Research Article

Cooperative Internal Conversion Process

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Abstract

This paper presents a theoretical discussion of a new phenomenon, called the 'cooperative internal conversion process', in which the coupling of bound-free electron and neutron (or proton) transitions due to the dipole term of their Coulomb interaction, which permits cooperation of two nuclei leading to neutron (or proton) exchange if that is allowed by energy conservation. General expressions of the cross section of the processes are reported in the case of one particle nuclear and spherical shell models as well as in the case of free atoms (e.g. noble gases). A half-life characteristic of the process is also determined. As numerical examples, the cooperative internal conversion process by neutron exchange in Ne and by proton exchange in Al are dealt with. The processes may have significance in fields of nuclear waste disposal and nuclear energy production. As a generalization, cooperative internal conversion process by heavy charged particle exchange is discussed, and as an example of it, the cooperative internal conversion process by triton exchange is also discussed.

1. Introduction

The issue of modifying nuclear processes by the surroundings has been a question of primary interest of nuclear physics from the very beginning. The best known and investigated process in respect of modifying nuclear processes by surroundings is the internal conversion process, in which an atomic electron around an excited nucleus takes away nuclear transition energy of an electromagnetic multipole transition which otherwise would be prohibited by angular momentum conservation [1].
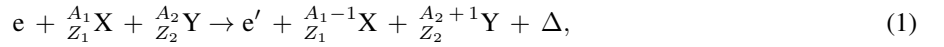
Motivated by observations of an anomalous screening effect (see e.g. [2]) we have searched for physical processes that may affect nuclear reactions in the solid state environment. We found theoretically [3,4] that the leading channel of the p + d → ^{3}He reaction in the solid environment is the so-called solid state internal conversion process, an adapted version of an ordinary internal conversion process. It was shown [3,4] that if the reaction p + d → ^{3}He takes place in solid material the nuclear energy is taken away by an electron of the environment instead of the emission of a γ photon.

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In the usual internal conversion process only one nucleus is involved. However, very many pairs of nuclei can be found which may go to a state of lower energy if they could cooperate exchanging e.g. a neutron (or proton) [5]. Therefore it is worth investigating the way the electronic environment could lead to such cooperation.

Let us take two initial nuclei $^{A_1}_{Z_1}X$, $^{A_2}_{Z_2}Y$ and two final nuclei $^{A_1-1}_{Z_1}X$, $^{A_2+1}_{Z_2}Y$ which may be formed by neutron exchange. If the sum E_{0i} of the rest energies of the initial nuclei is higher than the sum E_{0f} of the rest energies of the final nuclei, i.e. if $E_{0i} - E_{0f} = \Delta > 0$, then the two nuclei $\left(^{A_1}_{Z_1}X \text{ and } ^{A_2}_{Z_2}Y\right)$ are allowed to make a neutron exchange. $\Delta = \Delta_- + \Delta_+$, with $\Delta_- = \Delta_{A_1} - \Delta_{A_1-1}$ and $\Delta_+ = \Delta_{A_2} - \Delta_{A_2+1}$. Δ_{A_1}, Δ_{A_1-1}, Δ_{A_2}, Δ_{A_2+1} are the energy excesses of neutral atoms of mass numbers A_1, $A_1 - 1$, A_2, $A_2 + 1$, respectively [5]. As it was stated above there exist very many pairs of nuclei that fulfill the $\Delta > 0$ requirement. So it is a question of how these nuclei can realize the $^{A_1}_{Z_1}X$, $^{A_2}_{Z_2}Y \rightarrow ^{A_1-1}_{Z_1}X$, $^{A_2+1}_{Z_2}Y$ neutron exchange transition.

The process

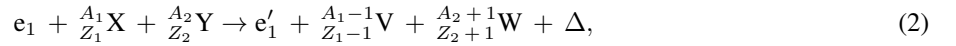$$e + ^{A_1}_{Z_1}X + ^{A_2}_{Z_2}Y \rightarrow e' + ^{A_1-1}_{Z_1}X + ^{A_2+1}_{Z_2}Y + \Delta, \tag{1}$$

that we call cooperative internal conversion process by neutron exchange (CICP-NE), will be discussed with bound-free electron transition and for atomic state only.

On the other hand, it can happen that the two final nuclei are $^{A_1-1}_{Z_1-1}X$, $^{A_2+1}_{Z_2+1}Y$ which may be formed by proton exchange from $^{A_1}_{Z_1}X$, $^{A_2}_{Z_2}Y$. If the sum E_{0i} of the rest energies of the initial nuclei is higher than the sum E_{0f} of the rest energies of the final nuclei then the two nuclei $\left(^{A_1}_{Z_1}X \text{ and } ^{A_2}_{Z_2}Y\right)$ are allowed to make a proton exchange.

Proton exchange can be realized in the process

$$e_1 + ^{A_1}_{Z_1}X + ^{A_2}_{Z_2}Y \rightarrow e'_1 + ^{A_1-1}_{Z_1-1}V + ^{A_2+1}_{Z_2+1}W + \Delta, \tag{2}$$

called cooperative internal conversion process by proton exchange (CICP-PE) which will be discussed theoretically in more detail too. (In processes (1) and (2) e and e' denote bound and free electrons, respectively.)

In Section 2, the transition rate (the transition probability per unit time) and in Section 3 the cross section of the cooperative internal conversion process by neutron exchange (CICP-NE) in the single electron-single nucleon model are derived. In Section 4, the cross section of CICP-NE in the spherical nuclear shell model is determined. Section 5 is devoted to cooperative internal conversion by proton exchange (CICP-PE). In Section 6, the total rate per unit volume of the sample, in Sections 7 and 8 as numerical examples the CICP-NE in Ne and CICP-PE in Al are given. In Section 9, other results and discussion and in Section 10, a summary can be found. In the Appendix the Coulomb factors $f_{2'3}$ and f_{34}, which are necessary in the case of CICP-PE, are determined.

2. Transition Rate of CICP-NE in Single Electron – Single Nucleon Model

The CICP-NE can be understood with the aid of a standard time independent second order perturbation calculation of quantum mechanics. There are two perturbations present which cause the effect and which have to be taken into account. The first is the electric dipole term V_{Cb}^{dip} of the Coulomb interaction between a bound electron and a neutron of an atom of nucleus $^{A_1}_{Z_1}X$ in which the effective charge of the neutron $q_n = -Z_1 e/A_1$ [6].

$$V_{Cb}^{dip} = \frac{Z_1 e^2}{A_1} \frac{4\pi}{3} x_1 x_e^{-2} \sum_{m=-1}^{m=1} Y_{1m}^*(\Omega_e) Y_{1m}(\Omega_1), \tag{3}$$

where Z_1 and A_1 are charge and mass numbers of the first nucleus, e is the elementary charge, x_1, x_e and Ω_1, Ω_e are magnitudes and solid angles of vectors $\mathbf{x}_1$, $\mathbf{x}_e$ which are the coordinates of the neutron and the electron in the first atom, respectively and Y_{1m} denotes spherical harmonics. The second is the nuclear strong interaction potential V_{st}

of nucleus $^{A_2}_{Z_2}Y$. For this nuclear potential a rectangular potential well is assumed, i.e. $V_{\mathrm{st}} = -V_0$ $(x_2 \leq R_{A_2})$ and $V_{\mathrm{st}} = 0$ $(x_2 > R_{A_2})$ where x_2 is the magnitude of vector $\mathbf{x}_2$, which is the coordinate of the neutron in the second nucleus and R_{A_2} is its radius.

