energy E,

$$\delta p = \sqrt{2M\langle V(q) - E \rangle}, \tag{4}$$

in the subbarrier region defined by the conditions $V(q) \leqslant E, \; R \leqslant q \leqslant L(E)$.

In particular, if the condition $L(E) \ll \hbar/2\delta p$ is satisfied, then the transparency coefficient of the Coulomb barrier surrounding the nucleus is close to 1.

In the opposite case where $L(E) \gg \hbar/2\delta p$, the transparency of the barrier D will be extremely small:

$$D \;=\; \exp\{-W(E)\} \;\ll\; 1,$$

$$W(E) \;=\; (2/\hbar) \int\limits_{R}^{R+L(E)} \sqrt{2MV(q) - E} dq \;=\; 2\,\delta p_q\, L(E)/\hbar \;\gg\; 1. \tag{5}$$

The very low transparency of the barrier is the main argument which is used in the proof of the impossibility (or the extreme inefficiency) for nuclear reactions to run at a low energy of colliding particles, which corresponds to a very large width of the barrier $L(E)$.

We note that the condition of smallness of the energy $E \ll \langle V(q) \rangle$ is a minor argument in the proof of the inefficiency of a reaction as compared with the main argument, namely the presence of a great width of the potential barrier at a low energy: $L(E) \gg \hbar/2\delta p$. This is related to the fact that the quantity $W(E)$ in (5) for the transparency of the barrier is proportional to the barrier width and depends on the energy only as a square root.

Not denying the obvious importance of the made estimates, we note that the particular (1) and general (3) Heisenberg uncertainty relations are not such universal laws as, for example, the energy conservation law. Their applicability is limited, as it is easy to prove, to only those quantum states, where the quantities A and B are mutually noncorrelated. We recall briefly the most strict derivation of the basic relation (3). It follows from the requirement that the expression

$$G = \int_{-\infty}^{\infty} |\alpha u(q) + i v(q)|^2 \mathrm{d}q \ \geqslant 0 \tag{6}$$

be nonnegative for any value of the parameter α and for any functions $u(q)$, $v(q)$. Relation (3) was obtain on the basis of the direct analysis of (6) under the conditions that α is any real parameter, and $u(q)$, $v(q)$ are the functions related to the variances:

$$\begin{aligned} u &= \ \Delta\widehat{A}\psi(q) \ \equiv \ (\widehat{A} \ - \ \langle A\rangle)\,\psi(q), \\ v &= \ \Delta\widehat{B}\psi(q) \ \equiv \ (\widehat{B} \ - \ \langle B\rangle)\,\psi(q), \\ \sigma_A &= \int |u(q)|^2 \mathrm{d}q,\ \sigma_B = \int |v(q)|^2 \mathrm{d}q \ . \end{aligned} \tag{7}$$

In 1930, Schrödinger and Robertson independently generalized the Heisenberg idea of the quantum-mechanical uncertainty of different dynamical quantities A and B on the basis of the more correct analysis of (6). If we remove the ungrounded limitation that the parameter α is purely real, then (6) yields the more universal condition

$$\sigma_A\sigma_B \ \geqslant \ |\langle\widehat{A}\widehat{B}\rangle|^2/4(1 - r^2) \tag{8}$$

called the Schrödinger–Robertson uncertainty relation [12, 13]. In this relation, the quantity

$$r = \sigma_{AB}/\sqrt{\sigma_A\sigma_B} \tag{9}$$

is the correlation coefficient which determines the degree of cross correlation of the quantities A and B in a specific state described by the wave function $\Psi(q)$. Respectively, the expression

$$\sigma_{AB} = \langle\{\Delta\widehat{A}, \Delta\widehat{B}\}\rangle/2 = (\langle\widehat{A}\widehat{B} + \widehat{B}\widehat{A}\rangle)/2 - \langle\widehat{A}\rangle\langle\widehat{B}\rangle \tag{10}$$

is the mean value of the anticommutator of the operators of errors $\Delta\widehat{K} \ = \ \widehat{K} \ - \ \langle K\rangle$ of the quantities A and B. By analogy with σ_A and σ_B, the quantity σ_{AB} can be called the cross variance of these quantities.

The Schrödinger–Robertson uncertainty relation (8) is an obvious generalization of the Heisenberg uncertainty relation (3) to the case of partially or completely correlated states. It clarifies the interrelation between the quantities A and B, by defining the lower bound for the product of variances.

