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Research Article

Lithium – An Important Additive in Condensed Matter Nuclear Science

Chang L. Liang, Zhan M. Dong, Yun P. Fu and Xing Z. Li*

Department of Physics Tsinghua University, Beijing 100084, People's Republic of China

Abstract

A (p+^{6}Li) low energy resonance state is found using the 3-parameter formula for fusion cross-section based on the selective resonant tunneling model. The electron capture is the possible weak interaction to make use of this low energy resonance. When the resonance energy level is close to zero, the width of the resonance peak in the fusion cross-section is much greater than the width of the resonance energy level; therefore, the absorption through this low energy resonance level is no longer a problem for resonant tunneling of Coulomb barrier at low energy. Both hot fusion data and the CMNS experiment data support this resonant tunneling concept. As a result, lithium turns out to be an important additive in both CMNS and Hot Plasma fusion research.

Keywords: Resonance and resonant tunneling, p+^{6}Li low energy resonance, Resonance accompanied with weak interaction, The width of resonance at low energy, 3-Parameter formula for fusion cross-section

1. Introduction

A formula for fusion cross-section has been presented in ICCF-14 (Washington D.C. 2008). It was based on selective resonant tunneling model [1], and has three parameters only [2–8]; however, it fits with a set of five major fusion cross-section experimental data much better than the 5-parameter old formula in the famous NRL Plasma Formulary [9,10] does. This 3-parameter formula has been further applied to almost all the comparable light nucleus fusion cross-section data (14 sets of data including p+p, p+D, p+^{6}Li, p+^{7}Li, p+^{7}Be, p+^{9}Be, p+^{10}B, d+d, d+T, d+^{3}He, d+^{6}Li, d+^{11}B, t+t and t+^{3}He), and the good agreement with those hot fusion data strongly confirmed the validity of this 3-parameter formula and its physics – selective resonant tunneling model.

Using this 3-parameter formula, we are going to discuss the following three issues:

(1) What is the resonance and resonant tunneling in CMNS phenomena? Although resonance has been mentioned almost everywhere in CMNS literature, nevertheless, what is the definition of a resonance in CMNS? Whenever one needs to overcome Coulomb barrier, a resonance is invoked as a Panacea, however, why does a resonance

*Corresponding author. E-mail: lxz-dmp@tsinghua.edu.cn

overcome the Coulomb barrier? How can a resonance make a resonant tunneling? What are the necessary conditions to have this important resonant tunneling? Is there really such a low energy nuclear resonance in the nature? How can we identify such a resonance while there is no experimental data for a proton projectile of thermal energy? This 3-parameter formula is able to answer these questions, and shows clearly that (p+^{6}Li) has just this resonance.

(2) Lithium has been used as an additive widely in CMNS research; however, it was based on the consideration of electrochemistry ("conductivity" for electrolyte), or based on chemical engineering ("catalyzer" in Ni–H systems), or based on thermodynamics ("long range coherence in liquid lithium" – Chemonuclear reaction). Since the year of 2002, we attempted to find an explanation for this lithium issue based on nuclear physics. This presentation is an effort to pursue this explanation using the 3-parameter formula for fusion cross-section.

(3) The width of a resonance peak in *cross-section* and the width of a resonance *energy level* are different concepts; however, there are some misunderstanding which would be clarified using the selective resonant tunneling model

2. 3-Parameter New Formula for Fusion Cross-section

For low energy projectile the cross-section, $\sigma(E)$ is used to be expressed by a phase-shift of S-partial wave function, δ_0, provided that S-partial wave is dominant,

$$\sigma(E) = \frac{\pi}{k^2}\left(1 - \left|\mathrm{e}^{i2\delta_0}\right|^2\right). \tag{1}$$

Here, k is the wave number in the centre of mass system. This expression does not show clearly where Gamow factor for charged particle interaction is, and how the resonance would overcome the Coulomb barrier. Therefore, we derived another expression which is identically equal to Eq. (1) [2]

$$\sigma(E) = \frac{\pi}{k^2}\frac{(-4W_\mathrm{i})}{W_\mathrm{r}^2 + (W_\mathrm{i} - 1)^2}. \tag{2}$$