The initial state, which is composed from the state of a bound electron of the atom having nucleus $^{A_1}_{Z_1}X$, the state of a bound neutron of the atomic nucleus $^{A_1}_{Z_1}X$ and the states of centers of mass motion, is changed to first order due to the perturbation $V^{\mathrm{dip}}_{\mathrm{Cb}}$ according to stationary perturbation calculation as

$$\psi_{\mathrm{i}} = \left[\psi^{(0)}_{\mathrm{i}} + \sum_k a_{\mathrm{bb},k} \psi^{(0)}_{k,\mathrm{bb}} + \sum_k \int a_{\mathrm{bf},k} \psi^{(0)}_{k,\mathrm{bf}} \, \mathrm{d}\nu_k + \int a_{\mathrm{ff},k} \psi^{(0)}_{k,\mathrm{ff}} \, \mathrm{d}\nu_k \right] \mathrm{e}^{-\frac{iE_{\mathrm{i}}t}{\hbar}}. \tag{4}$$

Here E_{i} is the total initial energy, which includes the sum $E_{0\mathrm{i}}$ of the rest energies of the initial nuclei $^{A_1}_{Z_1}X$, $^{A_2}_{Z_2}Y$. $\psi^{(0)}_{\mathrm{i}}$ is the product of the unperturbed bound electron and neutron states and two plane waves ψ_{CM,A_1} and ψ_{CM,A_2} which describe the motions of the centers of mass of the atoms having $^{A_1}_{Z_1}X$ and $^{A_2}_{Z_2}Y$ nucleus, respectively. $\psi^{(0)}_{k,\mathrm{bb}}$ is the product of other bound electron and neutron states, ψ_{CM,A_1} and ψ_{CM,A_2}. In $\psi^{(0)}_{k,\mathrm{bf}}$ one of the electron and neutron states is bound and the other is free which are multiplied by ψ_{CM,A_1} (in case of free electron) or by ψ_{CM,A_1-1} (in case of free neutron) and ψ_{CM,A_2}, where ψ_{CM,A_1-1} describes the motion of the center of mass of the atom having $^{A_1-1}_{Z_1}X$ nucleus. The last term is interesting from the point of view of our process and in $\psi^{(0)}_{k,\mathrm{ff}}$ both the electron and the neutron are free and their product is multiplied by ψ_{CM,A_1-1} and ψ_{CM,A_2}. Accordingly, the last term in Eq. (4) can be written as

$$\delta\psi_{\mathrm{i,free}} = \int \frac{V^{\mathrm{dip}}_{\mathrm{Cb},ki}}{E_{\mathrm{i}} - E_k} \psi^{(0)}_{k,\mathrm{ff}} \, \mathrm{d}\nu_k \mathrm{e}^{-\frac{iE_{\mathrm{i}}t}{\hbar}}. \tag{5}$$

The sum of the energy of the free electron, neutron and center of mass states is E_k, which contains the sum E_{0k} of the rest energies and the state density is $\mathrm{d}\nu_k$ [7].

Taking into account the interaction V_{st} between a free neutron and an other nucleus $^{A_2}_{Z_2}Y$ the second order transition rate has the form

$$W_{\mathrm{fi}} = \frac{2\pi}{\hbar} \int \left| \int \frac{V_{\mathrm{st},fk} V^{\mathrm{dip}}_{\mathrm{Cb},ki}}{E_{\mathrm{i}} - E_k} \, \mathrm{d}\nu_k \right|^2 \delta\left(E_{\mathrm{i}} - E_{\mathrm{f}}\right) \mathrm{d}\nu_{\mathrm{f}}. \tag{6}$$

Here $V^{\mathrm{dip}}_{\mathrm{Cb},ki}$ and $V_{\mathrm{st},fk}$ are matrix elements of $V^{\mathrm{dip}}_{\mathrm{Cb}}$ and V_{st} with states $\psi^{(0)}_{k,\mathrm{ff}}$, $\psi^{(0)}_{\mathrm{i}}$ and $\psi^{(0)}_{\mathrm{f}}$, $\psi^{(0)}_{k,\mathrm{ff}}$, respectively. $\psi^{(0)}_{\mathrm{f}}$ is the product of the free electron state of the nucleus $^{A_1-1}_{Z_1}X$, the bound neutron state of $^{A_2+1}_{Z_2}Y$ and free center of mass states of the two nuclei. The quantity $\mathrm{d}\nu_{\mathrm{f}}$ is the density of the final states of sum energy E_{f}, which comprises the sum $E_{0\mathrm{f}}$ of the rest energies of the final nuclei $^{A_1-1}_{Z_1}X$ and $^{A_2+1}_{Z_2}Y$. If the free states are plane waves of wave vectors $\mathbf{k}_{\mathrm{e}}$, $\mathbf{k}_1$ and $\mathbf{k}_2$ corresponding to the wave vectors of the free electron, the $^{A_1-1}_{Z_1}X$ nucleus which has lost the neutron and the nucleus $^{A_2+1}_{Z_2}Y$ which has taken up the neutron then

$$\mathrm{d}\nu_{\mathrm{f}} = \frac{V^3}{(2\pi)^9} \, \mathrm{d}\mathbf{k}_{\mathrm{e}} \, \mathrm{d}\mathbf{k}_1 \, \mathrm{d}\mathbf{k}_2, \, \mathrm{d}\nu_k = \frac{V}{(2\pi)^3} \, \mathrm{d}\mathbf{k}_{\mathrm{n}}, \tag{7}$$

where $\mathbf{k}_{\mathrm{n}}$ is the wave vector of the free (intermediate) neutron, and V is the volume of normalization. The initial wave vectors of atoms having nuclei $^{A_1}_{Z_1}X$ and $^{A_2}_{Z_2}Y$ are negligible. The nuclear energy difference Δ, that is the reaction energy, is distributed between the kinetic energies of the final free electron and the two final nuclei. All told, in process (1) the nucleus $^{A_1}_{Z_1}X$ loses a neutron which is taken up by the nucleus $^{A_2}_{Z_2}Y$ forming $^{A_2+1}_{Z_2}Y$ in this manner.

3. Cross Section of CICP-NE in Single Electron – Single Nucleon Model

The cross section σ_{bf} of the bound-free (bf) electron transitions of CICP-NE can be determined from the transition rate by the standard method. The evaluation of the transition rate is carried out in a one particle nuclear model. The hydrogen-like state of binding energy E_{Bi} and Coulomb-factor corrected plane wave are used as initial, bound and final, free electron states. The motion of the intermediate neutron and the two final nuclei are also described by plane waves. The rest masses of the two initial nuclei of mass numbers A_1 and A_2 are $m_1 = A_1 m_0$ and $m_2 = A_2 m_0$ where $m_0 c^2 = 931.494$ MeV is the atomic energy unit.