In the case of completely mutually noncorrelated quantities A and B, we have $\sigma_{AB} \ = \ 0$, $r \ = \ 0$ and (8) is reduced to (3), which corresponds to the minimization of the product of variances $\sigma_A\sigma_B$. In this case, it is possible to form such coherent superposition (a packet) of the eigenfunctions of a particle in a specific potential well, for which $(\sigma_A\sigma_B)_{\min} = \hbar^2/4$. This superposition is called a coherent state (CS), and the last equality is the condition of its realization. In the presence of a partial correlation of these quantities, $\sigma_{AB} \neq 0$, $0 < \ |r|^2 < 1$, and $(\sigma_A\sigma_B)_{\min} \ > \ \hbar^2/4$. Respectively, on their full correlation, we have $|r|^2 \rightarrow 1$ and

$$(\sigma_A\sigma_B)_{\min} \ \rightarrow \ \infty \tag{11}$$

Such correlated coherent state (CCS) can be created only on the basis of another coherent superposition of the same functions, for which the condition of maximization of the quantity $|\sigma_{AB}|$ is fulfilled.

For the correlated states, the specific uncertainty relations of type (1) are also changed and take the form

$$\delta q\ \delta p_q \geqslant \hbar/2\sqrt{1 - r^2}, \quad \delta E\ \delta t \geqslant \hbar/2\sqrt{1 - r^2}. \tag{12}$$

From the formal viewpoint, the influence of the correlation of dynamical quantities can be taken into account by the introduction of the modified Planck constant $\hbar^* = \hbar/\sqrt{1-r^2}$ in the formulas for the tunnel effect probability. In addition to the purely formal equivalency, such a substitution well reflects also the sense of these changes, because just the Planck constant defines the boundary separating the regions of adequacy of the classical and quantum-mechanical descriptions of the processes. It is obvious that the presence of a cross correlation of different processes extends significantly the region, where only the quantum-mechanical description is valid, and this fact corresponds directly to an increment of the Planck constant.

For particles in the completely correlated states, the barrier transparency will be always large ($D \to 1$ at $r^2 \to 1$) at any energy (including $E \ll V_{\max}$), which breaks at once the universality of the assertion on the inefficiency of nuclear reactions at a low energy for any states of charged particles interacting at a low energy. It is seen that this assertion is valid only for uncorrelated particles. In view of this conclusion, it is obvious that the question about the possibility and the conditions of a realization of the correlated states of particles with $r^2 \to 1$ becomes central.

3. Conditions of the Formation of a Correlated Coherent State of a Particle

From the formal viewpoint, the problem of the formation and the stability of a correlated state of particles is reduced to solving the inverse problem of quantum mechanics, i.e., to finding such a state of a specific particle in the given force field, for which the maximum value of the cross variance modulus σ_{AB} (10) of different physical characteristics of this particle is realized. The necessary properties can be inherent only in the superpositional function defining a phased superpositional state of a particle in the given force field. One of the simplest methods of the formation of the correlated coherent states of a particle under a parametric pumping of a nonstationary harmonic oscillator, being firstly in the ground state, is considered in Refs. 14–18.

The system of normed eigenfunctions $\Psi_\alpha(q,t)$, which describe the behavior of a particle in the field of a nonstationary harmonic oscillator with variable frequency $\omega(t)$ at an arbitrary time moment, can be found from the solution of a nonstationary Schrödinger equation and has the form

$$\Psi(q,t) = \int b(\alpha)\Psi_\alpha(q,t)\mathrm{d}\alpha,$$

$$\Psi_\alpha(q,t) \equiv= \frac{1}{\sqrt[4]{\pi\varepsilon^2}} \exp\left\{\left[\frac{i\xi^2}{\omega_0}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + \alpha\xi\sqrt{8} - \alpha^2\varepsilon^* - \varepsilon|\alpha|^2\right]\Big/ 2\varepsilon\right\}. \tag{13}$$

Here, $\zeta = q/q_0$ is the coordinate normed by the quantity $q_0 = \sqrt{\hbar/M\omega_0}$, $\omega_0 = \omega(t=0)$ is the base frequency of the harmonic oscillator at the time moment of the switching of a perturbation, α is any constant complex-valued number, and $\varepsilon(t)$ is a complex-valued solution of the classical equation of motion of the oscillator with variable frequency $\omega(t)$,

$$\frac{\mathrm{d}^2\varepsilon}{\mathrm{d}t^2} + \omega^2(t)\varepsilon = 0, \quad \varepsilon(0) = 1, \quad \left.\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right|_0 = i, \quad \omega(0) = \omega_0. \tag{14}$$

