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Research Article

Hydrogen-lithium Low Energy Resonant Electron-capture and Bethe's Solar Energy Model

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Abstract

Bethe's solar energy model is applied to the case of (p+^{6}Li) low energy resonance near 50 eV. The cross-section would be greatly enhanced to meet the experimental observations in Ni–H($LiAlH_4$) systems and in "Hydrogen–lithium fusion devices." The width of the resonance does not prevent low energy resonance from contributing to the cross-section.

Keywords: Bethe's solar energy model, Huizenga's three puzzles, $LiAlH_4$, (p +^{6}Li) Low energy resonance, New formula for fusion cross-section with three parameters, Resonant electron capture, Width of resonance peak

1. Introduction

A combination of strong interactions and weak interactions is the essence of Bethe's solar energy model, in which a nuclear potential of a strong interaction was used to calculate the elastic scattering of (p+p), and a nuclear transmutation of a weak interaction was used to calculate the probability of a transition from (p+p) continuum to (n+p) discrete energy level (deuteron) [1]. This model gave the correct fusion rate of p + p $\rightarrow$ D+ e^+ + neutrino, and made a good prediction of the solar temperature. In Bethe's model there was no low energy resonance between (p+p), and the positron emission was the dominant weak interaction process. The same model might be applied to (p+^{6}Li) low energy resonance if the effect of (p+^{6}Li) low energy resonance is fully included, and if the orbital electron capture of the beryllium excited state ($^7Be^*$) is introduced. The reaction rate is compared with the "excess power" in the Ni–H gas-loading experiments with $LiAlH_4$ as an additive [2–5], and is compared with Lipinski's measurement of the helium ion emission rate from low temperature (p+^{6}Li) plasma [6]. Both comparisons lead to a (p+^{6}Li) low energy resonance near 50 eV. This is consistent with (p+^{6}Li) low energy fusion cross-section as well [7].

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2. Bethe's Solar Energy Model and its Application to (p+^{6}Li) Low Energy Resonance

Bethe assumed that the formation of deuterons through (p+p) fusion reaction is an elastic scattering of (p+p) followed by a positron emission. The cross-section of this reaction may be written as:

$$\sigma(E) = \frac{gF(W_e)}{v}\left|\int \psi_f\psi_i \, d\tau\right|^2. \tag{1}$$

Here, g is the coupling constant of weak interaction, and $F(W_e)$ is a factor different for the case of K-capture or positron emission. $\left|\int \psi_f\psi_i \, d\tau\right|^2$ is the probability of transition from a mother state, ψ_i, to a daughter state, ψ_f. The overlapping of these two wave functions determines the amplitude of this transition. For (p+^{6}Li) low energy resonance, ψ_i is the wave function of (p+^{6}Li) relative motion. At large distance between p and ^{6}Li, ψ_i approaches to a plane wave, e^{ikz}, normalized to a unit density (where k is the wave number of relative motion in the z-direction in the center of mass system). Thus the current density of probability is ν, and ν is the speed of relative motion. At low energy, only the S-partial wave is important for the interaction; hence, in the Coulomb interaction region, the wave function may be written as:

$$\Psi_i = \frac{1}{kr}e^{i\delta}(\cos[\delta]F_0[r] + \sin[\delta]G_0[r]) \quad (r > r_0). \tag{2}$$

Here, r is the distance between p and ^{6}Li; δ is the phase shift of the S-partial wave due to the strong nuclear interaction. F_0 and G_0 are the regular and irregular Coulomb wave function for S-partial wave, respectively. r_0 is the radius of the nuclear interaction region. Inside the nuclear interaction region, the wave function is

$$\Psi_i = \frac{1}{kr}e^{i\delta}(\cos[\delta]F_0[r_0] + \sin[\delta]G_0[r_0])\frac{\sin[k_1 r]}{\sin[k_1 r_0]} \quad (r < r_0), \tag{3}$$

$$k = \sqrt{\frac{2\mu E}{\hbar^2}}, \quad k_1 = \sqrt{\frac{2\mu(E - U_{pp})}{\hbar^2}}.$$

k and k_1 are the wave numbers in the Coulomb field and inside the nuclear potential well U_{pp}, respectively. $\hbar$ is the Planck constant divided by 2π. ψ_f is the final state of the electron-capture process:

$$p + {}^6Li^* + e^- \to {}^7Li^* + \text{Neurino}, \tag{4}$$

^{7}Li* may be assumed as a confined state of a neutron in the potential well of ^{6}Li* (similar to neutron halo of Lithium we may call it neutron skin of ^{6}Li* in the case of the electron-capture in (p+^{6}Li) low energy resonance):

$$\psi_f = \frac{1}{\sqrt{2\pi b(1+\frac{r_0}{b})(1+\frac{\beta^2}{k_2^2})}}\frac{1}{r}e^{-\beta(r-r_0)}, \quad \beta = \sqrt{\frac{2\mu E_b}{\hbar^2}}, \quad b = \frac{1}{\beta} \quad (r > r_0), \tag{5}$$

$$\psi_f = \frac{1}{\sqrt{2\pi b(1+\frac{r_0}{b})(1+\frac{\beta^2}{k_2^2})}}\frac{1}{r}\frac{\sin[k_2 r]}{\sin[k_2 r_0]}, \quad k_2 = \sqrt{\frac{2\mu(-U - E_b)}{\hbar^2}} \quad (r < r_0). \tag{6}$$