Here $W \equiv \cot\delta_0 \equiv W_\mathrm{r} + iW_\mathrm{i}$ is introduced to replace δ_0. The imaginary part, W_i, describes the absorption in the nuclear potential well. This new formula clearly shows the physical meaning of a resonance and resonant tunneling: resonance corresponds to an energy which makes $W_\mathrm{r} = 0$, and resonant tunneling corresponds to both $W\mathrm{r} = 0$ and $W\mathrm{i} = -1$. Indeed, W is the coefficient of a linear composition of two independent solutions of Schrodinger equation. In case of charged nuclei collision, $\varphi(\mathrm{r}) = W \cdot F_0 + G_0$, here $\varphi(\mathrm{r})$ is the reduced radial wave function in the Coulomb field, F_0 and G_0 are the regular and irregular Coulomb wave function, respectively. Here r is the radial distance from the centre of nuclear potential well. When the resonant tunneling happens,

$$\varphi(r) = (W_\mathrm{r} + iW_\mathrm{i}) \cdot F_0 + G_0 \underset{r\to\infty}{\longrightarrow} (W_\mathrm{r} + iW_\mathrm{i})\sin(kr) + \cos(kr) \propto \frac{1 + W_\mathrm{i} - iW_\mathrm{r}}{1 - W_\mathrm{i} + iW_\mathrm{r}}\mathrm{e}^{ikr} + \mathrm{e}^{-ikr} \to \mathrm{e}^{-ikr}. \tag{3}$$

The resonant tunneling implies an incoming spherical wave without any reflection, or a *perfect absorption* of incoming wave by a nuclear potential boundary. This is the physical meaning of a resonant tunneling. Then, where is the Gamow factor? It is embedded in W. Based on the continuity of wave function at the interface between nuclear potential well and Coulomb barrier, we may further explore the meaning of $W_\mathrm{r} = 0$ as follows:

$$W = -\left(\frac{G_0}{F_0}\right)_a \left[\frac{D_\mathrm{L} - \frac{k}{G_0}\frac{\partial G_0}{\partial\rho}\Big|_a}{D_\mathrm{L} - \frac{k}{F_0}\frac{\partial F_0}{\partial\rho}\Big|_a}\right] = -\left(\frac{G_0}{F_0}\right)_a \left[\frac{D_\mathrm{L} - \frac{k}{G_0}\frac{\partial G_0}{\partial\rho}\Big|_a}{\left(D_\mathrm{L} - \frac{k}{G_0}\frac{\partial G_0}{\partial\rho}\Big|_a\right) - \frac{k}{F_0G_0}\Big|_a}\right], \tag{4}$$

where D_L is the logarithmic derivative of inner wave function at the interface between Coulomb field and nuclear interaction region, k is the wave number in the Coulomb field in the centre of mass system. $\rho = k \cdot r$, a is the radius of the nuclear interaction region. When the real part of numerator in Eq. (4) approaches to 0 (which implies that the inner wave function is mainly connected to G_0 at the interface), we have $W_r = 0$. This is the first requirement to have a resonant tunneling.

In order to clearly show where the Gamow factor is, we may separate W into two factors in Eq. (4): the fast varying factor in round brackets and the slow one in square brackets. Since G_0 is exponentially rising and F_0 is exponentially decreasing when r is approaching nuclear boundary, a, the ratio of

$$\left(\frac{G_0}{F_0}\right)_a \propto \left(\frac{e^{2\pi/(ka_c)} - 1}{2\pi}\right) \equiv \theta^2$$

is a fast varying factor at low energy, and is a very large number.

