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Research Article

Deep-electron Orbits in Cold Fusion

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Abstract

The lochon models of cold fusion, among others, propose deep-energy electrons as necessary for low-energy nuclear reactions (LENR). Relativistic Schrodinger equations, e.g., the Klein–Gordon (K–G) and Dirac equations, have 'irregular' solutions that predict such levels at ∼500 keV. The basis for such a level and its implications are presented.
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1. Introduction

One theory of cold fusion, CF, based on the lochon (a local charged Boson, e.g., paired-electrons) and the model's extension into the nucleus have proposed deep-energy electrons as the active agent in producing a fusionable result. While the model does not require stable electron orbits to mediate the fusion of colliding hydrogen nuclei, the existence of such orbits would support the deep-energy-electron portion of the lochon models, would increase the fusion probabilities, and would explain other results observed in CF experiments.

The Schrodinger equation is made relativistic in two forms: the Klein–Gordon (K–G) equation for spinless particles and the Dirac equation for spin-1/2 particles. These pairs of equations each have two solutions. In each case, for the $1/r$ Coulomb potential, one of the two equations has been rejected for various reasons (primarily having to do with the singularity at $r = 0$). In the rejection of these solutions, recognition of the existence of very-deep (∼500 keV) electron orbits has been lost.

Details and implications of the single-electron deep-orbits predicted by the 'anomalous' K–G and Dirac models are described below in terms of binding energy and electron orbital radius. The assumed applicability of both models to the lochon models is explained in terms of the tightly bound lochons (Bosons), and electrons (Fermions). The lochon model, for successful cold fusion, depends on the spatially paired electrons remaining together for as long as possible

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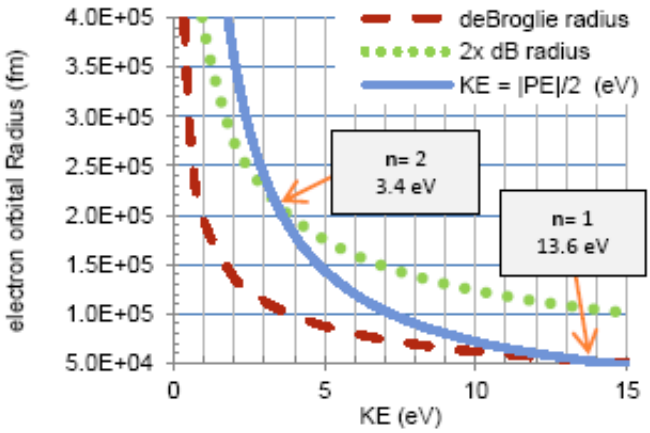

Figure 1. First two atomic orbitals (of many) associated with the common points of integer de Broglie radii (momentum based) and kinetic energy.

during the H–H collision process. The normally rejected K–G solution would allow the decay from a filled 1s orbital (e.g., the lochon in an H^- ion) to the boson-occupied nought orbit. The anomalous Dirac equation solution would allow deep-orbit electrons to combine two protons in fusion or a molecule.

Two multi-femtometer-spaced hydrogen atoms, as a femto-molecule, would share the electron(s). The transition pathways and probabilities for both possibilities must be explored. Cold fusion results provide experimental evidence for the proposed deep-orbit and their theoretical basis comes from relativistic quantum mechanics. The consequences for hydrogen of such transitions are indicated below and implications for transmutations are explored in other papers [1].

2. Resonances

What are electron orbitals? They are resonances; in some respects standing waves. In the case of atomic orbitals, they are orbits in which the electron de Broglie wavelengths are 'completed' within the cycle about the nucleus. This means that it is in the identical condition (position, energy, momentum, spin, etc.) at the same point in every cycle. This is an energy minimal point and, therefore, more stable than adjacent points. Figure 1 indicates this 'resonance' by having the known orbitals identified with the crossing of the electron's kinetic energy and the deBroglie radius that gives one-half the potential energy required to give the kinetic energy. This is a stable configuration according to the virial theorem.