When calculating σ_{bf}, it is reasonable to use the

$$\mathrm{e}^{i\frac{m_{\mathrm{e}}}{(A_1-1)m_0}\mathbf{k}_1\cdot\mathbf{x}_{\mathrm{e}}} = 1$$

long wavelength approximation in the electron part of the Coulomb matrix element since

$$\frac{m_{\mathrm{e}}}{(A_1-1)m_0} \ll 1.$$

The analysis of σ_{bf} shows that those processes give an essential contribution to the cross section in which $k_{\mathrm{e}} \ll k_1$ and $k_{\mathrm{e}} \ll k_2$ where k_{e}, k_1 and k_2 are the magnitudes of the wave vectors of $\mathbf{k}_{\mathrm{e}}$, $\mathbf{k}_1$ and $\mathbf{k}_2$. In this case as a consequence of momentum conservation the integration over $\mathbf{k}_1$ can be carried out resulting the substitution $\mathbf{k}_1 = -\mathbf{k}_2$. Thus E_{f}, which is the sum of the kinetic energies of the free electron, particle $_{Z_1}^{A_1-1}\mathrm{X}$ and particle $_{Z_2}^{A_2+1}\mathrm{Y}$, (in the Dirac-delta) and $E(k_{\mathrm{e}}, k_2)$, which is the sum of the kinetic energies in the intermediate state (in the energy denominator) become $E_{\mathrm{f}} = \hbar^2 k_2^2 / [2m_0 a_{12}] + \hbar^2 k_{\mathrm{e}}^2 / (2m_{\mathrm{e}})$ with $a_{12} = (A_1 - 1)(A_2 + 1)/(A_1 + A_2)$ and

$$E(k_{\mathrm{e}}, k_2) = \frac{A_1}{A_1 - 1}\hbar^2 k_2^2 / (2m_0) + \hbar^2 k_{\mathrm{e}}^2 / (2m_{\mathrm{e}}),$$

respectively (the intermediate neutron has wave vector $-\mathbf{k}_2$). These simplifications result

$$\sigma_{\mathrm{bf}} = \left(\frac{2Z_1}{3A_1}V_0\right)^2 \frac{\alpha_{\mathrm{f}}^2\hbar c^2}{v(2\pi)^3} \int\int\int \delta(E_{\mathrm{f}} - \Delta_{\mathrm{Bi}}) \frac{\left|\sum_{m=-1}^{m=1} I_{\mathrm{ef}}^m(\mathbf{k}_{\mathrm{e}}) I_1^m(\mathbf{k}_2)\right|^2 |I_2(\mathbf{k}_2)|^2 \, \mathrm{d}\mathbf{k}_{\mathrm{e}}\,\mathrm{d}\mathbf{k}_2}{[E(k_{\mathrm{e}}, k_2) + \Delta_{\mathrm{n}} - \Delta_- + E_{\mathrm{Bi}}]^2}, \tag{8}$$

where v is the relative velocity of the two atoms, c is the velocity of light (in vacuum),

$$I_{\mathrm{ef}}^m(\mathbf{k}_{\mathrm{e}}) = \int u_{\mathrm{i}}(\mathbf{x}_{\mathrm{e}}) Y_{1m}^*(\Omega_{\mathrm{e}}) x_{\mathrm{e}}^{-2} \mathrm{e}^{-i\mathbf{k}_{\mathrm{e}}\cdot\mathbf{x}_{\mathrm{e}}} \, \mathrm{d}\mathbf{x}_{\mathrm{e}}, \tag{9}$$

$$I_1^m(\mathbf{k}_2) = \int \phi_{\mathrm{i}}(\mathbf{x}_1) x_1 Y_{1m}(\Omega_1) \mathrm{e}^{-i\mathbf{k}_2\frac{A_1}{(A_1-1)}\cdot\mathbf{x}_1} \, \mathrm{d}\mathbf{x}_1, \tag{10}$$

and

$$I_2(\mathbf{k}_2) = \int_{\Delta V} \phi_{\mathrm{f}}^*(\mathbf{x}_2) \, \mathrm{e}^{i\mathbf{k}_2\cdot\mathbf{x}_2} \, \mathrm{d}\mathbf{x}_2. \tag{11}$$

Here $u_{\mathrm{i}}(\mathbf{x}_{\mathrm{e}})$ and $\phi_{\mathrm{i}}(\mathbf{x}_1)$ are the initial bound electron and neutron states and $\phi_{\mathrm{f}}(\mathbf{x}_2)$ is the final bound neutron state. α_{f} denotes the fine structure constant and $\hbar$ is the reduced Planck-constant. $\Delta_{\mathrm{n}} = 8.071$ MeV is the energy excess of the neutron. Furthermore, m_{e} is the rest mass of the electron and $\Delta_{\mathrm{Bi}} = \Delta - E_{\mathrm{Bi}}$. ΔV in $I_2(\mathbf{k}_2)$ is that volume of the surface of the second nucleus (of A_2) in which direct neutron capture may be assumed [8]. It can be considered as a shell of a sphere of radius R_{A_2} and of thickness L, where L is the mean free path of the ingoing neutron in the nucleus.

4. Cross Section of CICP – NE in Spherical Nuclear Shell Model

The bound, initial electron, the initial and final nuclear states, which are used, have the form:

$$u_i(\mathbf{x}_e) = R_i(x_e) Y_{js}(\Omega_e), \qquad \phi_i(\mathbf{x}_1) = \varphi_i(x_1) Y_{l_i m_i}(\Omega_1)/x_1 \quad \text{and} \quad \phi_f(\mathbf{x}_2) = \varphi_f(x_2) Y_{l_f m_f}(\Omega_2)/x_2.$$

Here j and s are orbital angular momentum and magnetic quantum numbers of the bound electron state. $\varphi_i(x_1)/x_1$ and $\varphi_f(x_2)/x_2$ are the radial parts of the one particle shell-model solutions of quantum numbers l_i, m_i and l_f, m_f. In the cases to be investigated the corresponding part $R_{0\Lambda} = b_k^{-1/2}\Gamma(\Lambda + 3/2)^{-1/2}2^{1/2}\rho_k^{\Lambda+1}\exp(-\rho_k^2/2)$ of 0Λ one particle spherical shell model states [9] is applied as $\varphi_i(x_1)$ and $\varphi_f(x_2)$. Here

$$\rho_k = x_k/b_k, \quad b_k = \left(\frac{\hbar}{m_0\omega_{sh,k}}\right)^{1/2} \quad \text{and} \quad \hbar\omega_{sh,k} = 41A_k^{-1/3}$$

(in MeV units, [6]) with $k = 1, 2$ corresponding to A_1 and A_2, and $\Gamma(x)$ is the gamma function.

Thus in the case of spherical shell model states

$$\sigma_{bf,sh} = \frac{32}{3v}\left(\frac{Z_1}{A_1}V_0\right)^2 (2l_f + 1)\int\left|J_2^{l_f}(k_2)\right|^2 \tag{12}$$

$$\times \int\sum_{l,\lambda}\frac{N_{l\lambda}\left|J_1^{\lambda}(k_2)\right|^2\left|J_e^l(k_e)\right|^2}{[E(k_e,k_2) + \Delta_n - \Delta_- + E_{Bi}]^2}\alpha_f^2\hbar c^2\delta(E_f - \Delta_{Bi})k_e^2\,dk_e k_2^2\,dk_2,$$

where in case of $0l_f$ final nuclear state

$$\left|J_2^{l_f}(k_2)\right|^2 = \frac{\pi L^2 \rho_f^{2l_f+3}e^{-\rho_f^2}J_{l_f+\frac{1}{2}}^2(k_2 R_{A_2})}{\Gamma\left(l_f + \frac{3}{2}\right)k_2} \tag{13}$$

with $\rho_f = R_{A_2}/b_2$ and $J_{l_f+\frac{1}{2}}$ is a Bessel-function of the first kind. In the case of $0l_i$ initial nuclear state

$$J_1^{\lambda}(k_2) = \int R_{0l_i}(x_1) j_{\lambda}\left(\frac{A_1}{A_1 - 1}k_2 x_1\right) x_1^2\,dx_1, \tag{14}$$

$$J_e^l(k_e) = F_{Cb}^{1/2}(k_e)\int R_i(x_e) j_l(k_e x_e)\,dx_e, \tag{15}$$

$$N_{l\lambda} = (2l + 1)(2\lambda + 1)\begin{pmatrix} j & l & 1 \\ 0 & 0 & 0 \end{pmatrix}^2\begin{pmatrix} l_i & 1 & \lambda \\ 0 & 0 & 0 \end{pmatrix}^2. \tag{16}$$

The parenthesized expressions are Wigner $3j$ symbols. In Eqs. (14) and (15) j_{λ} and j_l are spherical Bessel-functions of the first kind. (The suffix sh of any quantity denotes that it is calculated in the one particle spherical shell model.) In (15) $F_{Cb}(k_e)$ is the Coulomb factor.