$$a_c \equiv \frac{4\pi\varepsilon_0\hbar^2}{z_a z_b \mu e^2}.$$

ε_0 is the vacuum dielectric constant, $\hbar$ the Planck constant divided by2π, e the charge of proton, μ reduced mass, z_a and z_b are the charge numbers of the colliding nuclei, respectively. On the contrary, both

$$\left(D_L - \frac{k}{G_0}\frac{\partial G_0}{\partial \rho}\bigg|_a\right) \quad \text{and} \quad \frac{k}{F_0 G_0}\bigg|_a$$

are slowly varying factors when $k \to 0$; therefore, we may assume:

$$W \equiv \theta^2(w_r + iw_i) \approx \theta^2(C_1 + C_2 E_{lab} + iw_i) \tag{5}$$

and it leads to the $\sigma(E)$ expression with Gamow factor ($\frac{1}{\theta^2}$),

$$\sigma(E) = \frac{\pi}{k^2}\frac{1}{\theta^2}\frac{(-4w_i)}{w_r^2 + (w_i - \frac{1}{\theta^2})^2} \approx \frac{\pi}{k^2}\frac{1}{\theta^2}\frac{(-4w_i)}{(C_1 + C_2 E_{lab})^2 + (w_i - \frac{1}{\theta^2})^2}, \tag{6}$$

$$\theta^2 \equiv \left(\frac{e^{2\pi/(ka_c)} - 1}{2\pi}\right). \tag{7}$$

This new formula of fusion cross-section clearly shows that the Gamow factor would disappear only if $w_r = 0$ and $w_i \approx -\frac{1}{\theta^2}$. Indeed w_i is proportional to the imaginary part of D_L, i.e. the probability current density across the interface between the nuclear interaction region and the Coulomb field. Hence, $w_i \approx -\frac{1}{\theta^2}$ implies a very weak absorption rate inside the nuclear interaction region. This is the second necessary condition for a resonance to overcome the Coulomb barrier.

This assumption (5) is strongly supported by the experimental data for eight fusion cross-sections: d+T, d+^{3}He, d+D, t+T, t+^{3}He, p+D, p+^{6}Li and p+^{7}Li in Fig. 1. In the eight plots of Fig. 1, the solid lines are the fitting curves using Eq. (6) with three parameters: C_1 C_2 and w_i; circles are experimental data points from National Nuclear Data Center in Brookhaven National Lab. [11]. Using the least-squares method, we may find three parameters for each reaction as shown in Table 1. Logarithmic scales are used to show the good fit at very low energy region, and the famous resonance peak for d+T is shown in Fig. 1 in linear scale as well.

Recently this new formula has been further justified by 14 sets of light nuclei fusion data. However, we discuss first the fact that in Table 1, among the eight fusion data, only p+^{6}Li cross-section data might be fit by three parameters with diminishing C_1 and C_2. This is the first hint of low energy resonance in p+^{6}Li fusion, because it means that $w_r \to 0$ when $E_{lab} \to 0$, and the width of the resonance would be determined by the term of $(w_i - \frac{1}{\theta^2})^2$ in the denominator of Eq. (6).

Table 1. Three parameters in formula of cross-section for eight reactions

Reaction	C_1	C_2 (1/keV)	w_i	Norm(barn)/data point number	$(\sigma(E))_{max}$ (barn)	$(E)_{max}$ (keV) in Lab.
d+T	0.544	−0.00558	−0.390	0.227/24	5.0	280
d+^{3}He	1.13	−0.00304	−0.670	0.0520/800	0.8	1034
d+D	4.78	−0.00226	−0.186	0.00567/39	0.177	1045
t+T	36.8	−0.00928	−24.6	0.0129/757	0.115	4300
t+^{3}He	2.79	0.000959	−1.04	0.00331/225	0.0214	1000
p+D	8.04×10^7	-1.80×10^6	-5.31×10^7	3.35×10^{-8}/74	1.45×10^{-7}	48.1
p+^{6}Li	**0**	-5.02×10^{-9}	−6.62	0.00125/15	0.0197	190
p+^{7}Li	30.9	−0.00367	−4.18	0.000310/42	0.00633	998

3. The Width of this Resonance Energy Level near Zero Energy

The width of the resonance peak in the cross-section is very important after knowing the existence of this p+^{6}Li resonance near the zero energy, because it will determine the absorption integral of the injected proton in the CMNS experiments.