There are other stable configurations. Figure 2 shows some of these related to the kinetic energies associated with the Compton radius and de Broglie radius (small boxes). The large box includes common solutions (and large uncertainty) with the classical electron radius with and without relativistic correction. These last solutions are the ones that represent the deep-electron orbits predicted by the K–G and the Dirac equations.

3. Nought Orbits

A deep atomic orbital at a binding energy of about mc^2 is predicted from one solution of the K–G equation [2]. Earlier studies of the relativistic 1-D hydrogen atom [3], of a normally rejected solution to the Schrodinger equation [4], and in an independent development [5], had predicted a similar deep orbit (in some cases infinitely deep [3]). The deep-orbit

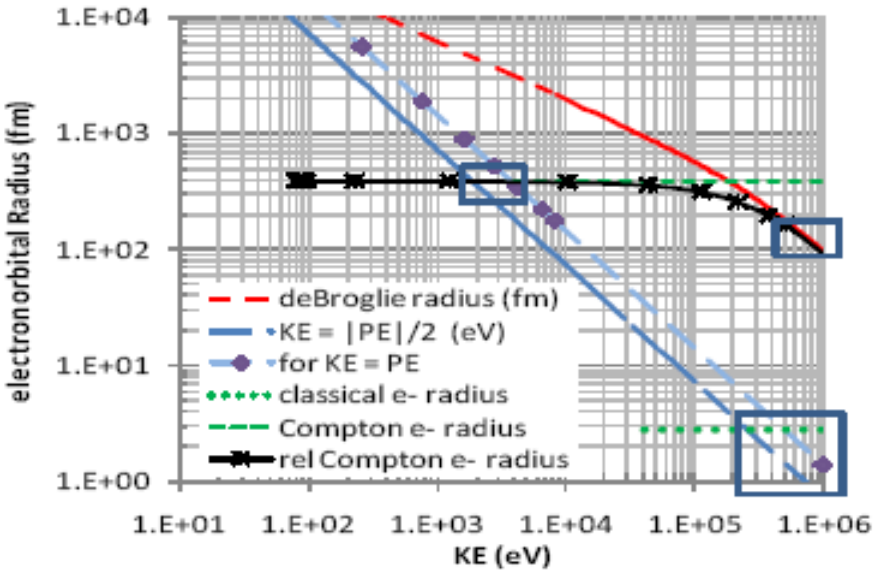

Figure 2. Possible atomic orbitals associated with the common points of de Broglie, Compton, and classical radii with higher kinetic energies.

solution of the K–G equation has been rejected by other authors [6,7] as: being from an equation that is not applicable to particles with spin, being non-square integrable, and being singular at the origin (and therefore not normalizable). Thus, by their definition, it cannot be applied to an electron and it could not be real even if it did pertain. Nevertheless, the same K–G model predicts two sets of energy levels. One set is 'acceptable' (even without considering spin) because it provides levels consistent with known values for bosons (e.g., pions) and is not singular. The other set is 'unacceptable' because, along with predicting the known atomic-electron levels to within milli-electron volts, it predicts a level at >500 keV that has not been observed.

The deep-orbital level predicted by the K–G equation is far below the $n = 1$ level, so we will call it interchangeably the $n = 0$, or 'nought', or 'naught' level. Some of its predicted properties (and problems?) are provided in Eqs. (**??**) and (**??**) (the fine-structure constant $\alpha =\sim 1/137$). The total nought-orbit energy, TE_0, equals the mass energy plus the binding energy (E_b = potential energy – a negative value – plus the kinetic energy). Therefore, the binding energy of the orbit (a negative value for an attractive potential and a single charge without spin) is $E_0 - m_0c^2$ and the proposed nought-orbit values from the K–G equation are:

$$TE_0 \cong m_0c^2\alpha \cong 3.7\text{keV} \Rightarrow \text{binding energy } = E_b = TE_0 - m_0c^2 \cong -507.3 \text{ keV} \tag{1}$$

and

$$r_0 \cong \hbar/m_0c \cong 390 \text{ fm}. \tag{2}$$

A more recent paper [8] has established theoretical criteria for accepting the deep-orbit solution in the K–G equation, but not in the Dirac equation for which such a solution had been claimed earlier [9].