Here, E_b is the binding energy of the neutron in the nuclear potential of excited ^{6}Li*, U is the depth of this nuclear potential. r_0 is the radius of this potential, b is the range of the extension of neutron wave function outside this nuclear interaction region. The coefficients in ψ_f normalize the final state in its confined region. ψ_f is a rapidly decrease function with r outside the nuclear interaction region. The overlapping between ψ_i and ψ_f is maximized when the shape of ψ_i is very similar to ψ_f, and the amplitude of ψ_i is greatly enhanced at the nuclear boundary. Hence, we are looking for the case where G_0 is dominant in the wave function in Eq. (2), because G_0 is a rapidly decrease function with r outside the nuclear interaction region and G_0 is greatly enhanced at the nuclear boundary as well (F_0 behaves in an opposite way). In order to make G_0 dominant in the wave function ψ_i, the coefficient of G_0 in Eq. (2), $\sin[\delta]$, should be enhanced. $\sin[\delta]$ is determined by the nuclear boundary condition at $r = r_0$, i.e.

$$\cot[\delta] = \left[-(\frac{G_0}{F_0})(\frac{D_\mathrm{L} - \frac{k}{G_0}\frac{\partial G_0}{\partial \rho}}{D_\mathrm{L} - \frac{k}{F_0}\frac{\partial F_0}{\partial \rho}}) \right]_{r=r_0}. \tag{7}$$

Here, D_L is the logarithmic derivative of the wave function at the boundary between nuclear interaction region and the Coulomb field. $\rho \equiv kr$. Usually, $\cot[\delta]$ is a very large number, because (G_0/F_0) is a very large number. At the very low energy,

$$\begin{aligned} &F_0 = C_0 \rho \Phi[y], \quad G_0 = C_0^{-1}\Theta[y], \\ &C_0 = \sqrt{\frac{2\pi\eta}{\mathrm{e}^{2\pi\eta} - 1}}, \quad \eta = \frac{1}{k a_\mathrm{c}}, \\ &a_\mathrm{c} = \frac{4\pi\varepsilon_0 \hbar^2}{z_a z_\mathrm{b} \mu e^2}, \end{aligned} \tag{8}$$

$$\Phi[y] \equiv \frac{1}{\sqrt{y}} I_1[2\sqrt{y}], \tag{9}$$

$$\Theta[y] \equiv 2\sqrt{y} K_1[2\sqrt{y}]. \tag{10}$$

Here, $y = 2r/a_\mathrm{c}$, $I_1[2\sqrt{y}]$ is the modified Bessel Function of first kind, and $K_1[2\sqrt{y}]$ is the modified Bessel Function of second kind. a_c is the length of Coulomb unit, ε_0 is the dielectric constant in vacuum, z_a and z_b are charge number of colliding particles, respectively; e is the charge of a proton. At low energy, $k \to 0$, η is very large; then, C_0 is a very small number. $\Phi[2r/a_\mathrm{c}]$ and $\Theta[2r/a_\mathrm{c}]$ are slow varying functions and equal to 1 when $r \to 0$. Therefore,

$$\left(\frac{G_0}{F_0}\right) = \frac{\Theta}{C_0^2 \rho \Phi} \to \left.\frac{\mathrm{e}^{2\pi\eta} - 1}{2\pi\eta\rho}\right|_{r-r_0} = \frac{\mathrm{e}^{2\pi\eta} - 1}{2\pi}\frac{a_\mathrm{c}}{r_0} \equiv \frac{a_\mathrm{c}}{r_0}\theta[E]^2. \tag{11}$$

The ratio (G_0/F_0) in Eq. (7) should be an exponentially increase number at low energy. Consequently,

$$\sin[\delta] = \sqrt{\frac{1}{1 + \cot[\delta]^2}} \to 0.$$

This makes F_0 dominant instead of G_0 dominant. However, there is a way to make G_0 dominant. Notice that at low energy three of logarithmic derivatives at the nuclear boundary condition in Eq. (7) approach to three constants [8].