The direct calculation of the correlation coefficient (8) with the use of the wave function (13) yields

$$r = \mathrm{Re}\left\{\varepsilon^*\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right\}\Big/\left|\varepsilon^*\frac{d\varepsilon}{\mathrm{d}t}\right|, \quad r^2 = 1 - \omega_0^2\Big/\left|\varepsilon^*\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right|^2. \tag{15}$$

Formulas (13)–(15) show that wave function (13) of a correlated state can be written as

$$\Psi_\alpha(q,t) = \frac{1}{\sqrt[4]{2\pi\sigma_q}} \exp\left[-\frac{q^2}{4\sigma_q}\left(1 - \frac{ir(t)}{\sqrt{1-r(t)^2}}\right) + \frac{\alpha q}{\sqrt{\sigma_q}} - \frac{1}{2}(\alpha^2 + |\alpha|^2)\right], \tag{16}$$

$$\langle q\rangle = 2\sqrt{\sigma_q}Re\alpha, \qquad \langle p\rangle = \frac{\hbar}{\sqrt{\sigma_q}}\left\{Im\alpha + \frac{r}{\sqrt{1-r^2}}Re\alpha\right\}$$

Here, α is a parameter determining the symmetry of the system and the direction of the drift of the coherent correlated state. In a symmetric nonstationary parabolic potential well with $\alpha = 0$, $\langle q\rangle = 0$, and $\langle p\rangle = 0$, the wavefunction of the correlated state of a particle has the form

$$\Psi_0(q) = \frac{1}{\sqrt[4]{2\pi\sigma_q}}\exp\left[-\frac{q^2}{4\sigma_q}\left(1-\frac{ir}{\sqrt{1-r^2}}\right)\right]. \tag{17}$$

Based on relations (14)–(17), we obtain the connection between the given change of the correlation coefficient $r(t)$ and the necessary law of variations in the actual frequency $\omega_r(t)$ of the harmonic oscillator [17]:

$$\omega_r(t) = A\sqrt{\frac{1}{(g(t))^2(1-r^2)}\left\{1-\frac{g(t)}{\omega_0\sqrt{1-r^2}}\frac{\mathrm{d}r}{\mathrm{d}t}\right\}},$$
$$g(t) = \frac{r(t)}{|r(t)|} + 2\omega_0\int\limits_0^t \frac{r(\tau)\mathrm{d}\tau}{\sqrt{1-r(\tau)^2}}. \tag{18}$$

The coefficient A can be found from the condition $\omega_0 = \omega(t=0)$. In particular, we have $A = \omega_0$ under the initial conditions

$$r(0) = 0, \quad \mathrm{d}r/\mathrm{d}t|_{t=o} = 0. \tag{19}$$

Equations (14)–(19) determine a solution of the problem of the formation of a correlated coherent state on the basis of the parametric swinging of a harmonic oscillator.

Relation (18) for $g(t)$ implies that the sign of the function $g(t)$ is defined by the sign of the correlation coefficient $r(t)$ and the product $g(t)(\mathrm{d}r/\mathrm{d}t)$ is a positive value increasing with the correlation coefficient modulus $|r(t)|$. In this case, it follows from the explicit formula (18) for $\omega_r(t)$ that an increase in the correlation degree ($\mathrm{d}r/\mathrm{d}t > 0$) corresponds always to a decrease in the frequency of a parametric pumping $\omega_r(t)$.

Formula (18) yields also easily the explicit form of the law of variations of the frequency of a parametric pumping

$$\omega_{r_0}(t) = \omega_0/(1+2\omega_0|r_0|t/\sqrt{1-r_0^2}) \tag{20}$$

for which the correlation coefficient remains constant and is equal to r_0. In particular, in order that the system be all the time in a state close to that with maximum correlation (for $|r_0| \approx 1$), it is necessary that the resonant frequency of the oscillator vary by the law

$$\omega_{r_0}(t) \approx \sqrt{1-r_0^2}/2t, \qquad |r_0| < 1. \tag{21}$$

These results have a great heuristic meaning. In particular, we can make the following important conclusion on their basis. In order that $|r(t)|$ will grow, it is necessary that the frequency $\omega_r(t)$ will decrease faster than $\omega_{r_0}(t)$ (21) with increase in time.

There is additional condition for creation of optimal correlated state of the particle.