4. Deep-Dirac Levels (DDLs)

In 1993, recognizing that deep-orbit electrons were an important element in the cold-fusion dilemma, Maly and Va'vra [9] made a break from one of the traditional mathematical physicists' 'mind games' that had been going on since the early recognition of deep orbits was presented in 1957 [10]. Instead of the obviously idealized $1/r$ dependence of the Coulomb potential, with its point-charge singularity at $r = 0$, Maly and Va'vra selected a modified Coulomb potential that nuclear-physicists had been using for years. This non-singular potential reflected a charge distribution within the nucleus (rather than a point charge) and still matched the $1/r$ Coulomb potential beyond the surface of the nucleus. With a non-singular potential, the solution of the relativistic Schrodinger and Dirac equations, which had been rejected by the mathematical physicists for over three decades, must now be considered as valid as the accepted solution.

While the 1993 paper is largely ignored today because it is relativistic quantum mechanics, and therefore 'suspect' on two counts, it was not ignored at the time. In a challenging argument [11], Rice, Kim, and Rabinowitz did not suggest on mathematical grounds that the orbit could not exist. They argued that the equilibrium population was too low to be significant (2×10^{-9} times that of the accepted atomic orbitals). However, they then discount the possibility of the DDLs, because the fusion rate would be too high not to be observable.

If their calculation is within orders of magnitude of a real value, we could consider that the density of these DDLs for hydrogen in a metal matrix might be in the range of $10^{-13}/\text{cm}^3$. However, the Dirac equation does not consider nuclear fusion or even the weak interaction; therefore, the equilibrium value between the regular and irregular solutions will be quite different from that predicted from the simple Dirac equations with Coulomb potential. The number of populated DDLs will be much lower, because the transition rate is very low between atomic levels and DDLs (i.e., from a weak overlap of the respective wave functions) and the steady flow of electrons from the atomic to the nuclear level and its consequences must be considered.

The deep-orbit solutions for the relativistic Schrodinger and the Dirac equations have multiple deep levels [9,12] with higher angular-momentum. These additional orbits also have higher binding energy for hydrogen ($509 < E_b < 511$) than the nought-orbit level, or ground-state DDL, as distinct from the atomic orbits that have lower binding energies as n increases.

5. Deep-electron Orbits and QM

A point not mentioned in the nought-orbit papers is the need for the electron orbit to be in the Fermi range, not the picometer range, to attain the ~1 MeV energies required for the solution. This important concept is addressed briefly in [12] and below.

The non-relativistic QM predictions are based on the de Broglie wavelengths and the associated resonances of a bound electron. The addition of a (perhaps) single Compton wavelength for the electron (Fig. 2) could give a new orbit (a solution or resonance) and still not violate the old QM models (they may just not be the general solution). Since the strict Coulomb potential has no minimum and nothing except the perceived singularity at $r = 0$ prevents the electron from passing through the nucleus, the normal ground state is a minimal (not a minimum) only as a result of 'mechanical' resonance states of the electrons in a potential well. The photon and its requirements for $E = h\nu$ and $\ell = 1$, which are critical to a proper understanding of the atom, do not directly enter the standard Schrodinger-equation picture.

This resonance of an electron in a Coulomb potential is a consequence of the de Broglie relation ($\lambda_{dB} = h/mv$) that is often used, but seldom explained. Alternatively, it is answered in the form of a mathematical solution of the Schrodinger equation with 'no further discussion required' (allowed?). The relativistic addition of the Compton relation ($\lambda_C = h/mc$) does not alter the former solutions. However, instead of multiple resonances (as in Fig. 1), made possible by a variable (v, the electron velocity) in the *de Broglie* wavelength definition, there may be only a single K–G resonance for the single *Compton* wavelength ($v \leq C$). On the other hand, just as there are beat frequencies when two frequencies are combined, it is possible that there are multiple resonances (and higher angular-momentum states) that

can be associated with the relativistic-electron (near-nuclear) regime [9].