We restrict ourselves to $1s$ initial electronic state of $R_i(x_e) = 2a^{-3/2}\exp(-x_e/a)$ with $a = a_0/Z_{eff}$, where a_0 is the Bohr-radius, $Z_{eff} = \sqrt{E_{Bi}/Ry}$ and Ry is the Rydberg energy and use the $F_{Cb}(k_e) = 2\pi/(k_e a)$ approximation. Since

$$j = 0 \quad \text{and} \quad (2l + 1)\begin{pmatrix} 0 & 1 & l \\ 0 & 0 & 0 \end{pmatrix}^2 = \delta_{1,l},$$

$$N_{1\lambda} = (2\lambda + 1) \begin{pmatrix} l_i & 1 & \lambda \\ 0 & 0 & 0 \end{pmatrix}^2. \tag{17}$$

Keeping the leading term of $J_e^1(k_e)$, introducing $k_2 = k_0 x$, and carrying out integration over k_e with the aid of the energy-Dirac-delta and in the case of $l_i =$ even $[l_i = 2; \text{Ne}(3/2^+, 0d)]$ to be investigated one obtains

$$\sigma_{\text{bf,sh}} = \frac{2^{10}\pi^3}{3v} \frac{Z_1^2 V_0^2}{A_1^2 \hbar^3} \frac{b_1^5 L^2}{a_0^2} m_0 a_{12} \left(2l_f + 1\right) \frac{\rho_f^{2l_f + 3} e^{-\rho_f^2}}{\Gamma\left(l_f + \frac{1}{2}\right)} \sum_{\lambda = l_i \pm 1} \frac{N_{1\lambda} (k_0 b_1)^{2\lambda}}{\Gamma\left(\lambda + \frac{3}{2}\right)} S_\lambda. \tag{18}$$

Here $k_0 = \sqrt{2m_0 c^2 a_{12} \Delta_{\text{Bi}}}/(\hbar c)$,

$$S_\lambda = \int_0^1 f(x) g_\lambda(x) \, dx, \text{ with } x = k_2/k_0, \tag{19}$$

$$f(x) = \frac{\left(1 - x^2\right) x^{2\lambda + 1} e^{-(k_0 b_1)^2 x^2} J_{l_f + \frac{1}{2}}^2 (x k_0 R_{A_2})}{\left[1 + \frac{\Delta_{\text{Bi}}}{E_{\text{B}}}\left(1 - x^2\right)\right]^2 \left[\frac{A_1 a_{12}}{A_1 - 1} x^2 + 1 + \xi\right]^2} \tag{20}$$

and $\xi = (\Delta_n - \Delta_- + E_{\text{Bi}})/\Delta_{\text{Bi}}$. In Eq. (19) $g_\lambda(x) = 1$ if $\lambda = l_i + 1$ and

$$g_\lambda(x) = (2l_i + 1)^2 - 2(2l_i + 1)(k_0 b_1 x)^2 + (k_0 b_1 x)^4 \tag{21}$$

if $\lambda = l_i - 1$.

The differential cross section $d\sigma_{\text{bf,sh}}/dE_2$ of the process can be determined with the aid of

$$P(x) = \sum_{\lambda = l_i \pm 1} \frac{N_{1\lambda} (k_0 b_1)^{2\lambda}}{\Gamma\left(\lambda + \frac{3}{2}\right)} \frac{f(x) g_\lambda(x)}{x} \tag{22}$$

as $d\sigma_{\text{bf,sh}}/dE_2 = K_{\text{bf}} [P(x)]_{x=\sqrt{z}}/(2E_{20})$ where $z = E_2/E_{20}$ with $E_{20} = (A_1 - 1)\Delta_{\text{Bi}}/(A_1 + A_2)$, which is the possible maximum of the kinetic energy E_2 of particle $_{Z_2}^{A_2 + 1}Y$ created in the process, K_{bf} stands for the whole factor which multiplies the sum in (18). $d\sigma_{\text{bf,sh}}/dE_2$ has accountable values near below $z = 1$, i.e. if $E_2 \sim E_{20}$.

$d\sigma_{\text{bf,sh}}/dE_e = K_{bf} [P(x)]_{x=\sqrt{1-z}}/(2\Delta_{\text{Bi}})$ can also be determined with the aid of $P(x)$ where $z = E_e/\Delta_{\text{Bi}}$, E_e is the kinetic energy of the electron and K_{bf} is defined and is given above. $d\sigma_{\text{bf,sh}}/dE_e$ has accountable values near above $z = 0$, i.e. if $E_e \sim 0$.

5. Cooperative Internal Conversion Process by Proton Exchange

Now the cooperative internal conversion process by proton exchange (CICP-PE, process (2)) will be discussed. In this process (see Fig. 1) a bound proton of an atomic nucleus $\left(_{Z_1}^{A_1}X, \text{particle } 2\right)$ is virtually excited into a free state (particle 3) due to the dipole term $V_{\text{Cb}}^{\text{dip}}$ of its Coulomb-interaction (in electric dipole coupling the proton has effective charge $q_p = (1 - Z_1/A_1)e$ [6]) with one of the bound atomic electrons (e_1) of the atom containing the $_{Z_1}^{A_1}X$ nucleus while the electron becomes free (e_1'). The free, virtual proton is captured by an other nucleus $_{Z_2}^{A_2}Y$ (particle 4) due to its nuclear potential V_{st} (created by strong interaction) forming the nucleus $_{Z_2 + 1}^{A_2 + 1}W$ (particle 5) in this way. The sum of the rest energies of the initial nuclei is E_{0i} and the sum of the the rest energies of the final nuclei is E_{0f}. If $E_{0i} - E_{0f} = \Delta > 0$, i.e. if $E_{0i} > E_{0f}$, then the process is energetically allowed and proton exchange is possible. The nuclear energy difference Δ, which is the reaction energy, is shared between the kinetic energies of the final, free electron and the two final nuclei $[_{Z_1 - 1}^{A_1 - 1}V$ (particle $2'$) is the nucleus which has lost the proton].
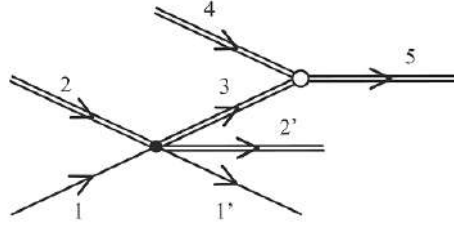
Figure 1. Graph of the cooperative internal conversion process by a heavy charged particle (e.g. proton) exchange. Particle 1 (bound) and 1'(free) are electrons, particle 2 is the nucleus which loses the heavy charged particle (e.g. proton) and becomes particle 2'. Particle 3 is the intermediate heavy charged particle (e.g. proton). Particle 4 is the nucleus which absorbs the heavy charged particle (e.g. proton) and becomes particle 5. The filled dot denotes (in case of a proton the dipole term of) the Coulomb-interaction and the open circle denotes nuclear (strong) interaction.