$$\frac{k}{G_0}\frac{\partial G_0}{\partial \rho} \to -\sqrt{\frac{2}{a_\mathrm{c} r_0}}; \tag{12}$$

$$\frac{k}{F_0}\frac{\partial F_0}{\partial \rho} \to \sqrt{\frac{2}{a_\mathrm{c} r_0}}; \tag{13}$$

$$D_{\mathrm{L}} = k_1 \cot[k_1 r_0] \to \sqrt{\frac{2\mu(-U_{\mathrm{pp}})}{\hbar^2}} \cot\left[\sqrt{\frac{2\mu(-U_{\mathrm{pp}})}{\hbar^2}} r_0\right]. \tag{14}$$

For a certain specific nuclear potential, U_{pp}, it is possible to make the difference of two logarithmic derivatives in numerator of Eq. (7) as small as needed for a resonance to appear: e.g.

$$\frac{D_{\mathrm{L}} - \frac{k}{G_0}\frac{\partial G_0}{\partial \rho}}{D_{\mathrm{L}} - \frac{k}{F_0}\frac{\partial F_0}{\partial \rho}} \to \frac{r_0}{\theta[E_0]^2 a_{\mathrm{c}}} \ll 1. \tag{15}$$

Then, for

$$E > E_0, \quad |\cot[\delta]| \sim \frac{\theta[E]^2}{\theta[E_0]^2} \ll 1 \quad \text{and} \quad \sin[\delta] = \sqrt{\frac{1}{1+\cot[\delta]^2}} \to 1.$$

Thus G_0 is dominant. It should be noticed that $\theta^2(E)$ defined in Eq. (11) is different from $\frac{1}{C_0^2}$ in Eq. (8). When $E \to \infty, \theta^2(E) \to 0$, but $1/C_0^2 \to 1$. The question remaining is: Is there really a nuclear boundary which makes $|\cot[\delta]| \ll 1$?

3. Search for the Low Energy Resonance use (p+^{6}Li) Fusion Reaction

We can use the (p+^{6}Li) fusion reaction data [9] to investigate if there is a low energy resonance, because we have derived a tool to extract information about $\cot[\delta]$ from the fusion cross-section data. That is a new formula for light nuclei fusion cross-section, $\sigma_0[E]$, at low energy [10–15].

$$\sigma_0[E] = \frac{\pi}{k^2}\left(1 - \left|\mathrm{e}^{\mathrm{i}2\delta}\right|^2\right) \equiv \frac{\pi}{k^2}\frac{(-4W_{\mathrm{i}})}{W_{\mathrm{r}}^2 + (W_{\mathrm{i}} - 1)^2}. \tag{16}$$

Here, $W \equiv \cot[\delta] = W_{\mathrm{r}} + \mathrm{i}W_{\mathrm{i}}$, which determines the coefficient in front of $G_0[r]$ in Eq. (2). We may calculate W_{r} based on experimental data $\sigma_0[E]$ if W_{i} is assumed:

$$W_{\mathrm{r}}^2 = \frac{\pi}{k^2}\frac{(-4W_{\mathrm{i}})}{\sigma_0[E]} - (W_{\mathrm{i}} - 1)^2. \tag{17}$$

According to Eqs. (7) and (11), we may extract the fast varying part in W and define $(w_{\mathrm{r}} + \mathrm{i}w_{\mathrm{i}})$ as

$$W = \left[-\left(\frac{a_{\mathrm{c}}}{r_0}\theta[E]^2\right)\left(\frac{D_{\mathrm{L}} - \frac{k}{G_0}\frac{\partial G_0}{\partial \rho}}{D_{\mathrm{L}} - \frac{k}{F_0}\frac{\partial F_0}{\partial \rho}}\right)\right]_{r=r_0} \equiv \theta[E]^2(w_{\mathrm{r}} + \mathrm{i}w_{\mathrm{i}}). \tag{18}$$

At very low energy, $(w_{\mathrm{r}} + \mathrm{i}w_{\mathrm{i}})$ is a slowly varying function of energy; particularly, w_{i} is almost a constant because it is proportional to the ratio of flight-time to life-time $\tau_{\text{flight}}/\tau_{\text{life}}$ [12]. Here, τ_{flight} is the flight time of proton inside the nuclear interaction region; τ_{life} is the life-time of the composite nuclear state, (p+^{6}Li). Use Eq. (18), we have w_{r}^2 as a function of energy E:

$$w_{\mathrm{r}}^2 = \frac{\pi}{k^2}\frac{(-4w_{\mathrm{i}})}{\theta[E]^2\sigma_0[E]} - \left(w_{\mathrm{i}} - \frac{1}{\theta[E]^2}\right)^2 \tag{19}$$