We have given a logical basis for the existence of very-deep energy levels for atomic electrons that are predicted by the relativistic corrections to the Schrodinger equation. What keeps electrons from filling it and precipitating high rates of fusion in H-rich material? The Heisenberg Uncertainty Principle (HUP) is often used to explain the lack of energy levels below the known atomic orbitals. However, this ignores some important features of the atom (see Appendix A). A partial key, which is never discussed in the QM development of atomic orbitals, is in the inability to transfer photonic energy without the simultaneous transfer of angular momentum. This limitation exists in energy transfer via photons between one or more bound electrons or between a bound electron and a proton. Both photons and phonons are bosons and have angular momentum of 1 ($\hbar$). Therefore, they are not candidates for energy transfer in this $\ell = 0$ case (ground state and below). To fit theoretical results, but without momentum transfer, all the Coulomb potential energy difference between levels must be converted into the electron's kinetic energy or a portion ($\sim$50%) must be used to do work within a fundamentally conservative system.

6. Field Nature of DDL Electrons

The Maxwellian radiation, bound to the electron, cannot exceed the velocity of light in vacuum. The DDL electron is moving at >0.9 C in a multi-fermi orbit. The radius that includes $\sim$99% of its EM field (by some criterion) is nearly 400 fm. Thus, the field cannot keep up with the electron and the field lines must be tightly 'wrapped' about the charge pair. They cannot stay there. They must terminate on a positive charge. In a static, or a non-relativistic, or a single pass-by case (e.g. scattering experiment), this 3-D 'flow' of field lines is not too difficult to picture. For a bound, relativistic-electron, orbit, the picture is less clear.

The cyclic 'disturbance-front' in the dipole field propagates out less than the distance around the DDL orbit circumference before the next front is initiated. Since the DDL electron may take 100 orbital cycles to return to the exact orientation of the equivalent point in its de Broglie cycle, resonance cannot occur in one or even a few cycles. This 100-cycle time frame establishes the resonant condition in the multi-hundred fermi EM field about the tightly bound electrons.

At the close-in DDL range, most of the field lines are closed between the electron and the local nuclear proton(s). The dipole potential drops off as $1/r^2$, rather than the $1/r$ of the Coulomb potential. (Thus, in the presence of a proton, an electron, defined by its potential or field, 'shrinks' in size.) Field lines are a potential gradient. As such, they are 'directional' and cannot cross one another without violating the limitations of 3-dimensional space. They must 'close' on the adjacent proton(s); but, since the distributed potential is dependent on the relative position of the two charges, both it and the associated field lines may be highly dynamic.

A problem with the simple field-line model exists for the dipole field lines extending to >10 fm and with the return time of a change in the source being greater than the cycle time. How does the DDL electron avoid its own outward-radiating field lines (on their return to the proton) and find, over time, a path of minimum energy? The answer must lie in the ability of the charge's potential to change the field lines at both the source region and then again at all other places locally, perhaps even before the original perturbation reaches them. This does not violate any causality; but it does set up some interesting conditions for resonance between the DDL electron and its bound EM radiation. It means that the potentials and field lines have a short-term 'memory' and, at least to some extent, this memory can affect the EM fields at a given location.

The dipole's far-field radiation ($r \gg 2\pi r_{\mathrm{DDL}}$) is seen only as a fluctuating bipolar field and closed field lines cannot make sense under that condition. Another unanswered question is, "while this field derives its energy from the dipole and is in a sense bound to the dipole, does it contribute to the electron's kinetic energy and effective mass?" (The proton's mass to too great relative to that of the electron to be affected by the field.) This same question can be posed about atomic electrons. However, the energy tied up in that far-field radiation is too small to be of concern. This may

not be so, in the case of DDL electrons where the field energy may exceed the mass energy of the source electron.