The diminishing C_1 and C_2, and the large value of $(-w_i)$ for p + ^{6}Li $\rightarrow$ ^{3}He + ^{4}He cross-section data makes an unexpected behaviour of the resonance peak. It does not show a sharp peak near the zero energy, instead it keeps climbing up the in the range of 50–190 keV without any peak. Indeed, in the range of 50–190 keV, we have

$$(C_1 + C_2 E_{lab})^2 \ll \left(w_i - \frac{1}{\theta^2}\right)^2 \tag{8}$$

because in this range, θ^2 varies from 97 150 to 148, hence, $\frac{1}{\theta^2}$ is much smaller than $(-w_i)$ such that $(w_i - \frac{1}{\theta^2})^2 \approx (-6.62)^2$, and $(C_1 + C_2 E_{lab})^2 < (0 - 5.02 \times 10^{-9} \times 190)^2 \ll (w_i - \frac{1}{\theta^2})^2$.

Thus we may write the expression of cross-section as

$$\sigma(E) \approx \frac{\pi}{k^2}\frac{1}{\theta^2}\frac{(-4w_i)}{(w_i - \frac{1}{\theta^2})^2} = \frac{\pi}{k^2}\frac{(-4\theta^2 w_i)}{(\theta^2 w_i - 1)^2} = \frac{\pi}{k^2}\frac{(4x)}{(x+1)^2}. \tag{9}$$

Here, $x \equiv -\theta^2 w_i$. It should be noticed that if $E \rightarrow 0$, $x \rightarrow \infty$ then, $\frac{(4x)}{(x+1)^2} \rightarrow 0$ and if $E \rightarrow \infty$, $x \rightarrow 0$; then, $\frac{(4x)}{(x+1)^2} \rightarrow 0$ as well. Only if $x = 1$, then $\frac{(4x)}{(x+1)^2} \rightarrow 1$ reaches its peak value. Its peak position is determined by

$$x \equiv -\theta^2 w_i = 1. \tag{10}$$

For p + ^{6}Li $\rightarrow$ ^{3}He + ^{4}He, $w_i = -6.62$, in the range of 50–190 keV,

$$x \equiv -\theta^2 w_i \sim 10^5 \text{ to } 10^2 \gg 1. \tag{11}$$

Therefore, $\sigma(E)$ would not show any peak until $x \equiv -\theta^2 w_i = 1$. In order to have a peak in this energy region it is necessary to reduce the value of $(-w_i)$. In Fig. 2, four curves with different values of w_i are drawn using Eq. (6) for p+^{6}Li reaction. The thin solid (red) line (the curve for $w_i = -6.62$) is for the real p + ^{6}Li $\rightarrow$ ^{3}He + ^{4}He data, the curve is almost overlapping with the abscissa without any peak before 800 keV. However, if there is any other weaker interaction which would make $(-w_i)$ much smaller; then, there might be a resonance peak before 800 keV (dotted line, dashed line, and the thick dash–dotted line). It is noticed that: the width of the resonance peak in cross-section is decreasing while $(-w_i)$ is getting smaller and smaller. Then one may ask if the width of resonance becomes extremely small when the peak position is approaching zero energy?

Fortunately, the resonance peak position of the cross-section and its width approaches zero slowly in the scale of log$[-w_i]$. It is very slow; hence, the absorption integral near this resonance energy level would approach to zero very