The final correlated state with $r(t)$ cannot be attained for an arbitrarily small time interval. This corresponds to the fact that the derivative $|\mathrm{d}r(t)/\mathrm{d}t|$ should not be large at all the time moments. Formally, this condition follows immediately from solution (18) defining the law of variations of the frequency of a parametric swinging. This frequency cannot become imaginary, which yields the condition

$$\frac{\mathrm{d}r(t)}{\mathrm{d}t} \leqslant \omega_0\sqrt{1-r(t)^2}\Big/\left\{1+2\omega_0\int\limits_0^t \frac{r(\tau)\mathrm{d}\tau}{\sqrt{1-r(\tau)^2}}\right\}. \tag{22}$$

This condition corresponds to the requirement for the process of variation of the frequency $\omega_r(t)$ to be adiabatic. One of the possible solutions [17], that satisfy the condition of adiabaticity and ensure the fulfillment of the condition $|r(t\to\infty)|\to 1$ has the form

$$r(t)=\frac{t/\tau}{\sqrt{F(t)+(t/\tau)^2}},\quad F(t\to 0)\to\infty,\quad F(t\to\infty)\to \mathrm{const},$$
$$\tau = T_{1/2}\sqrt{2/F(T_{1/2})} \tag{23}$$

for any monotonously decreasing function $F(t)$.

In Fig. 1, the temporal dependence of the optimum frequency $\omega_r(t)$ of a parametric pumping of a harmonic oscillator and the correlation coefficient $r(t)$ for a function

$$F(t) = c^2 1/c + (\tau/t)^{32/3} \tag{24}$$

is presented.

The physical reason for the increase of the probability of tunneling effect is related to the fact that the formation of a coherent correlated state leads to the cophasing and coherent summation of all fluctuations of the momentum $\Delta\vec{p}(t) = \sum_n^N \Delta\vec{p}_n(t)$ for various eigenstates forming the superpositional correlated states (16), (17). This leads to a very great dispersion of the momentum

$$\sigma_p = \langle \left\{\sum_n^N \Delta\vec{p}_n(t)\right\}^2\rangle = N\left\langle(\Delta\vec{p}_n)^2\right\rangle + N^2\left\langle\Delta\vec{p}_n\Delta\vec{p}_m\right\rangle \tag{25}$$

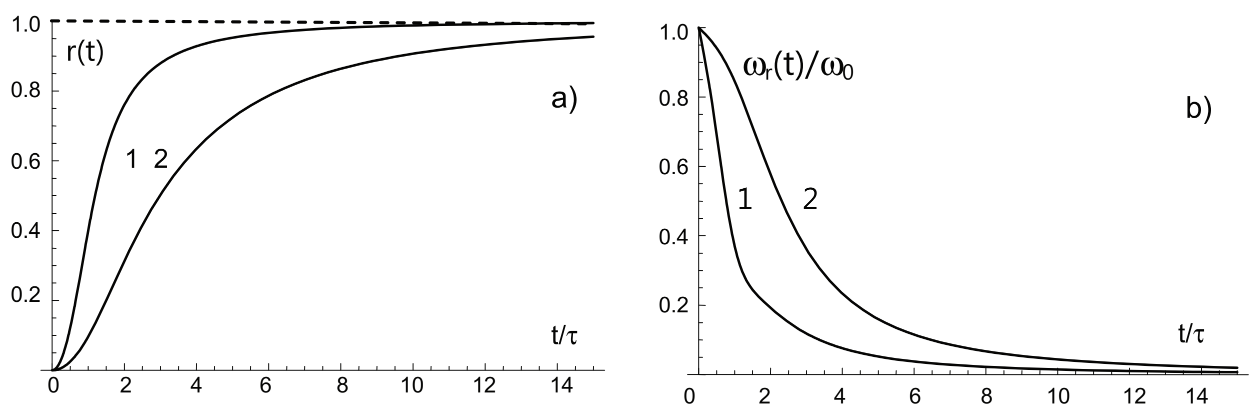

Figure 1. Change of correlation coefficient $r(t)$ of the particle in nonstationary harmonic oscillator (a) connected with the change of frequency $\omega_r(t)$ of the oscillator (b) for function $F(t)=c^2 1/c+(\tau/t)^{32/3}$, $c=2$ (case 1) , $c=10$ (case 2).