The point of this section is to indicate that a bound electron is quite different from an energetic incident electron. Its creation of, and interaction with, its own radiant field is complicated and perhaps the source of some aspects of nuclear and quantum physics not presently being explained. Conservation and symmetry considerations are able to provide much of the predictive power available in mathematical physics today. Nevertheless, we must not be bound by preconceived ideas that may not pertain to the specific system that we are seeking to describe.

7. Deep Orbits and Cold Fusion

Deep-orbit electrons are the ultimate Coulomb screen. Atomic or 'free' electrons cannot spend sufficient time close to a proton to shield its positive charge from another proton. The deep-orbit electrons can. They make a proton look like a 'fat-neutron'. Fusion becomes inevitable; however, there are many kinds of fusion possible with this proton combined with a DDL electron, $p^{\#}$ or $H^{\#}$ [1]. We will only sample the cases of hydrogen.

7.1. Interaction of $p^{\#}$ with H

The major differences between $p^{\#}$ and H are that the proton has given up about 1.5–2 MeV of its mass (in the form of potential energy) to give the DDL electron about 1–1.5 MeV of kinetic and mass energy and a binding energy of $\sim$0.5 MeV. This means that there is insufficient mass energy to form a neutron despite the electron's >1 MeV of kinetic energy. On the other hand, if $p^{\#}$ meets H in a low-energy, low-angular-momentum encounter, then production of a deuteron is probable.

The DDL electron draws the two protons together (dipole–monopole attraction). Both protons do work through the DDL electron and lose mass. The electron gains kinetic energy as it moves to deeper (now femto-*molecular*) levels. The deep-orbit electron shifts toward the heavier proton and, in the sharing process, the proton masses are equalized. However, there is now sufficient total mass energy to form a deuteron. (We can ignore any atomic electron in this, because it is not in the action.)

7.2. Interaction of $p^{\#}$ with $p^{\#}$(or H^{+} with $H^{\#\#}$)

It is unlikely that two $p^{\#}$ would encounter unless there were a strong nuclear active environment (NAE) capable of producing this femto-atom in large quantities. However, the lochon model, utilizing $p^{+} + p^{-} \Longrightarrow p_2^{\#\#}$, produces this entity by preference, unless fusion occurs.

Again, there is not sufficient mass energy in the potential-energy-depleted protons alone to form a deuteron. Nevertheless, with the DDL electron energies and proximity available, fusion is expedited. If fusion does not occur, the resultant neutral, but polarizable, femto-molecule is highly penetrating of both atomic-electron screens and of nuclei. Thus, $H_2^{\#\#}$ is a strong transmutation agent.

8. Interaction of $p^{\#}$ with D or $d^{\#}$ with H

While both $^3He^{\#}$ and ^3H can result from either of these combinations, the end product will be ^3He in both cases. In the first case, the DDL electron will assist in dissipating the excess nuclear energy and will finally be ejected from the deep orbit – to leave the ^3He nucleus in its ground state. In the second case, the tritium formed, with conversion of a proton into a neutron and neutrino, would ultimately decay into a ^3He atom.

8.1. Interaction of $d^+ + d^- \Longrightarrow {}^4He^*$, ${}^4He^{*\#}$ or ${}^4He^{*\#\#}$

This starting point is the principle candidate for the formation of DDLs. If the lochon process does not achieve the proper depth of the bound electrons, it is generally reversible. However, even during a reversible process, there is still a slight probability for the deuterons to tunnel through their mutual Coulomb barrier, without DDL formation, and produce ${}^4He^*$ by the conventional hot-fusion process. The result, however, would be the expected 50/50 split in the n and p fragmentation levels.