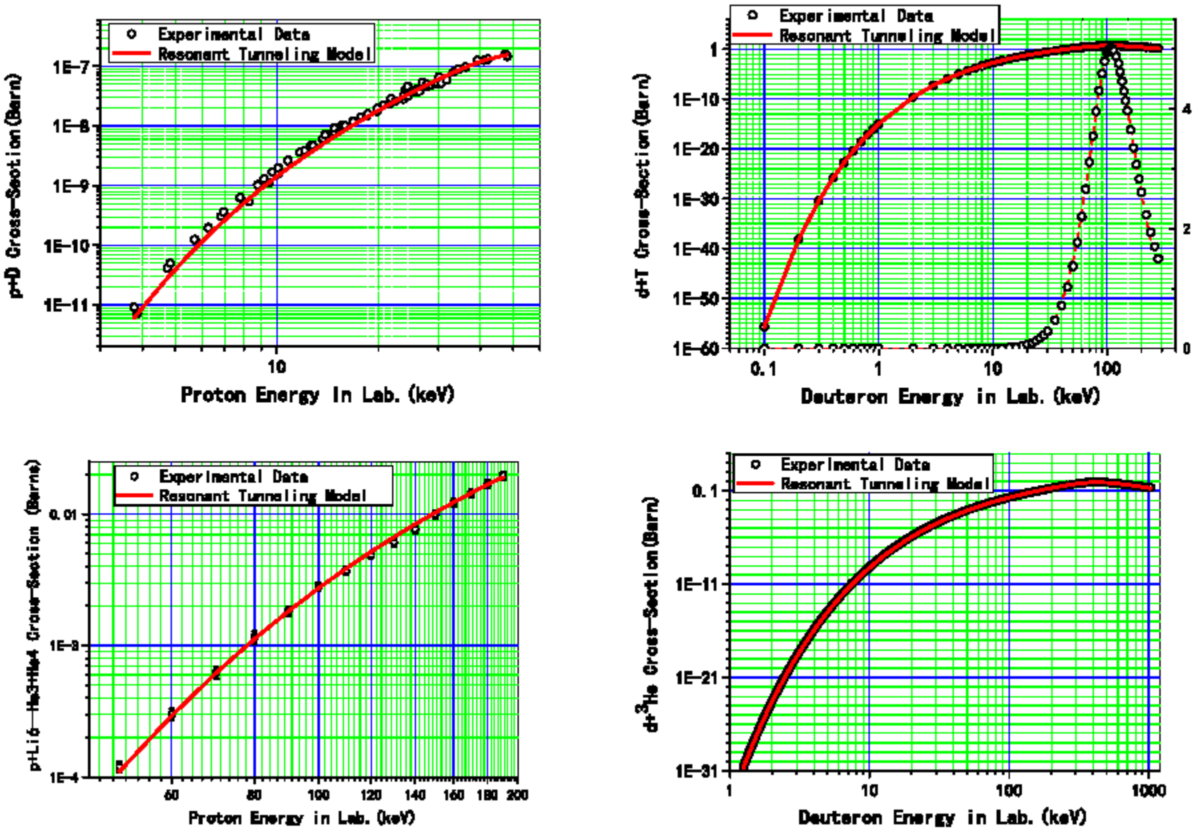

Figure 1. 3-Parameter formula, Eq. (6) is justified by eight sets of fusion cross-section data.

slow as well. Figure 3 shows that while the peak position of $\sigma(E)$ in Eq (9) is determined by $x \equiv -\theta^2 w_\mathrm{i} = 1$, the width of this peak is determined by $x = 3 + 2\sqrt{2}$ and $x = 3 - 2\sqrt{2}$. Therefore, when $-w_\mathrm{i} \to 0$, the corresponding energy of resonance peak and width are approaching zero in logarithmic scale as $\log[E]$ due to Eq. (7)

$$\left(\theta^2 \approx \left(\frac{\mathrm{e}^{2\pi/(\sqrt{E}a_c)}}{2\pi}\right)\right).$$

Indeed, what affects the absorption of injected beam is the integration under the curve between $x = 3 - 2\sqrt{2}$ and $x = 3 + 2\sqrt{2}$ (Fig. 3). The absorption integral – the shaded area under the cross-section curve in the Full Width Half Peak (FWHP) decreases with $(-w_\mathrm{i})$ slowly in a logarithmic scale (Fig. 4) as well. Consequently, we do not worry about the width of the resonance peak of the cross-section for this low energy resonance induced by a weak interaction.