The transition rate and the cross section σ_{bf} of CICP-PE with bound-free (bf) electron transitions can also be determined with the aid of a standard second-order perturbation calculation of quantum mechanics. (The cross section of CICP-PE with bound-bound electron transition is neglected since it has proved to be much smaller than σ_{bf}.) The calculation of σ_{bf} is similar to the calculation of CICP-NE. The difference is the appearance of the square of the two Coulomb factors $f_{2'3}$ and f_{34} which ($f_{2'3}^2$ and f_{34}^2) multiply the cross section and Δ_{n}, $\frac{Z_1}{A_1}$ are changed to Δ_{p}, $\left(1 - \frac{Z_1}{A_1}\right)$, respectively. $\Delta_{\mathrm{p}} = 7.288969$ MeV is the energy excess of the proton. ($f_{2'3}$ and f_{34} are determined in the Appendix, see $(A.1)$.) The dipole term of the Coulomb interaction reads as

$$V_{\mathrm{Cb}}^{\mathrm{dip}} = \left(1 - \frac{Z_1}{A_1}\right) e^2 \frac{4\pi}{3} x_1 x_{\mathrm{e}}^{-2} \sum_{m=-1}^{m=1} Y_{1m}^*(\Omega_{\mathrm{e}}) Y_{1m}(\Omega_1),$$

where x_1 and Ω_1 are magnitude and solid angle of vector $\mathbf{x}_1$ which is the coordinate of the proton in the first atom. (The order of magnitude of the cross section produced by the L-th pole coupling is $(R/r)^{2L-2}$ times smaller than the cross section produced by the dipole coupling where R and r are the nuclear and atomic radii. Therefore the leading term to the cross section is produced by the dipole coupling.) The motion of the intermediate proton and the two final nuclei are also described by plane waves.

Introducing the wave vectors $\mathbf{k}_{\mathrm{e}}$ and $\mathbf{k}_1$, $\mathbf{k}_2$ of the free electron and particles $_{Z_1-1}^{A_1-1}V$ (particle 2') and $_{Z_2+1}^{A_2+1}W$ (particle 5), respectively, the analysis of σ_{bf} shows that, similarly to the CICP-NE, those processes give essential contribution to the cross section in which $k_{\mathrm{e}} \ll k_1$ and $k_{\mathrm{e}} \ll k_2$ where k_{e}, k_1 and k_2 are the magnitudes of the wave vectors of $\mathbf{k}_{\mathrm{e}}$, $\mathbf{k}_1$ and $\mathbf{k}_2$. (In this case as a consequence of momentum conservation $\mathbf{k}_1 = -\mathbf{k}_2$, furthermore the intermediate proton has wave vector $-\mathbf{k}_2$.)

The initial and final nuclear states are one particle shell-model solutions of quantum numbers l_{i}, m_{i} and l_{f}, m_{f}. The case of spherical shell model states of $0l_{\mathrm{i}}$ initial nuclear state and of $0l_{\mathrm{f}}$ final nuclear state is investigated.

Also keeping the leading term of $J_{\mathrm{e}}^1(k_{\mathrm{e}})$, in the one particle spherical shell model and in the case of $l_{\mathrm{i}} = $ even $[l_{\mathrm{i}} = 2; \mathrm{Al}(5/2^+, 0d)]$ to be investigated one obtains

$$\sigma_{\mathrm{bf,sh}} = \frac{2^{10}\pi^3}{3v} \left(1 - \frac{Z_1}{A_1}\right)^2 \frac{V_0^2}{\hbar^3} \frac{b_1^5 L^2}{a_0^2} m_0 a_{12} (2l_{\mathrm{f}} + 1) \frac{\rho_{\mathrm{f}}^{2l_{\mathrm{f}}+3} e^{-\rho_{\mathrm{f}}^2}}{\Gamma\left(l_{\mathrm{f}} + \frac{1}{2}\right)} \sum_{\lambda=l_{\mathrm{i}}\pm 1} \frac{N_{1\lambda} (k_0 b_1)^{2\lambda}}{\Gamma\left(\lambda + \frac{3}{2}\right)} S_{\lambda 2}. \tag{23}$$

Here

$$S_{\lambda 2} = \int_0^1 f(x) g_\lambda(x) h_{2'3}(x) h_{34}(x) \, dx, \tag{24}$$

$$f(x) = \frac{(1-x^2) x^{2\lambda+1} e^{-(k_0 b_1)^2 x^2} J_{l_f+\frac{1}{2}}^2 (x k_0 R_{A_2})}{\left[1 + \frac{\Delta_{Bi}}{E_B}(1-x^2)\right]^2 \left[\frac{A_1 a_{12}}{A_1-1} x^2 + 1 + \xi\right]^2}, \tag{25}$$

$x = k_2/k_0$, and $\xi = (\Delta_p - \Delta_- + E_{Bi})/\Delta_{Bi}$. In (24) $h_{jk}(x) = d_{jk}(x)/\left[\exp(d_{jk}(x)) - 1\right]$, $j, k = 2', 3$ and $3, 4$ ($f_{2'3}^2$ and f_{34}^2, see $(A.1)$ in the Appendix) with

$$d_{2'3}(x) = 2\pi(Z_1 - 1)\alpha_f \frac{1}{x} \sqrt{\frac{(A_1 + A_2) m_0 c^2}{2A_1(A_2+1)\Delta_{Bi}}} \tag{26}$$

and

$$d_{34}(x) = 2\pi Z_2 \alpha_f \frac{1}{x} \sqrt{\frac{(A_1 + A_2) m_0 c^2}{2(A_1-1)(A_2+1)\Delta_{Bi}}} \tag{27}$$

(see $(A.4)$ and $(A.6)$ in the Appendix).

The quantities $d\sigma_{bf,sh}/dE_2$ and $d\sigma_{bf,sh}/dE_e$ can also be determined with the aid of

$$P(x) = \sum_{\lambda=l_i\pm 1} \frac{N_{1\lambda}(k_0 b_1)^{2\lambda}}{\Gamma\left(\lambda + \frac{3}{2}\right)} \frac{f(x) g_\lambda(x) h_1(x) h_2(x)}{x} \tag{28}$$

but now K_{bf} stands for the whole factor which multiplies the sum in (23). $d\sigma_{0bf,sh}/dE_2$ has accountable values if $E_2 \sim E_{20}$ and $d\sigma_{0bf,sh}/dE_e$ has accountable values if $E_e \sim 0$.

6. Total Rate Per Unit Volume

The transition rate λ_1 of CICP of one nucleus of mass number A_1 of an atomic gas of number density n created by all those isotopes of mass number A_2 for which CICP is allowed, is expressed in the spherical shell model in the $\sigma_{bf} = \sigma_{bf,sh}$ approximation as $\lambda_1 = n \sum_{A_2} r_{A_2} v \sigma_{bf,sh}$. Here r_{A_2} is the relative natural abundance of isotope of mass number A_2 (in the cases of Ne and Al to be investigated the $A_2 = A_1$ event is only possible). Furthermore $\tau_{1/2,1} = \ln 2/\lambda_1$, which can be considered as a half-life.