If the $d^+ + d^-$ process is only partially successful, but one electron had to absorb the other electron's energy so that one of them could attain the DDL, then a single-DDL-electron ${}^4He^{*\#}$ state could result. This state is beneath the ${}^4He^*$ neutron fragmentation level, but not that of the proton fragmentation. Few or no neutrons would be observed; but a goodly number of energetic protons and tritium would result. The single DDL electron could still provide an alternative path to the ${}^4He$ ground state and so a significant amount of heat would be produced along with the fragments and ${}^4He$.

A fully successful $d^+ + d^- \Longrightarrow {}^4He^{*\#\#}$ process would produce a 'decaying' neutral nucleus with many MeV excess nuclear energy. It is also a powerful transmutant. Since it would penetrate nuclei easily and has four nucleons to contribute, the extra energy available could split many of the isotopes that it would encounter. On the other hand, with its many bodies, including the DDL electrons, it could also create many combinations of decay paths to allow the excess nuclear energy to be dissipated without highly energetic particles being emitted [4].

8.2. Interaction of $d^+ + d^- \Longrightarrow d_2^{\#+} + e$ or $d_2^{\#\#}$ or ${}^4He^{*\#\#}$

Two deuterons bound by DDL electron(s) cannot have too much angular momentum to fuse. Nevertheless, they may be metastable in the femto-molecular or fused, but excited, state, where they can exist as such in the lattice for a brief period (fs?). The $d_2^{\#\#}$ neutral femto-molecule or ${}^4He^{*\#\#}$ would be a powerful transmutant.

9. Conclusions

The deep-orbit electron levels, which have been rejected for more than five decades because of mathematical physicists' inability to see the inappropriate use of a singular potential, were properly revealed (again) two decades ago. Since this information was published in the context of cold fusion, it was ignored outside of the CF field. Similarly, no experimental evidence, outside of the CF literature, has been accepted to validate these levels and this knowledge has remained dormant.

With the recognition that a non-singular Coulomb potential provides acceptable deep-orbit solutions of the K–G, Schrodinger, and the Dirac equations, we must also look at the consequences and any experimental evidence to validate and refine these solutions and/or the input potential. The K–G nought orbits and deep-Dirac levels (DDLs) provide properties that explain many of the experimental observations of low-energy nuclear reaction (LENR) research and condensed-matter nuclear science (CMNS). The strongest support for these deep-Dirac levels is the number of transmutation products observed in the cold fusion work. It would appear that experimental support for these levels already exists and validation may not be far away.

Accepting the reality of these deep-orbit electrons and utilizing them will provide a century of new growth in both Physics and Chemistry (and perhaps in Biology). The practical benefits could far exceed those in the last century from QM and Relativity combined.

Appendix A. Appendix: Deep-Dirac Levels and the HUP

The HUP, or Relation, is sometimes an obstacle to those asked to consider a deep-orbit or nuclear electron. The HUP indicates that to confine an electron to the fermi range requires a high kinetic energy (>100 MeV) so that the uncertainty in momentum is large enough to satisfy the relation. How is this challenge solved?

Classically, the nought-orbit electron can have angular momentum. However, it will be much lower than Planck's constant divided by 2π (where $h/2\pi = \hbar$). Therefore, since $0 < \ell \ll \hbar$ and the uncertainty in angular momentum is $\hbar/2$, it would be classified as an $\ell = 0$ orbit. The nought orbit is thus both an $n = 0$ and an $\ell = 0$ orbit.

There are other interesting points about this orbit relative to normal atomic orbitals. 'Circumferences' of the Bohr orbits are integer multiples of the electrons' *de Broglie* wavelength ($\lambda_{dB} = h/p$, where p is the electron momentum). However, the naught orbit in one development [2] has a predicted circumference close to that of the electron's *Compton* wavelength (Eq. (**??**) $\Longrightarrow \lambda_C = h/m_0c$). As such, nought-orbit electrons still fit within the wave mechanics regime and are consistent with the Heisenberg Uncertainty Relation (see below).