There have been two misunderstanding about the width of this low energy resonance: (1) The life-time of the resonance would be very long due to the weak interaction; therefore, the energy level would be extremely narrow because of the energy uncertainty relation. As a result, it would be very difficult for the injected beam to be collimated

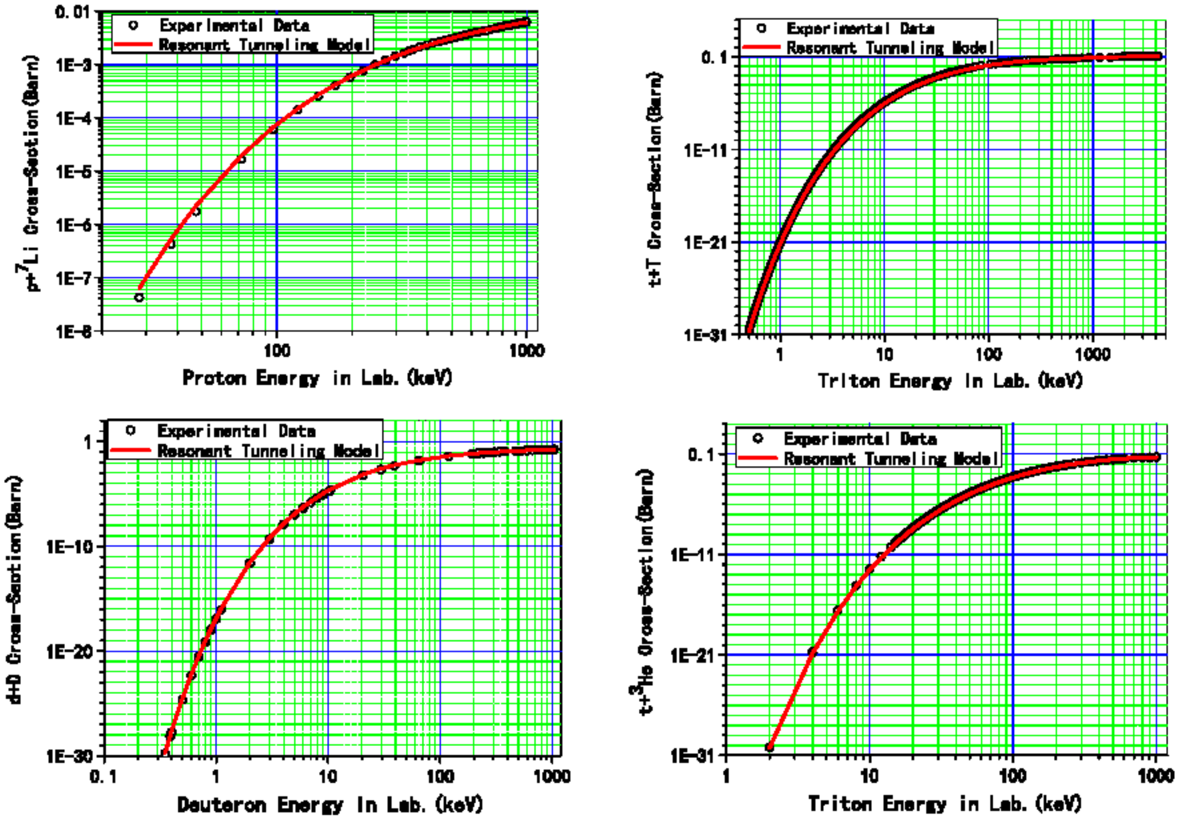

Figure 1 (continued).

in such a narrow energy width. (2) In Eq. (4), the difference between two logarithmic derivatives,

$$\left(D_{\mathrm{L}} - \frac{k}{G_0} \frac{\partial G_0}{\partial \rho} \bigg|_a \right) = 0,$$

might be valid only at a single value of energy, $E = E_0$. Any deviation from E_0 would introduce a non-zero value $(w_{\mathrm{r}})^2$ which would be enlarged by the big factor, θ^2; therefore, the width of resonance in E would be extremely narrow, such that there would be no chance for any resonance to happen in a real world.