The quantity $r_{A_1} n \lambda_1$ is the rate per unit volume of the sample produced by the nuclei of mass number A_1 which take part as initial nuclei in CICP, and $r_{tot} = \sum_{A_1} r_{A_1} n \lambda_1$, which is the total rate per unit volume of the sample.

If the participating atoms are in the same solid then the cross sections are multiplied by the so-called Debye–Waller factor [10]. The analysis of (8), (12), (18) and (23) modified accordingly shows that the cross section and consequently the transition rate drastically drop due to the Debye–Waller factor and the phenomenon (CICP) becomes unobservable. In the case of diatomic molecules the appearance of a factor $\exp(-\beta^2 r_a^2/4)$ in the cross section and the transition rate leads to the same result. (Here r_a is the distance between the two nuclei in equilibrium position and $\beta = \sqrt{\mu\omega/\hbar}$ were μ is the reduced mass of the two vibrating atoms and ω is the angular frequency of vibration.) One can draw the conclusion that the phenomenon (CICP) is mainly related to atomic state, i.e. CICP can have an observable rate if either of the participating atoms is in atomic state or they belong to different clusters (solids, molecules).

7. Numerical Calculation in Case of Ne (CICP-NE)

In the numerical calculation $V_0 = 50$ MeV is used [6]. In the case of Ne only the $e + {}^{21}_{10}Ne + {}^{21}_{10}Ne \rightarrow e' + {}^{20}_{10}Ne + {}^{22}_{10}Ne + \Delta$ reaction of $\Delta = 3.603$ MeV is allowed. CICP-NE does not work in Ar since all the possible channels are energetically forbidden. On the other hand in the case of Kr and Xe nuclei the applicability of the spherical shell model may be in doubt. For Ne the transition rate is estimated as $\lambda_1 > \lambda_1(K)$, which is the transition rate of the bound-free CICP-NE from the K shell of Ne. The initial and final states of ${}^{21}Ne$ and ${}^{22}Ne$ are supposed to be $0d$ spherical shell model states of $l_i = l_f = 2$, $r_{A_1} = r_{A_2} = r_{21} = 0.0027$, and $\Delta_- = 1.310$ MeV. The electron binding energy in the K shell is $E_{Bi} = 870.1$ eV. ($2E_{20} = 3.26$ MeV and $K_{bf}/(2E_{20}) = 4.77 \times 10^{-25}$ cm^{3}s^{-1}MeV$^{-1}/v$). $v\sigma_{bf,sh}(K) = 2.48 \times 10^{-34}$ cm^{3}s^{-1} is obtained in the case of bound-free CICP-NE from the K shell of Ne. Taking this, the $\lambda_1 > 1.8 \times 10^{-17}$ s^{-1} and $\tau_{1/2,1} < 3.8 \times 10^{16}$ s (1.2×10^9 y) and $r_{tot} > 1.25$ cm^{-3}s^{-1} for a gas of normal state $(n = 2.652 \times 10^{19}cm^{-3})$. The estimated half life is so long that the decay through CICP-NE does not alter natural abundance of Ne in observable measure. However, the rate is high enough to be measurable. ${}^{20}_{10}Ne$ and ${}^{22}_{10}Ne$ are mostly formed with energy near below

$$E_{10} = \frac{A_1 + 1}{A_1 + A_2}\Delta_{Bi} = 1.97 \text{ MeV} \quad \text{and} \quad E_{20} = \frac{A_1 - 1}{A_1 + A_2}\Delta_{Bi} = 1.63 \text{ MeV}$$

and with wave vectors of opposite direction. Therefore, it is plausible to observe their creation in coincidence measurement.

8. Numerical Calculation in Case of Al (CICP-PE)

It is a special case of process (2) when the two initial nuclei are identical. In this case the CICP-PE reads as

$$e_1 + {}^{A_1}_{Z_1}X + {}^{A_1}_{Z_1}X \rightarrow e'_1 + {}^{A_1-1}_{Z_1-1}V + {}^{A_1+1}_{Z_1+1}W + \Delta. \tag{29}$$

For example of such a case the reaction $e + {}^{27}_{13}Al + {}^{27}_{13}Al \rightarrow e' + {}^{26}_{12}Mg + {}^{28}_{14}Si + \Delta$ is considered when the reaction starts from the K shell. The initial and final nuclear states are supposed to be $0d$ spherical shell model states of $l_i = l_f = 2$, $\Delta = 3.31362$ MeV. The electron binding energy in the K shell is $E_{Bi} = 1.5596$ keV and $\Delta_- = -0.98235$ MeV. In this case $v\sigma_{bf,sh}(K) = 7.23 \times 10^{-36}$ cm^{3}s^{-1} is obtained in the case of bound-free CICP-PE from the K shell of Al. If one compares this result with $v\sigma_{bf,sh}(K) = 2.48 \times 10^{-34}$ cm^{3}s^{-1} obtained in the case of CICP-NE in Ne one can recognize that the ratio of the two cross sections is only 0.029. At first sight it seems to be larger then expected since two Coulomb factors appear in the cross section. But as it was said earlier the intermediate proton has wave vector $-\mathbf{k}_2$ and thus its energy $E_3 = \hbar^2\mathbf{k}_2^2/(2m_0)$ with $\hbar k_2 = \hbar k_0 x = x\sqrt{2m_0\Delta_{Bi}a_{12}}$. It gives $E_3 = x^2\Delta_{Bi}A_1/2$ since $a_{12} = A_1/2$ if $A_1 = A_2$ and near below $x = 1$ the value of E_3 is large enough to result moderately small Coulomb factors.

For a gas of atomic Al and of number density n the rate $\lambda_1 = n\sum_{A_2} r_{A_2}v\sigma_{bf,sh} = nv\sigma_{bf,sh}$ since the relative natural abundance r_{A_2} of the initial ${}^{27}_{13}Al$ isotope equals unity. λ_1 is estimated as $\lambda_1 > \lambda_1(K)$, which is the transition rate of the bound-free CICP-PE from the K shell of Al ($\lambda_1(K) = nv\sigma_{bf,sh}(K)$), resulting $\lambda_1 > 1.92 \times 10^{-16}$ s^{-1} and $r_{tot} > 5.08 \times 10^3$ cm^{-3}s^{-1} for a gas of normal state ($n = 2.652 \times 10^{19}$cm^{-3} and $r_{tot} = n\lambda_1$, which is the total rate per unit volume of the sample, in this case since $r_{A_1} = 1$).

9. Other Results and Discussion

Although the λ_1 obtained for Ne and Al is rather small, the Weisskopf-estimation of the cross section indicates that it, and consequently λ_1, may be increased very much with the increase of the atomic number. Moreover, the magnitudes of σ_{bf} and λ_1 are very sensitive to the model applied, e.g. if neutron or proton capture of the nucleus of A_2 is not

Table 1. Data for cooperative internal conversion process by neutron exchange of long lived nuclear fission products. (Data to reaction (1).) $A-1$ and $A+1$ are the mass numbers of the two final isotopes, τ is the half-life of the fission product in year units. For the definition of Δ_- and Δ_+ see the text.