The problem with a Compton radius ($\sim$380 fm) for the deep-electron orbit is that it does not provide the potential energy needed to establish the binding energy predicted by either the K–G or the Dirac equations. That must come from a radius in the low fermi range. How are these apparently mutually exclusive conditions reconciled? Part of the answer comes from recognition that Δx does not represent the uncertainty in position of a point particle (or even its centroid of distribution) but the extent of the wave function (wave packet?) associated with that particle. Since a wave packet has no well-defined edges, Δx can be much larger than an average, a weighted-mean, or an otherwise-defined position distribution. The 'distribution of what' is also not defined. Do we define a particle by its mass, by its field, by its potential? Therefore, the value cannot be single valued. This concept allows both values for orbital radius.

Looking at the 1 MeV kinetic energy region of Fig. 2, one can see two resonances. Both the Compton and classical electron radii are involved. Both involve the electron mass, but at different radii: one with the electron momentum, the other with the kinetic energy as defined by the electrostatic potential. It would appear that this is a region of significance, where an electron's KE and PE are close to twice its mass energy.

The HUP implies that to confine an electron to the nuclear region would require such a high momentum that the Coulomb potential cannot contain it. However, an electron, to be confined to a nucleus, will have enough kinetic energy (from the Coulomb potential) to create electron–positron pairs that will spread the effective mass energy – and therefore increase the uncertainty in position – as the positron is ejected by the nuclear Coulomb field. Since the electrons are identical particles, when one electron combines with the positron, it is impossible to determine which electron it is. Again, the uncertainty in position is increased sufficiently to fulfill the HUP. A 'modern-physics' view of the model, when extended to lower energies, would say that virtual electron–positron pairs are constantly being created and annihilated about every electron. The nuclear Coulomb field polarizes the virtual pair further increasing the uncertainty of location, because, once again, there is no means of distinguishing the virtual from the real electron.

The use of virtual particles is a QM technique for using 'quantized' entities, rather than fields, to describe phenomena. A stationary electron or positron can be described by only one of the two Dirac equations. As soon as an electron begins to move, a contribution to the combined wave function from the second coupled equation begins to grow. This second equation describes the positron (or a 'portion' thereof). Thus the concept of virtual 'particles', a portion of the linear superposition of 'eigen states' (the electron and positron in this case), can be used in place of Maxwell's equations to describe the EM-energy field that grows with velocity and acceleration of a charge. The 'range', a limit of existence, of the virtual particles corresponds to the evanescent wave from a classical photon of the electron's mass energy. When the energy of the EM field reaches $2mc^2$, then real particles, can be created This transition from virtual to real has dramatic and, perhaps, not-easily explicable consequences [13].

Classically, a moving electron creates a magnetic field (a description of the relativistic changes to the electrostatic **E** field – even at low velocities). If it is not moving linearly, additional field components are required to describe the resulting disturbance to the radial Coulomb field. Since another charge is required to accelerate an electron (any **E** field is created by, or composed of, charges), the field lines of both charges are greatly distorted during this process. Once the source of acceleration is removed, the field lines return to their original configuration (in the rest frame of the electron). Viewed from another frame moving relative to the electron, the fields appear distorted to reflect a higher energy level. When bound to a nucleus, the electron is constantly accelerating and therefore its field lines reflect the

constant change. This is the basis of the 'bound' EM field described by Maxwellian radiation. It is bound both to the electron and to the positive charge(s) to which the electron is attracted.

Only under special conditions can the Maxwellian radiation 'leave' the electron. Maxwell did not believe in the corpuscular nature of light and therefore did not include the option of photons in his formulation. QM also ignores the photon in its time-independent calculation of the hydrogen atom. The difference is that Maxwell predicts EM fields extending to infinity (radiation) and, strictly, QM says there is no radiation at all (that is covered in QED!).