To answer the first question, we should notice that the *width of the resonance energy level* is not necessary equal to the *width of the resonance peak of the cross-section*. Only in the case of single energy level far away from the zero energy and far away from the other energy levels, we might use the relation,

$$\mathrm{e}^{-i(E_0 - i\Gamma)t/\hbar} \propto \int_{-\infty}^{\infty} \frac{\Gamma/2}{(E - E_0) + i(\Gamma/2)} \mathrm{e}^{-iEt/\hbar} \mathrm{d}E;$$

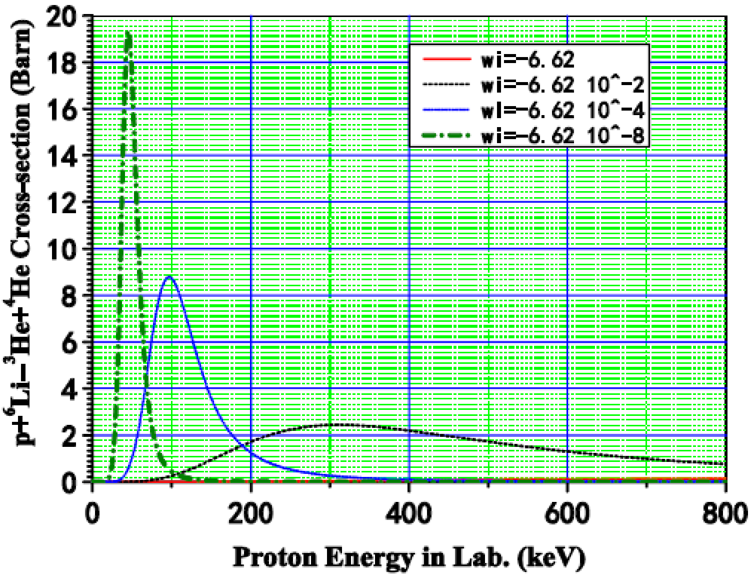

Figure 2. The low energy resonance peak in the cross-section would appear only if $(-w_\mathrm{i})$ is small enough.

$$\sigma(E) = \frac{4\pi}{k^2}\frac{(\Gamma/2)^2}{(E-E_0)^2+(\Gamma/2)^2}. \tag{12}$$

to derive that the width of the resonance peak in the cross-section would be equal to the width of the energy level, Γ while the energy dependence of Γ is ignored. For the case of tunneling Coulomb barrier near zero energy, only $(-w_\mathrm{i})$ is related to the ratio of the life-time to bouncing time inside the nuclear interaction region, but the width of the cross-section peak is proportional to the $\log[-w_\mathrm{i}]$, when the energy dependence of Γ is considered explicitly.

To answer the second question, we should notice that when the resonance energy approaches zero, the cross-section is no longer determined by $w_\mathrm{r}^2 \approx (C_1 + C_2E)^2$ term, instead, it is determined by $(w_\mathrm{i} - \frac{1}{\theta^2})^2$ in the denominator of Eq. (6), because

$$w_\mathrm{r}^2 \ll \left(w_\mathrm{i} - \frac{1}{\theta^2}\right)^2.$$

θ^2 affects only on the larger one among

$$w_\mathrm{r}^2 \quad \text{and} \quad \left(w_\mathrm{i} - \frac{1}{\theta^2}\right)^2$$

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Consequently, we do not worry about the absorption near this low energy resonance level, but we have to identify a weak interaction channel which makes use of this low energy resonance level.