Isotope	τ (year)	$A-1, A+1$	Δ_- (MeV)	Δ_+ (MeV)
^{113m}Cd	14.1	112, 114	1.795	1.235
^{121m}Sn	55	120, 122	1.907	0.749
^{151}Sm	90	150, 152	2.475	0.186
^{79}Se	6.5×10^4	78, 80	1.108	1.842
^{93}Zr	1.53×10^6	92, 94	1.337	0.149
^{107}Pd	6.5×10^6	106,108	1.533	1.149

restricted to the direct reaction then the integral in $I_2(\mathbf{k}_2)$ must be carried out over the whole volume of the nucleus. In this case $\sigma_{\rm bf,sh}$ and λ_1 are increased by a factor of about 240 for Ne. Furthermore, λ_1 can essentially increase e.g. with the increase of n. Therefore, contrary to the smallness of the value of λ_1 obtained in the cases of Ne and Al, CICP-NE and CICP-PE could play a role in the problem of nuclear waste disposal.

In Tables 1 and 2 some long lived fission products are listed which can take part in CICP-NE and CICP-PE, respectively. The values of Δ_- and Δ_+ indicate that each pair of isotopes chosen from either of the two tables can produce CICP-NE or CICP-PE since $\Delta_- + \Delta_- = \Delta > 0$ in every case. Consequently, it seems there may be a practical chance to accelerate the decay of the listed isotopes if they are collected in appropriately high concentration and density in atomic state. This is, however, a great technical challenge.

A special family of CICP-NE reactions – the $e + d + {}^A_Z X \rightarrow e' + p + {}^{A+1}_Z X + \Delta$ reaction family – is worth mentioning. The quantity $\Delta_-(dp) = \Delta_d - \Delta_p = 5.846$ MeV, which is characteristic of neutron loss of the deuteron. Here Δ_d and Δ_p are mass excesses of d and p. The energy of the reaction is $\Delta = \Delta_-(dp) + \Delta_+$ and some specific reactions together with their Δ_+ and Δ values are listed in Table 3. (This is not a complete list.) Chances are that these reactions may open new perspectives in the field of nuclear energy production.

In Table 4. the Δ_-, Δ_+ and Δ data of some cooperative internal conversion processes by proton exchange (data to reaction (2)) can be found. In the first column the initial stable isotope of relative natural abundance unity and in the second column the reaction products are listed.

There are other possibilities to realize CICP, when a charged heavy particle (such as d, t, ${}^3_2$He and ${}^4_2$He) is exchanged instead of proton exchange. The process is called cooperative internal conversion process by heavy charged particle exchange (CICP-HCPE) and it can be visualized with the aid of Fig.1 too. Denoting the intermediate particle (particle 3 in Fig. 1) by ${}^{A_3}_{z_3}$w, which is exchanged, the cooperative internal conversion process by heavy charged particle exchange reads as

$$e + {}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}Y \rightarrow e' + {}^{A_1-A_3}_{Z_1-z_3}V + {}^{A_2+A_3}_{Z_2+z_3}W + \Delta. \tag{30}$$
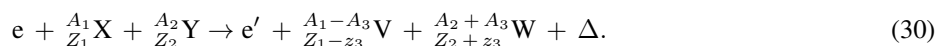
Table 2. Data for the cooperative internal conversion process by proton exchange of long lived nuclear fission products. (Data to reaction (2).) Products are the two stable final isotopes, τ is the half-life of the fission product in year units. For the definition of Δ_- and Δ_+ see the text.

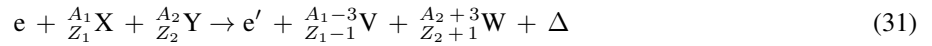| Isotope	τ (year)	Products	Δ_- (MeV)	Δ_+ (MeV)
^{99}Tc	2.11×10^5	^{98}Mo, ^{100}Ru	0.789	1.896
^{129}I	1.57×10^7	^{128}Te, ^{130}Xe	0.491	1.378
^{135}Cs	2.3×10^6	^{134}Xe, ^{136}Ba	0.538	1.305
^{137}Cs	30.07	^{136}Xe, ^{138}Ba	-0.126	1.716
^{155}Eu	4.7611	^{154}Sm, ^{156}Gd	0.637	0.717

Table 3. The values of the quantities $\Delta_+(A)$ and $\Delta = \Delta_-(dp) + \Delta_+(A)$ of the $e + d + {}^A_Z X \rightarrow e' + p + {}^{A+1}_Z X + \Delta$ reaction. r_A is the natural abundance of the isotope. $\Delta_-(dp) = \Delta_d - \Delta_p = 5.846$ MeV, Δ_d and Δ_p are mass excesses of d and p.

Isotope	r_A	$\Delta_+(A)$ (MeV)	Δ (MeV)
${}^9_4$Be	1.0	−1.259	4.587
${}^{10}_5$B	0.199	3.382	9.228
${}^{11}_5$B	0.812	−4.701	1.145
${}^{12}_6$C	0.989	−3.125	2.721
${}^{13}_6$C	0.011	0.105	5.951
${}^{14}_7$N	0.99634	2.762	8.608
${}^{15}_7$N	0.00366	−5.581	0.265
${}^{16}_8$O	0.99762	−3.928	1.918
${}^{17}_8$O	0.00038	−0.027	5.819

Here e and e′ denote bound and free electron and Δ is the energy of the reaction, i.e. the difference between the rest energies of initial $\left({}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}Y\right)$ and final $\left({}^{A_1-A_3}_{Z_1-z_3}V + {}^{A_2+A_3}_{Z_2+z_3}W\right)$ states. $\Delta = \Delta_- + \Delta_+$, with $\Delta_- = \Delta^{A_1}_{Z_1} - \Delta^{A_1-A_3}_{Z_1-z_3}$ and $\Delta_+ = \Delta^{A_2}_{Z_2} - \Delta^{A_2+A_3}_{Z_2+z_3}$. $\Delta^{A_1}_{Z_1}, \Delta^{A_1-A_3}_{Z_1-z_3}, \Delta^{A_2}_{Z_2}, \Delta^{A_2+A_3}_{Z_2+z_3}$ are the energy excesses of neutral atoms of mass number–charge number pairs A_1, Z_1 ; $A_1 - A_3, Z_1 - z_3$; A_2, Z_2; $A_2 + A_3, Z_2 + z_3$, respectively [5].

In process (30) the initial bound electron (particle 1) Coulomb interacts with the nucleus ${}^{A_1}_{Z_1}X$ (particle 2). A free electron (particle 1′), the intermediate particle ${}^{A_3}_{z_3}w$ (particle 3) and the nucleus ${}^{A_1-A_3}_{Z_1-z_3}V$ (particle 2′) are created due to this interaction. The intermediate particle ${}^{A_3}_{z_3}w$ (particle 3) is captured due to the strong interaction by the nucleus ${}^{A_2}_{Z_2}Y$ (particle 4) forming the nucleus ${}^{A_2+A_3}_{Z_2+z_3}W$ (particle 5) in this manner. So in process (30) the nucleus ${}^{A_1}_{Z_1}X$ (particle 2) looses a particle ${}^{A_3}_{z_3}w$ which is taken up by the nucleus ${}^{A_2}_{Z_2}Y$ (particle 4). The process is energetically forbidden if $\Delta < 0$.