There are three features in Eq. (19): (1) w_{r}^2 is a positive number by definition; (2) w_{i} must be a negative number to keep $w_{\mathrm{r}}^2 > 0$; (3) we may adjust w_{i} to keep the minimum of w_{r}^2 equal to zero. This zero point of w_{r}^2 is just the

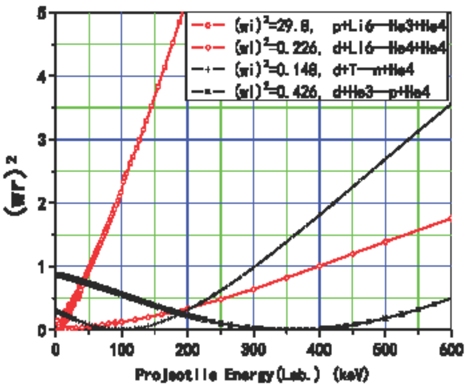

Figure 1. The minimum of w_r^2 approaches zero for four pairs of fusion reactions when w_i is assigned a specific number (data points are from ENDF/B-VII.1(2011)[9]).

resonance point, E_0. Figure 1 shows a slow varying w_r^2 when w_i is assumed as a constant for four pairs of nuclear reactions.

Even though the data near the low energy limit might be not very accurate, it is enough to judge qualitatively whether there is a resonance; i.e. whether $w_\mathrm{r}^2 = 0$ or $w_\mathrm{r}^2 \ll 1$ is below or above the lower limit of the experimental data for energy. Figure 1 demonstrates that there are resonances, $w_\mathrm{r}^2 = 0$, at $E_0 \sim 95$ keV and $E_0 \sim 375$ keV for d+T and d+He3 (+ and x signs), respectively. These are the famous resonances in hot fusion reactions. On the other hand, Fig. 1 reveals also that there are possible resonances near zero energy for (p+^{6}Li) and (d+^{6}Li) fusion reactions as well (open squares and circles). These are very important to understand the "anomalous heat effect" in early heavy water electrolysis experiments with lithium ions in electrolyte [16–24], and "anomalous heat effect" in Ni–H gas-loading systems with $LiAlH_4$ as an additive [2–5]. Recently, the publication of Lipinski patent [6], entitled as "Hydrogen-lithium fusion device, " strongly suggested that lithium is just the fuel per se.

4. Average over Maxwell Velocity Distribution

In order to compare with the experimental data of anomalous heat effect (e.g. [2–6]), we have to average the cross-section over Maxwell velocity distribution of temperature *T*. Use Eq. (1), we have the average reaction rate

$$
\begin{aligned}
\langle \sigma[E]\, v\rangle &= \left(\frac{\mu}{2\pi T}\right)^{3/2} \int_0^\infty gF(W) \left| \int \Psi_\mathrm{f}\Psi_\mathrm{i}\, \mathrm{d}\tau \right|^2 \exp\left[-\frac{\mu v^2}{2T}\right] 4\pi v^2 \mathrm{d}v \\
&= \left(\frac{\mu}{2\pi T}\right)^{3/2} gF[W] \frac{(4\pi b^2)^2}{2\pi b} \Lambda^2 \int_0^\infty \cos[\delta]^2 C_0^2 \exp\left[-\frac{\mu v^2}{2T}\right] 4\pi v^2\, \mathrm{d}v \\
&= \frac{32\pi^2\hbar}{\sqrt{\mu}} \left(\frac{1}{2\pi T}\right)^{3/2} \frac{gF[W]}{a_\mathrm{c}} b^3 (w_\mathrm{r})^2 \Lambda^2 \int_0^\infty \frac{\theta[E]^2}{1+(w_\mathrm{r}\theta[E]^2)^2} \exp\left[-\frac{E}{T}\right] \mathrm{d}E,
\end{aligned}
\qquad (20)
$$

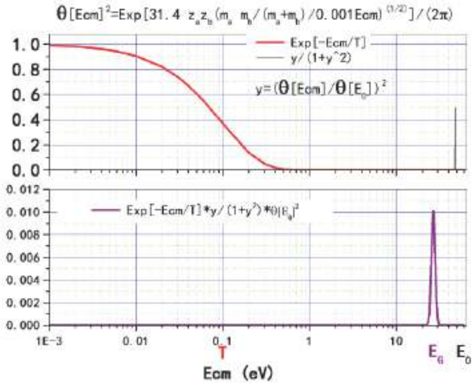

Figure 2. A Gamow peak is formed at E_{G} between T and E_0.