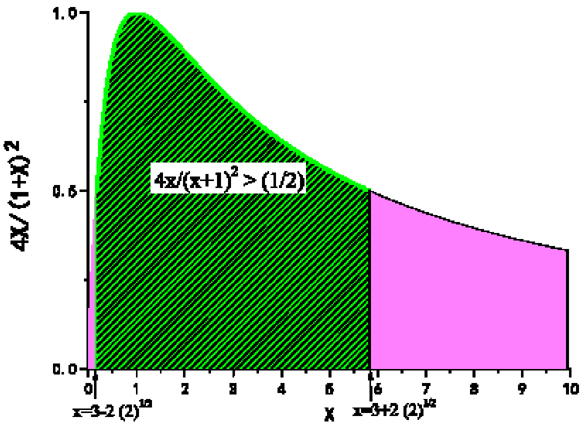

Figure 3. The width of the resonance peak in cross-section is $4(2)^{1/2}$.

4. A Possible Weak Interaction Channel accompanied with Low Energy Resonance for p+^{6}Li Fusion

In order to see the resonance peak in cross-section, it is necessary to identify a weak reaction channel which satisfies the second resonance condition: $-\theta^2 w_{\mathrm{i}} = 1$. Indeed there is an orbital electron capture (weak interaction) which may be accompanied with the resonant tunneling.

$$\begin{aligned} \mathrm{p} + {}^6\mathrm{Li} + e &\rightarrow {}^7\mathrm{Li}^* + \text{neutrino} \\ {}^7\mathrm{Li}^* &\rightarrow T + {}^4\mathrm{He} + (\sim 4\,\mathrm{MeV}). \end{aligned} \tag{13}$$

This reaction is very slow in comparison with the fast fusion reaction accompanied with the strong interaction:

$$\mathrm{p} + {}^6\mathrm{Li} \rightarrow {}^3\mathrm{He} + {}^4\mathrm{He} + 4.001\,\mathrm{MeV}. \tag{14}$$

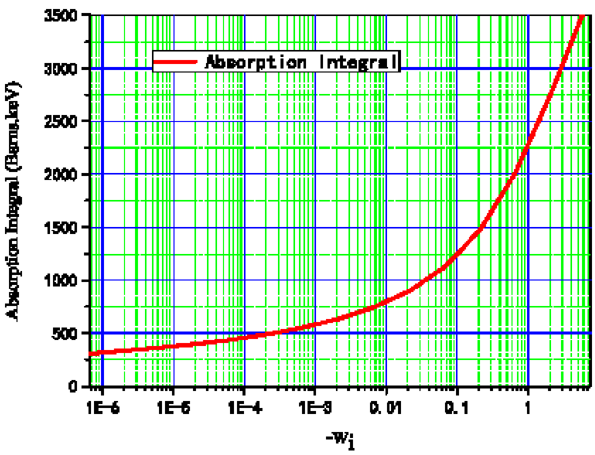

Figure 4. The absorption integral decreases with the $(-w_{\mathrm{i}})$ slowly in a logarithmic scale.

Based on the life-time of ^{7}Be due to electron capture, 53.22 days, it is possible to make a rough estimate of the $w_{\rm i} = -6.62 \times 10^{-29}$. Using Eqs. (7) and (10), we may show that the resonance peak of the cross-section would be close to 2 keV, and the width of this peak in cross-section would be 0.2 keV. As we pointed out in ICCF-17 [12], this excited state ^{7}Li* keeps most of the reaction energy, and this neutrino emission carry away only a small part of the reaction energy. Eventually the final fusion products, T + ^{4}He, which are all charged particles, would deposit the most of fusion energy in the crystal lattice and form the "excess heat" in CMNS experiments.

5. Conclusion Remarks

The hot fusion data for p+^{6}Li have shown the existence of a low energy resonance using the new 3-parameter formula for the fusion cross-section at low energy. It explains not only the importance of lithium additives in the series of CMNS experiments, but also consistent with the large cross-section of ^{6}Li for absorbing thermal neutron. Indeed, the solar energy is from a fusion reaction accompanied by a weak interaction as well [13]. Recently, in the hot fusion experiments, the hydrogen plasma pressure and confinement were greatly enhanced at the edge of the plasma when a little lithium was added into plasma [14]. This might be another evidence to show the existence of the p+^{6}Li low energy resonance.

Acknowledgments

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