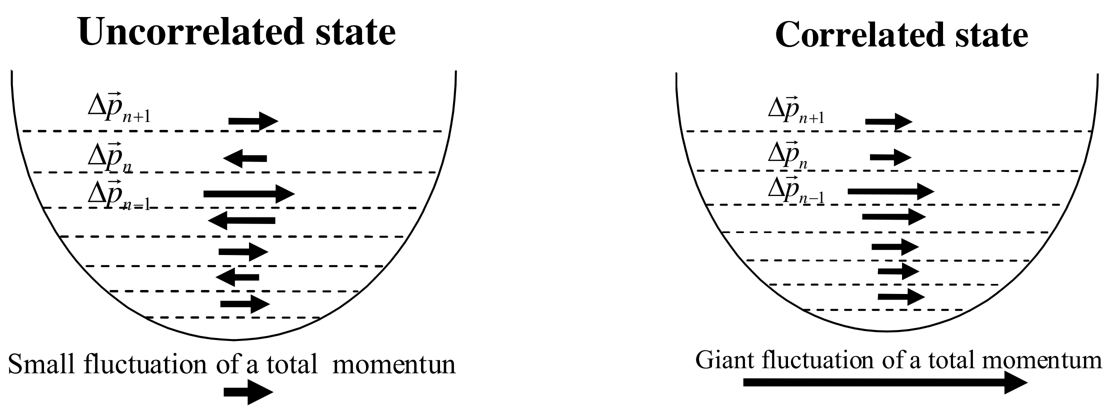

Figure 2. Formation of total (superpositional) fluctuating momentum of a particle in potential well in uncorrelated and correlated states.

correlated state, very great fluctuations of kinetic energy

$$\langle \Delta T\rangle = \langle \Delta \vec{p}(t)^2\rangle /2M = N^2 \langle \Delta \vec{p}_n \Delta \vec{p}_m\rangle /2M + N \left\langle (\Delta \vec{p}_n)^2 \right\rangle /2M \sim N^2 \tag{26}$$

of the particle in the potential well and increasing of potential barrier penetrability. This situation is presented in symbolic form in Fig. 2.

4. Change of the Probability of the Barrier Transparency for Particles in Coherent Correlated State

Let us consider simple model [17] which allows to demonstrate the process of barrier transparency increase during the increase of correlation coefficient $r(t)$.

Let us suppose that the particle are presented in the one-dimensional deforming parabolic well, and the characteristic frequency $\omega_r(t)$of that well decreased to the value $\omega_0$to the moment $t = 0$ in a specific way, and at that moment the particle state was characterised by the correlation coefficient r_s. After that moment (at $t > 0$) well-deformation stops.

For such one-dimensional Cartesian system we have (see (17))

$$\Psi_{\rm corr}(x, r_s, t = 0) = \frac{1}{\sqrt[4]{2\pi\sigma_x}} \exp\left[-\frac{x^2}{4\sigma_x}\left(1 - \frac{i r_s}{\sqrt{1 - r_s^2}}\right)\right]. \tag{27}$$

From the other hand the standard wave function of the particle in stationary parabolic well in any time (including $t \geqslant 0$ is

$$\Psi(x, r_s, t \geqslant 0) = \sum_n B_n(r_s)\varphi_n(x) e^{-iE_n t/\hbar}, \quad \varphi_n(x) = C_n H_n(x)\, e^{-x^2/2x_0^2}, \tag{28}$$

$$C_n = \frac{1}{\sqrt{x_0 2^n n\sqrt{\pi}}}, \quad x_0 = \sqrt{\hbar/M\omega_0}, \quad E_n = \hbar\omega_0(n + 1/2), \quad x_0^2 = 2\sigma_{x0}. \tag{29}$$

Population coefficients

$$B_n(r_s) = \int_{-\infty}^{\infty} \Psi_{\rm corr}(x, r_s, t = 0)\varphi_n^*(x) {\rm d}x \tag{30}$$

can be find from the condition of continuity for the total wave function at $t \leqslant 0$ and $t \geqslant 0$.

The resulting expression for the full wave particle function in partially correlated state with r_s in the stationary well at $t \geqslant 0$ has the form

$$\Psi(x, r_s, t \geqslant 0) = \sum_n \left\{ \int_{-\infty}^{\infty} \Psi_{\text{corr}}(x', r_s, t = 0)\varphi_n^*(x')\mathrm{d}x' \right\} \varphi_n(x) \exp(-iE_n t/\hbar). \quad (31)$$

The results of calculations of probability density $|\Psi(x, r_s, t \geqslant 0)|^2 = \rho$ of particle localization within a parabolic well at any moment of time $t \geqslant 0$ depending on the correlation coefficient r_s are presented below.

Figure 3 shows the results of calculation of this probability density for the localization of the particle in the parabolic well $V(x, t \geqslant 0) = (\hbar\omega_o/2)(x/x_0)^2$.

Figure 4 shows periodic modulation of probability density $|\Psi(x, r_s, t \geqslant 0)|^2 = \rho$ for states with different coefficient of correlation for different values of transversal coordinates.