$$\Lambda = \frac{\Lambda_1 + \Lambda_2 + \Lambda_3}{\sqrt{\left(1+\frac{r_0}{b}\right)\left(1+\frac{\beta^2}{k_2^2}\right)}}, \tag{21}$$

$$\Lambda_1 = \frac{\frac{1}{2}x_0^2\left(\Phi\left[\frac{2r_0}{a_{\mathrm{c}}}\right] + \frac{a_{\mathrm{c}}}{r_0 w_{\mathrm{r}}}\Theta\left[\frac{2r_0}{a_{\mathrm{c}}}\right]\right)}{\sin\left[\frac{k_1}{\beta}x_0\right]\sin\left[\frac{k_2}{\beta}x_0\right]}\int_0^{x_0}\sin\left[\frac{k_1}{\beta}x\right]\sin\left[\frac{k_2}{\beta}x\right]\mathrm{d}x,$$

$$\Lambda_2 = \int_{x_0}^{\infty} x\,\exp\left[-(x-x_0)\right]\Phi\left[\frac{2bx}{a_{\mathrm{c}}}\right]\mathrm{d}x, \tag{22}$$

$$\Lambda_3 = \frac{a_{\mathrm{c}}}{r_0 w_{\mathrm{r}}}x_0\int_{x_0}^{\infty}\exp\left[-(x-x_0)\right]\Theta\left[\frac{2bx}{a_{\mathrm{c}}}\right]\mathrm{d}x.$$

In the case of low energy resonance, w_{r} is very small (see Eqs. (15) and (18)); therefore, only the terms with $1/w_{\mathrm{r}}$ in Λ_1 and Λ_3 need to be kept in Λ^2. Thus

$$(w_{\mathrm{r}})^2\Lambda^2 \approx \frac{1}{\left(1+\frac{r_0}{b}\right)\left(1+\frac{\beta^2}{k_2^2}\right)}\left\{\frac{\frac{1}{2}x_0^2\left(\frac{a_{\mathrm{c}}}{r_0}\Theta\left[\frac{2r_0}{a_{\mathrm{c}}}\right]\right)}{\sin\left[\frac{k_1}{\beta}x_0\right]\sin\left[\frac{k_2}{\beta}x_0\right]}\left(\frac{\sin\left[\left(\frac{k_1}{\beta}-\frac{k_2}{\beta}\right)x_0\right]}{\frac{k_1}{\beta}-\frac{k_2}{\beta}} - \frac{\sin\left[\left(\frac{k_1}{\beta}+\frac{k_2}{\beta}\right)x_0\right]}{\frac{k_1}{\beta}+\frac{k_2}{\beta}}\right) + \frac{a_{\mathrm{c}}}{r_0}x_0\int_{x_0}^{\infty}\exp\left[-(x-x_0)\right]\Theta\left[\frac{2bx}{a_{\mathrm{c}}}\right]\mathrm{d}x\right\}^2. \tag{23}$$

The integrand of integral

$$\int_0^{\infty}\frac{\theta[E]^2}{1+(w_{\mathrm{r}}\theta[E]^2)^2}\exp\left[-\frac{E}{T}\right]\mathrm{d}E$$

has two factors competing with each other: $\exp[-\frac{E}{T}]$ decreases rapidly with E, when E is greater than T. On the other hand,

$$\frac{\theta[E]^2}{1+(w_\mathrm{r}\theta[E]^2)^2}$$

increases rapidly with E, before its peak at E_0 (if we assume $w_\mathrm{r} = \frac{1}{\theta[E_0]^2}$). Because the "excess heat" in experiments is always increase with temperature, E_0 should be greater than T. As a result, Gamow peak is formed between T and E_0. In Fig. 2 upper plot, the thick line (red) shows that the $\exp[-\frac{E}{T}]$ decreases rapidly with E when $E > T$, and the thin line (black) shows a sharp peak of function,

$$\frac{y}{1+y^2}, \quad \text{where } y \equiv \theta[E]^2 w_\mathrm{r}.$$

The integrand

$$\frac{\theta[E]^2}{1+(w_\mathrm{r}\theta[E]^2)^2}\exp\left[-\frac{E}{T}\right] = \frac{1}{w_\mathrm{r}}\frac{y^2}{1+y^2}\exp\left[-\frac{E}{T}\right]$$

would have a new peak at E_G shown in Fig. 2 lower plot. Therefore, this integral may be estimated by the steepest descent method.