As an example we deal with the

$$e + {}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}Y \rightarrow e' + {}^{A_1-3}_{Z_1-1}V + {}^{A_2+3}_{Z_2+1}W + \Delta \tag{31}$$

reaction in which a triton is exchanged. It is called CICP by triton exchange (CICP-TE). A special type of reaction (31) is

$$e + {}^{A_1}_{Z_1}X + {}^{A_1}_{Z_1}X \rightarrow e' + {}^{A_1-3}_{Z_1-1}V + {}^{A_2+3}_{Z_2+1}W + \Delta. \tag{32}$$
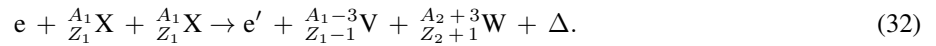
Table 4. Data for cooperative internal conversion process by proton exchange. (Data for reaction (2).) In the first column the initial stable isotope and in the second column the reaction products can be found. For the definition of Δ_-, Δ_+ and Δ see the text.

Isotope	Products	Δ_- (MeV)	Δ_+ (MeV)	Δ (MeV)
^{19}F	^{18}O, ^{20}Ne	−0.705	5.555	4.850
^{23}Na	^{22}Ne, ^{24}Mg	−1.505	4.404	2.899
^{27}Al	^{26}Mg, ^{28}Si	−0.982	4.296	3.314
^{31}P	^{30}Si, ^{32}S	−0.008	1.575	1.567
^{45}Sc	^{44}Ca, ^{46}Ti	0.400	3.056	3.456
^{55}Mn	^{54}Cr, ^{56}Fe	−0.778	2.895	2.117
^{59}Co	^{58}Fe, ^{60}Ni	−0.075	2.245	2.170
^{103}Rh	^{102}Ru, ^{104}Pd	1.076	1.369	2.445
^{127}I	^{126}Te, ^{128}Xe	1.083	0.873	1.956
^{133}Cs	^{132}Xe, ^{134}Ba	1.204	0.879	2.083

Table 5.   Data for the cooperative internal conversion process by triton exchange. (Data to reaction (32).) In the first column the initial stable isotope and in the second column the reaction products can be found. For the definition of Δ_-, Δ_+ and Δ see the text.

Isotope	Products	Δ_- (MeV)	Δ_+ (MeV)	Δ(MeV)
^{19}F	^{16}O, ^{22}Ne	3.250	6.537	9.787
^{23}Na	^{20}Ne, ^{26}Mg	−2.488	6.685	4.197
^{27}Al	^{24}Mg, ^{30}Si	−3.263	7.236	3.973
^{31}P	^{28}Si, ^{34}S	−2.948	5.491	2.543
^{45}Sc	^{42}Ca, ^{48}Ti	−2.522	7.418	4.896
^{55}Mn	^{52}Cr, ^{58}Fe	−2.294	4.443	2.149
^{59}Co	^{56}Fe, ^{62}Ni	−1.623	4.519	2.896
^{103}Rh	^{100}Ru, ^{106}Pd	1.197	1.882	3.079
^{127}I	^{124}Te, ^{130}Xe	1.536	0.893	2.429
^{133}Cs	^{130}Xe, ^{136}Ba	1.806	0.816	2.622

In Table 5, the Δ_-, Δ_- and Δ data of some cooperative internal conversion processes by triton exchange (data to reaction (32)) can be found.

10. Summary

In this paper the cooperative internal conversion process (CICP) is discussed. It is shown that the Coulomb interaction between bound electrons and the nucleons of a nucleus may lead to exchange of one or more nucleons between two nuclei which are located in different atoms. The cross sections of CICP with neutron and proton exchange are calculated and the qualitatively extended mechanism also shows the possibility of many nucleon (such as d, t, $^3_2$He and $^4_2$He) exchange. There is a chance that CICP may accelerate the decay of long lived isotopes, e.g. long-lived nuclear fission products. Moreover, it was found that special types of CICP with neutron exchange of deuterons with other nuclei may open new perspectives in the field of nuclear energy production. In conclusion, the results indicate that CICP seems to be able to significantly modify nuclear processes.

Appendix A. Square of Coulomb Factors ($f^2_{2'3}$ and f^2_{34})

Since particles $2'$, 3 and 4 all have positive charge, and furthermore they are all heavy, the square of the two essential Coulomb factors $f_{2'3}$ and f_{34} appears in the cross section. Since $f^2_{2'3}$ and f^2_{34} determine the order of magnitude of the cross section of the process (as it is proportional to $f^2_{2'3}f^2_{34}$) we treat them in more detail in the case of CICP-PE in the following.

The wave function $\varphi(\mathbf{r})$ of a free particle of charge number z_j in a repulsive Coulomb field of charge number z_k [11] (called Coulomb solution) has the following property: $\varphi(\mathbf{r}) \sim e^{-\pi\eta_{jk}/2}\Gamma(1 + i\eta_{jk})$, where Γ is the Gamma function. The quantity

$$\left| e^{-\pi\eta_{jk}/2}\Gamma(1 + i\eta_{jk}) \right| = \sqrt{\frac{2\pi\eta_{jk}(E)}{\exp\left[2\pi\eta_{jk}(E)\right] - 1}} = f_{jk}(E) \tag{A.1}$$

is the Coulomb factor. Here

$$\eta_{jk}(E) = z_j z_k \alpha_{\mathrm{f}} \sqrt{a_{jk}\frac{m_0 c^2}{2E}} \tag{A.2}$$

is the Sommerfeld parameter.

$$a_{jk} = \frac{A_j A_k}{A_j + A_k} \tag{A.3}$$

is the reduced mass number of particles j and k of mass numbers A_j, A_k and E is taken in the center of mass (CM) coordinate system.

If initial particles have negligible initial momentum then in the final state $\mathbf{k}_1 = -\mathbf{k}_2$ ($\mathbf{k}_{\text{particle},2'} = -\mathbf{k}_{\text{particle},5}$) because of momentum conservation. (We have determined that the process has an accountable cross section if the momentum of the final electron can be neglected.) In this case the momentum and energy of the virtual particle 3 (e.g. proton) are $\mathbf{k}_{\text{particle},3} = -\mathbf{k}_{\text{particle},2'} = \mathbf{k}_{\text{particle},5} \equiv \mathbf{k}_2$ and $E_3 = \hbar^2 \mathbf{k}_2^2 / (2m_3)$, where $\hbar$ is the reduced Planck-constant. Calculating the Coulomb factor $f_{2'3}$ (see (A.1)) between particles $2'$ and 3 the energy E_3 is given in their CM coordinate system (since $\mathbf{k}_{\text{particle},3} = -\mathbf{k}_{\text{particle},2'}$) thus E_3 can be substituted directly in (A.2) producing

$$\eta_{2'3} = (Z_1 - 1)\, \alpha_{\text{f}} \frac{1}{x} \sqrt{\frac{(A_1 + A_2)\, m_0 c^2}{2 A_1 (A_2 + 1)\, \Delta_{\text{Bi}}}} \tag{A.4}$$

in case of proton exchange. Here the $k_2 = k_0 x$ substitution is also used. Calculating f_{34}, the energy E_3 of particle 3 is now given in the laboratory frame of reference since particle 4 is at rest. In the CM system of particles 3 and 4 the energy $E_3(\text{CM})$ is

$$E_3(\text{CM}) = \frac{A_{\text{particle},4}}{(A_{\text{particle},3} + A_{\text{particle},4})} E_3. \tag{A.5}$$

Substituting it into (A.2)

$$\eta_{34} = Z_2 \alpha_{\text{f}} \frac{1}{x} \sqrt{\frac{(A_1 + A_2)\, m_0 c^2}{2 (A_1 - 1)(A_2 + 1)\, \Delta_{\text{Bi}}}} \tag{A.6}$$

in case of proton exchange.

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