The dispersion of the coordinate for an uncorrelated state remains unchanged and equal to the initial value σ_{x0}. The probability of penetration of the particle in the subbarrier region for $|x| \gg x_0$ is very low. It can be seen from the results of calculation that the value of the probability density in the subbarrier region (at $|x| > 2x_0$) for correlated states in the delocalization phase considerably exceeds the steady-state value for uncorrelated states.

It can be seen from the figure that the very sharp increase in the coordinate dispersion of a particle and in the probability of its subbarrier penetration for the correlated state in the delocalization phase (see Fig. 4c) is not compensated and is much larger than the decrease in these parameters in the localization phase.

The results of averaging of function $|\Psi(x, r_s, t \geqslant 0)|^2 = \rho$

$$\langle\rho\rangle \equiv \langle|\Psi(x, r_s, t \geqslant 0)|^2\rangle_t = \frac{1}{T}\int_t^{t+T} |\Psi(x, r_s, t \geqslant 0)|^2\mathrm{d}t, \quad T = 2\pi/\omega_0 \quad (32)$$

for various values of the initial correlation coefficient in the same well are presented in Fig. 5.

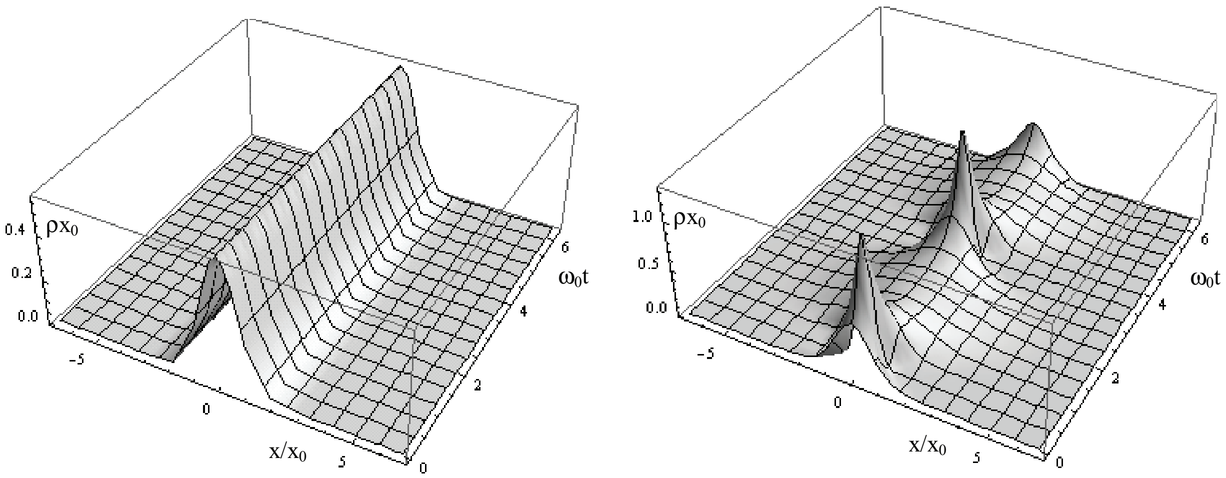

Figure 3. Changes of the cross structure of the distribution $|\Psi(x, r_s, t \geqslant 0)|^2 = \rho$ in an uncorrelated state $r_s = 0$(a) and in a state with the coefficient of correlation $r_s = 0.9$(b) in the course of time.

Figure 6 shows the resultant change (increase) in the averaged probability density of particle localization (tunneling probability D) $D = \langle|\Psi(x, r_s, t \geqslant 0)|^2\rangle_t = \langle\rho(x, r_s)\rangle$ for various values of the correlation coefficient in the interval r_s = [0, 0.988] and for coordinate $x = 10x_0$ in the region under the same potential barrier $V(x)$.

It can be seen that the tunneling probability for correlated states always increases considerably; the variation in the tunneling probability deep in the subbarrier region (e.g. for r_s = 0.988 and $x = 10x_0 \approx 14\sqrt{\sigma_{x0}}$ it increases by 30–40 orders of magnitude in relation to the uncorrelated state!).