$$\begin{aligned}\langle\sigma v\rangle =& \frac{32\pi^2\hbar}{\sqrt{\mu}}\left(\frac{1}{2\pi T}\right)^{3/2}\frac{gF[W]}{a_\mathrm{c}}b^3\frac{1}{(1+\frac{r_0}{b})\left(1+\frac{\beta^2}{k_2^2}\right)}\left\{\frac{\frac{1}{2}x_0^2(\frac{a_\mathrm{c}}{r_0}\Theta\left[\frac{2r_0}{a_\mathrm{c}}\right])}{\sin\left[\frac{k_1}{\beta}x_0\right]\sin\left[\frac{k_2}{\beta}x_0\right]}\right.\\ &\times\left(\frac{\sin\left[\left(\frac{k_1}{\beta}-\frac{k_2}{\beta}\right)x_0\right]}{\frac{k_1}{\beta}-\frac{k_2}{\beta}}-\frac{\sin\left[\left(\frac{k_1}{\beta}+\frac{k_2}{\beta}\right)x_0\right]}{\frac{k_1}{\beta}+\frac{k_2}{\beta}}\right)\\ &\left.+\frac{a_\mathrm{c}}{r_0}x_0\int_{x_0}^{\infty}\exp\left[-(x-x_0)\right]\Theta\left[\frac{2bx}{a_\mathrm{c}}\right]\mathrm{d}x\right\}^2\frac{1}{w_\mathrm{r}^2\theta[E_\mathrm{G}]^2}\exp\left[-\frac{E_\mathrm{G}}{T}\right]\sqrt{\frac{4E_\mathrm{G}^2\sqrt{2\mu E_\mathrm{G}a_\mathrm{c}}}{3\hbar}}. \end{aligned} \tag{24}$$

Here,

$$E_\mathrm{G} = \left(\frac{hT}{2\sqrt{2\mu}a_\mathrm{c}}\right)^{2/3}$$

is the Gamow peak energy and $h = 2\pi\hbar$. We notice that the position of Gamow peak is determined by temperature T only, and the height of the Gamow peak is greatly enhanced by $\frac{1}{w_\mathrm{r}} = \theta[E_0]^2$.

For the case of $\mathrm{p} + {}^6\mathrm{Li}^* + \mathrm{e}^- \rightarrow \mathrm{T} + {}^4\mathrm{He} + \text{neutrino} + 4\,\mathrm{MeV}$, $m_\mathrm{a} = 1$, $m_\mathrm{b} = 6$, $z_\mathrm{a} = 1$, $z_\mathrm{b} = 3$, $gF(W) \approx 0.9 \times 10^{-4}\mathrm{s}^{-1}$. The radius of nuclear potential is assumed same for both $(\mathrm{p} + {}^6\mathrm{Li})$ and for $(\mathrm{n} + {}^6\mathrm{Li}^*)$; then, $r_0 = 1.746\times10^{-15}(m_\mathrm{a}^{1/3}+m_\mathrm{b}^{1/3}) = 4.9\times10^{-15}$ m. The depth of the nuclear potential between $(\mathrm{p}+{}^6\mathrm{Li})$, $-U_\mathrm{pp}$, is assumed to keep the resonance condition: Eqs. (14) and (15). Thus, for $(\mathrm{p} + {}^6\mathrm{Li})k_1 = 4.08 \times 10^{14}\mathrm{m}^{-1}$. The depth of the nuclear potential between $(\mathrm{n}+{}^6\mathrm{Li}^*)$, $-U$, is assumed to have the binding energy $E_\mathrm{b} = 782$ keV which is the necessary condition for turning a free proton into a bound neutron. Thus, $k_2 = 4.04 \times 10^{14}\mathrm{m}^{-1}$. Then, $\beta = 1.80 \times 10^{14}\mathrm{m}^{-1}$, $b = 5.56 \times 10^{-15}$ m, and $x_0 = 0.884$.

For H. Zhang's Ni -H ($\mathrm{LiAlH_4}$) system [5], $T = 1000°\mathrm{C} = 1273 \times 1.38 \times 10^{-23}$ J,

$$E_{\mathrm{G}}=\left(\frac{hT}{2\sqrt{2\mu}a_{\mathrm{c}}}\right)^{2/3}=4.55\times10^{-18}\mathrm{J}=28.4\,\mathrm{eV} \tag{25}$$

$$w_{\mathrm{r}}^{2}=\frac{1.61\times10^{-20}}{\langle\sigma v\rangle}\left(\frac{1}{T}\right)^{2/3}\exp\left[-\frac{8416}{T^{1/3}}\right]\quad(T\text{ is in units of Kelvin}). \tag{26}$$

When

$$w_{\mathrm{r}}\equiv\frac{1}{\theta[E_0]^2}$$

is assumed, $E_0=48$ eV. This value is very close to the lower limit of the proton beam energy, 50 eV, in Lipinski's patent [6]. After seven years of effort with 25 runs of experiments, Lipinski claimed that he was able to detect the helium when a low energy proton beam was injected into the lithium vapor. The lower limit of proton beam energy was 50 eV.