The HUP, based on wave mechanics, has often been misused by assuming that the uncertainty in location of a particle is only defined by its distribution as predicted by its wave function. If it is recognized that particles are composed of waves, then the HUP applies to them as well. The electron is the easiest example of this. If we recognize that the electron with its $1/r$ potential can be considered as infinite in extent, then the HUP is a tautology. If $\Delta x = \infty$, then, $\Delta x\, \Delta p = \infty > \hbar/2$. This is not very useful. If, however, we consider the electron to be composed of a photonic shell structure with $E = h\nu = hc/\lambda = mc^2 = 511$ keV, and having a radius $r = \lambda/2\pi = \lambda\!\!\!^{-}$ and $p = E/c$, then the uncertainty in location is $2\pi r \geq \Delta x \geq 2r$ and in momentum is $2\mathrm{p} \geq \Delta\mathrm{p} \geq p = E/c = h/\lambda$. The HUP becomes $2\pi\lambda\!\!\!^{-}2h/\lambda \geq \Delta x\, \Delta p \geq 2\lambda\!\!\!^{-}h/\lambda = 2\hbar > \hbar/2$ and is then satisfied. This is a useful and instructive definition and condition. However, since the electron has no strict boundaries, the use of a Compton radius ($r_\mathrm{C} = \lambda\!\!\!^{-}/p$) is no more valid than that of the classical electron radius ($r_\mathrm{c} = e^2/mc^2$). Is full-width half-maximum (FWHM) a better description of a pulse shape than half-width half-maximum (HWHM), or full-width at $1/e$ of maximum value? Nevertheless, usage of the Compton radius when imposing the HUP has merit in that both are related to the wave-mechanical (photonic) description of an electron.

Too often, competent physicists use the HUP with little or no understanding of what it means or even represents. They use the de Broglie wavelength as the uncertainty in position because there is a QM wave function related to it. Recognition of what the wave function represents, or how it relates to the physical world is generally missing (it is seldom taught). Therefore, like engineers, they use the equations and assume that this is proper, always giving the correct answer.

In atomic physics, such procedure gives the electron orbitals to great accuracy. This gives great assurance of the procedure's correctness in using the de Broglie wavelength and its multiples. Approaching the nuclear case, where the Compton wavelength is more important if in resonance, the de Broglie wavelength of an electron is only a perturbation in an ill-defined region. Thus, the sum and difference between the Compton and de Broglie frequencies ($\nu_\mathrm{C} \pm \nu_\mathrm{dB}$) may be the dominant feature. Furthermore, interaction of an electron's frequencies with the nuclear frequencies can play a major role.

For high-energy scattering experiments (e.g., 100 MeV), the electron deBroglie frequencies greatly exceed the Compton frequency. This allows them to be used in the HUP as a useful indicator of limitations in 'resolution' of experiments. For DDLs, the electron kinetic energy ($\sim$1 MeV each) and momentum is small relative to that of scattering experiments, but large relative to the rest mass. Thus, the positional limitation imposed by the de Broglie wavelength would appear to forbid such nuclear orbits. However, just as the $n\lambda_\mathrm{dB}$ criterion provides sharp resonances within the atomic-electron orbits (e.g., Fig. 1), a λ_dB/n criterion, where n may be $\sim$100, could provide a very broad resonance (composed of many states) within the deep-electron orbits. The point is that, as in the atomic orbitals, the electron orbit is not defined by HUP with Δx equal to the DDL electron's radial distance from the nucleus. It is defined by the integer circumferential lengths of the electron orbit and $\Delta x = 2\pi rn = \lambda_\mathrm{dB}$, where n can be large and therefore r can be small.

When referring to the Coulomb attraction between nucleus and electron, the distance between centers of charge is the critical feature, unless the charge is distributed over a larger volume than that occupied by the centers of two bodies. The fact that the electron charge is distributed over many fermi and the proton charge over nearly a fermi puts the DDL electrons in this category. Some weighted-average distance between charges determines the potential energy between them. At some point ($r = 1$ fm?), the electron may no longer be distinguishable as a separate entity and then the HUP

must apply to the modified proton rather than to the electron. To say that confinement to an average distance violates the HUP, unless the momentum is sufficiently high, is mixing and misunderstanding physical and resonant sources of the HUP.

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