The rate of nuclear synthesis (probability of reaction per unit of time for one pair of interacting nuclei situated at $x = 0$ and $x = x^{(n)}$) is equal

$$\lambda_{r_s} = \Lambda\langle|\Psi(x^{(n)}, r_s, t)|^2\rangle_t. \tag{33}$$

Here, $\Lambda = S(E)\hbar/\pi \mathrm{Me}^2$ is the constant of nuclear synthesis for a specific pair of nuclei;$S(E)$ is the astrophysical nuclear factor depending on a matrix element of the interaction energy of nuclei. $S(E)$is the slowly changing function of energy, which with low relative energy of interacting particles in the case of non-resonance nuclear reactions, is constant $S(E) = S_0$ For example, $S_0 \approx 0.11$ Mev *bn* and $\Lambda_{\mathrm{dd}} \approx 1,5 \times 10^{-16}\,\mathrm{cm^3 s^{-1}}$for a non-resonance dd-fusion.

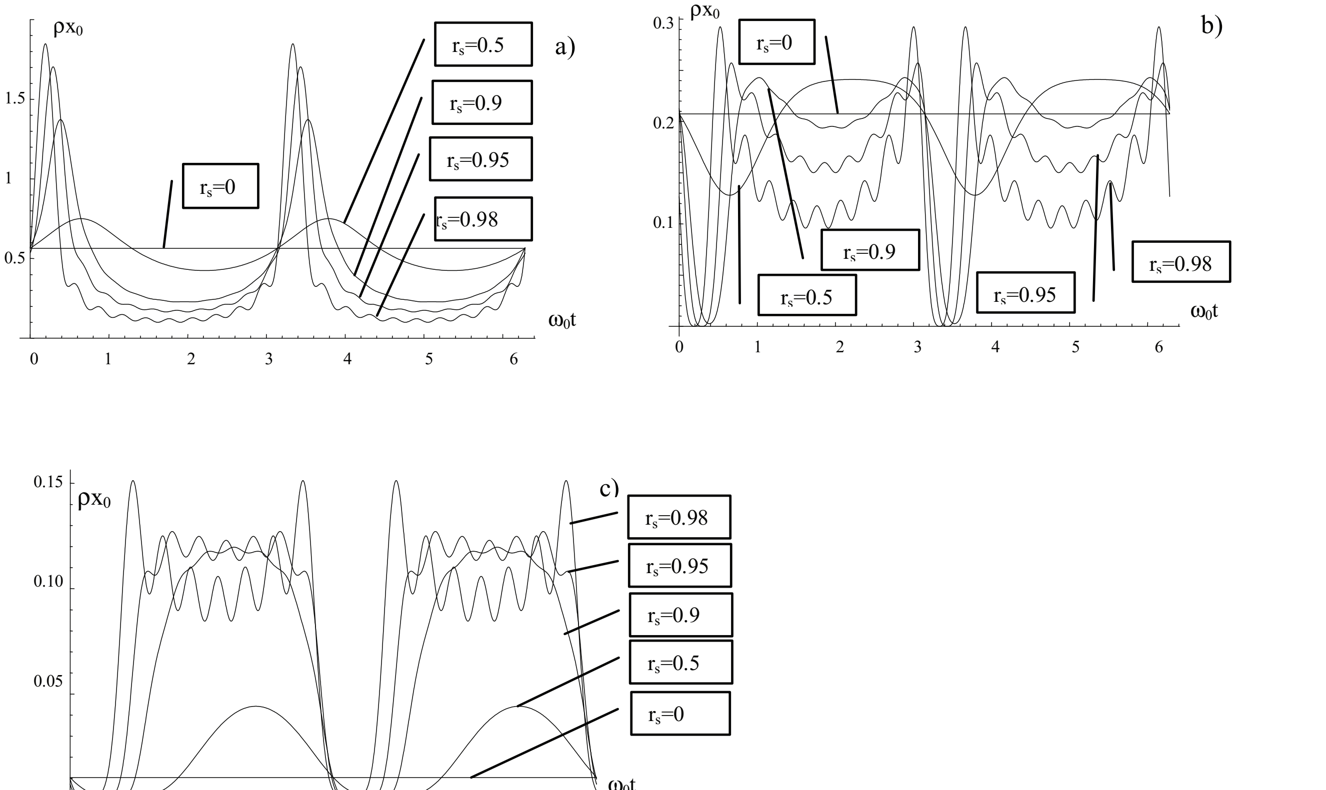

Figure 4. Periodic modulation of probability density $|\Psi(x, r_0, t \geqslant 0)|^2 = \rho$ for states with the coefficient of correlation r_s =0; 0.5; 0.9; 0.95; 0.98 in the same potential well for different values of transversal coordinates x in the channel $V(x)$: (a) $x = 0$; (b) $|x| = x_0$; (c) $|x| = 2x_0$.

The "giant" increasing of the barrier transparency $\langle|\Psi(x^{(n)}, r_s, t)|^2\rangle_t$ leads to the similar increasing the probability (33) of nuclear reaction with participation of nearest nuclei situated at the same distances $x^{(n)}$.