5. The Width of the Low Energy Resonance

The rapid decrease factor, $\exp[-\frac{E}{T}]$ and the rapid increase factor,

$$\frac{\theta[E]^2}{1+(w_{\mathrm{r}}\theta[E]^2)^2},$$

solved the problem of the width of the low energy resonance. The down-hill slope meets the up-hill slope at $E=E_{\mathrm{G}}$, where the first derivative of the product of

$$\exp\left[-\frac{E}{T}\right]\times\left(\frac{\theta[E]^2}{1+(w_{\mathrm{r}}\theta[E]^2)^2}\right)$$

reaches zero. The width of this Gamow peak is determined by its second derivative, i.e.

$$\Delta E=2\sqrt{\frac{2\log[2]}{f''[E_{\mathrm{G}}]}},\qquad f=-\frac{E}{1.38\times10^{-23}T}-31.4z_{\mathrm{a}}z_{\mathrm{b}}\sqrt{\frac{m_{\mathrm{a}}m_{\mathrm{b}}}{0.625\times10^{16}(m_{\mathrm{a}}+m_{\mathrm{b}})E}}. \tag{27}$$

Here, E is in units of Joules, and T is in units of Kelvin. $\Delta E=3.4$ eV is a function of T only. Therefore it is independent of the resonance energy E_0. Thus we need not worry about the width of Gamow peak while resonance energy E_0 approaches zero as long as $E_0\gg E_{\mathrm{G}}$.

Even if the width of resonance peak itself is not very small as we might see from Fig. 2 upper plot, because the resonance peak of

$$\frac{y}{1+y^2}\text{ is at }y=1,$$

and the width of this resonance peak is determined by the equation $y=2\pm\sqrt{3}$. Therefore the width of resonance peak approaches zero like

$$\left(\frac{1}{\log[E_0]}\right)^2$$

as we discussed in ICCF-17 [25].

There was a mistake that the width of resonance peak would approach zero like $\frac{1}{\theta[E]^2}$, because it was assumed that the position of resonance peak is determined by $W_\mathrm{r}^2 = 0$ in Eq. (16), and the width of resonance peak is assumed to be determined by the equation of $W_\mathrm{r}^2 \approx 1$. In most of the literature, $W_\mathrm{r}^2 = \theta[E]^2 w_\mathrm{r}$ is used to be expanded as

$$W_\mathrm{r} = \theta[E_0]^2 \left(w_\mathrm{r}[E_0] + \left.\frac{\partial w_\mathrm{r}}{\partial E}\right|_{E=E_0} (E - E_0) \right) = \theta[E_0]^2 \left.\frac{\partial w_\mathrm{r}}{\partial E}\right|_{E=E_0} (E - E_0); \tag{28}$$

therefore, the width of resonance peak would be:

$$\Delta E = \frac{2}{\theta[E_0]^2 \left.\frac{\partial w_\mathrm{r}}{\partial E}\right|_{E=E_0}} \propto O\left[\frac{1}{\theta[E_0]^2}\right].$$

It should be noted that $w_\mathrm{r}[E_0] = 0$ is assumed in Eq. (16) instead of

$$w_\mathrm{r}[E_0] = \frac{1}{\theta[E_0]^2}.$$

Numerically, there is no big difference between these two different assumptions; however, they are very different at very low energy where $w_\mathrm{r}[E_0]$ approaches a constant according to Eqs. (12)–(14). At very low energy, $\theta[E]^2$ varies much faster than $w_\mathrm{r}$does; therefore, the assumption in Eq. (28) is no longer valid. The width of resonance should be determined by the fast variation of $\theta[E]^2$ instead of the very slow variation of w_r.

The width of resonance blocked the resonance approach in the early days of condensed matter nuclear science. Now we understand that at very low energy this block was just a mistake. The behavior of $\theta[E]^2$at 100 keV (~10^9 K) is not supposed to be the same as the behavior of $\theta[E]^2$near 0.1 eV (~1000 K). The resonance contribution to the reaction rate does not diminish when the resonance approaches zero energy, because the resonance peak height is increase much faster than the resonance width is shrinking.

6. The Energy Carried away by Neutrino

If during the resonant electron-capture-process, the daughter state, Ψ_f, is the ground state of ^{7}Li instead of the excited state ^{7}Li*; then the neutrino might have carried away most of the reaction energy in the process of electron-capture: $\mathrm{p} + {}^6\mathrm{Li} + \mathrm{e}^- \rightarrow {}^7\mathrm{Li} + \text{neutrino} + 4.001\ \mathrm{MeV}$.

Consequently, the "anomalous heat effect" would have not been detected. However, during the resonant electron-capture the mother state, Ψ_i, is dominant by G_0 because it is formed in a low energy resonant process. The transition probability,

$$\left| \int \Psi_\mathrm{f} \Psi_\mathrm{i}\, \mathrm{d}\tau \right|^2,$$

requires that the daughter state Ψ_f should be very similar to the mother state, Ψ_i: i.e. the wave function, Ψ_f, should be extended to outside the nuclear core, and decrease quickly. Hence, Ψ_f should be a state very similar to neutron halo instead of the ground state of ^{7}Li. Fortunately, lithium is a famous halo nuclide. ^{9}Li and ^{11}Li may have a halo state [26] with one or two neutrons away from the nuclear core. Thus, the excited ^{7}Li*may just have a neutron skin as a daughter state, Ψ_f, of the resonant electron-capture process. As a result, the neutrino carries away just a little reaction energy and ^{7}Li* keeps most of the reaction energy in its excited state. When ^{7}Li* decays into ^{3}T + ^{4}He, both of the charged products carry away the reaction energy and transfer the kinetic energy to the surrounding electrons while they are slowing down. The energy carried by ^{7}Li* is then detected as the anomalous heat effect.

7. The Analytical Property of the Elastic Scattering Amplitude Determined by Inelastic Scattering

In the discussion above, we used the inelastic fusion process to determine the low energy resonance of elastic scattering. This is based on the analytical property of scattering amplitude on the complex energy E-plane. According to Landau [27], the scattering amplitude might be extended to the complex energy plane as an analytical function. Its property is determined by its poles and the residues of these poles. Indeed, the positions of these poles determine the resonance energy, and the residues of these poles determine the life-time of these resonance states. The elastic scattering and inelastic scattering share the same scattering amplitude at resonance energy. Therefore, we may use the inelastic fusion process to determine the pole of elastic scattering.

8. Concluding Remarks

The 3-parameter equation for low energy fusion cross-section, Eq. (16), reveals a way to overcome the Coulomb barrier suppression. Provided that there is a low energy level to keep $W_\mathrm{r} = \theta[E]^2 w_\mathrm{r} \leq 1$, and a tiny interaction to keep $|W_\mathrm{i}| = \left|\theta[E]^2 w_\mathrm{i}\right| \leq 1$; then the Gamow factor would disappear in the formula of fusion cross-section. This has led us to an elastic process followed by a weak interaction. The formula for a low energy elastic scattering cross-section is

$$\sigma_{\text{elastic}} = \frac{\pi}{k^2}\frac{4}{W_\mathrm{r}^2 + 1}. \tag{29}$$

Provided that there is a low energy level to keep $W_\mathrm{r} = \theta[E]^2 w_\mathrm{r} \leq 1$, σ_{elastic} would be free of Coulomb barrier suppression, because there would be no Gamow factor in the expression of σ_{elastic}. It is not necessary to require $W_\mathrm{r} = \theta[E]^2 w_\mathrm{r} = 0$, in order to overcome the Coulomb barrier, although $W_\mathrm{r} = 0$ does make the cross-section maximized.

It is also interesting to note that the accompanied weak interaction—electron capture introduces an additional favorable factor $\theta[E]^2$ from G_0; then,

$$\frac{\theta[E]^2}{1 + \left(w_\mathrm{r}\theta[E]^2\right)^2}$$

would have a resonance peak at E_0 where $W_\mathrm{r} = \theta[E_0]^2 w_\mathrm{r} = 1$, and the peak height would be enhanced a factor of $1/w_\mathrm{r}$, which is a very large number in the case of resonance.

Although this new resonance peak would appear at the phase shift $\delta_0 = \frac{\pi}{4}$: i.e. $\cot[\delta_0] = W_\mathrm{r} = \theta[E]^2 w_\mathrm{r} = 1$, the Gamow peak still appears at some phase shift $\delta \ll \delta_0$ after averaging over a Maxwell distribution of velocity, because usually the temperature T is still too low to compare with the resonance energy , E_0. However, the enhancement factor $1/w_\mathrm{r}$ is enhancing the integrand in Eq. (20) over the whole energy region (including the Gamow peak E_G), even if $\delta \ll \delta_0$.

The reaction rate after averaging is heavily dependent on the temperature, T:

$$\langle\sigma v\rangle = \frac{1.61 \times 10^{-20}}{w_\mathrm{r}^2}\left(\frac{1}{T}\right)^{2/3} \exp\left[-\frac{8416}{T^{1/3}}\right] \tag{30}$$

(T in Kelvin, $\langle\sigma v\rangle$ in m^3/s). It quickly increases with temperature. This would have a positive feedback effect as an exothermic process in experiments. The micro-melting spots in a palladium cathode with heavy water electrolysis [28] are exactly the kind of evidence we would expect for this positive feedback effect.

In conclusion, the combination of a selective resonant tunneling model and the weak interaction has answered Huizenga's three puzzles, and the anomalous heat effect may be achieved without commensurate neutron and gamma radiation.

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