5. Conclusions

Presented results clearly demonstrate the "giant" increases (by many order of magnitude) of localization density under the potential barrier and also the possibility of very effective under the barrier penetrations of particles at the increase of correlation coefficient. The regime of potential well deformation (parametric buildup of a harmonic oscillator) ensuring the most optimal adiabatic mode of the increase in the correlation factor that approaches its limiting value $|r| \to 1$ is determined, and this leads to an unlimited increase in the dispersion of the particle coordinate and to complete penetrability of the potential barrier.

One more nontrivial result seems to be important.

At increase of the correlation coefficient $|r| \to 1$ the giant increase the dispersion of the nucleus coordinate takes place. At such condition the mean square uncertainty $\delta x = \sqrt{\sigma_x}$ of the of this nucleus coordinate can significantly exceed the mean distance between nuclei $\langle a\rangle$. In this case, the volume of the three-dimensional region, where a nucleus in the correlated state is localized, will contain

$$N_c \approx (\sqrt{\sigma_x}/\langle a\rangle)^3 \gg 1 \quad (34)$$

nearest nuclei, with which a nuclear reaction is possible.

This result opens the way to the possibility for correlated collective many-nucleus reactions to run. The probability of such reactions increases by N_c times as compared with the probability of the running of binary reactions in the absence of a correlation.

These effects may play the important role in processes of controlled nuclesynthesis in different potential holes and systems with nonstationary action on active systems (e.g. in experiments with high-current electron driver in Kiev Laboratory "Proton-21" and in experiments on nuclear transmutation in biological systems).

The same effect of giant increasing of subbarrier transparency (by $10^{40} - 10^{60}$and more times!) takes place during the increase of correlation coefficient at special high-frequency periodic action on quantum system. Such effects take

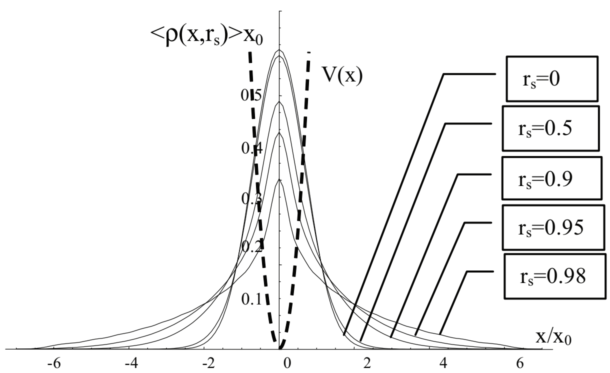

Figure 5. Averaged distributions of the probability density $\langle|\Psi(x, r_s, t \geqslant 0)|^2\rangle_t = \langle\rho(x, r_s)\rangle$ for a particle in the same potential well in an area under the barrier for correlated and uncorrelated states of particles with $r_s = 0; 0.5; 0.9; 0.95; 0.98$.

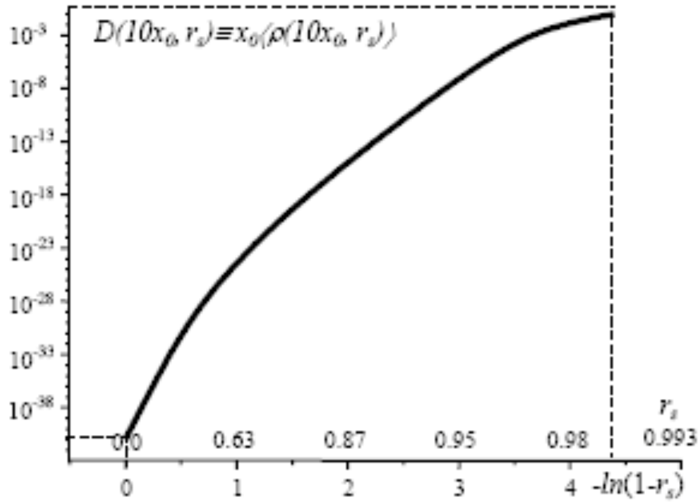

Figure 6. Change of the averaged barrier transparency (tunneling probability D) $D = \langle|\Psi(x, r_s, t \geqslant 0)|^2\rangle_t = \langle\rho(x, r_s)\rangle$ for selected coordinate $x = 10x_0$ under the same parabolic barrier V(x) for different coefficients of correlation.

place in different nonstationary CF experiments with release of energy (including action of ultrasound and the method of "SuperWave" [19]. These result will be presented in our next papers.

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