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Research Article

Nature of the Deep-Dirac Levels

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Abstract

Maly and Va'vra (M&V) in 1993 and 1995 presented a computational evaluation of the Dirac equations that included the 'anomalous' solution. The regular solutions of these equations are the basis for modern quantum mechanical predictions for comparison with the experimental values of atomic-electron orbital energies. The other solution, discussed in the literature for over 55 years, is relativistic and considered anomalous because its predicted levels are very deep (up to 511 keV) and have never been observed. Nevertheless, the existence of these deep levels provides a ready explanation of the mechanism for penetration of the Coulomb barrier and the means of D–D fusion below the ^{4}He fragmentation levels. Since these levels also provide the basis for all of the other cold fusion observations (both PdD and NiH systems), it is important that arguments for and against the Dirac model be examined. The theoretical support for this anomalous solution is provided in a companion paper in this conference. This presentation seeks: to update the deep-orbit information provided in a poster at ICCF-17, to describe the nature of these deep–Dirac levels (DDLs), to report on additional, but unpublished, results presented by Va'vra in 1998, and to correct some interpretations of the model that Va'vra has provided in 2013. There are some unusual properties of the DDLs relative to those of the known atomic orbitals. Interpretation of the DDL properties, based on the non-relativistic solutions, leads to misunderstandings and further rejection of the concept of the deep levels. We hope to clarify this situation and indicate the importance of the calculations for cold fusion models. Cold fusion results provide a basis for understanding the DDLs and the proposed new fields of femto-physics and femto-chemistry.
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1. Atomic Orbitals VS. the Relativistic Schrodinger Electron Deep Levels (EDLS) and the Deep Dirac Levels (DDLS)

Atomic-electron orbitals are very well known and understood. Until Maly and Va'vra (M&V, [1,2]) did so in the early 1990s, the multiplicity of relativistic electron deep levels (EDLs) had not been explored in the literature. The basic

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premise of *any* deep levels has been improperly challenged for over 45 years (see companion paper [3]); therefore no one has 'bothered' to look beyond the 'ground-state' of these predicted orbitals.

With the advent of powerful computers, the task of solving the relevant equations became much easier and M&V did so in the context of cold fusion (CF) [1,2]. In another context, their results might have been acceptable. Even within the CF community, the results and implications were misinterpreted and ignored for over 15 years. Their works are still not accepted, with few exceptions, despite the obvious answers they provide for the theory of cold fusion [4,5].

What are the properties of the DDLs and why are they important to CF? First, why do we refer to the electron deep levels as Dirac Deep levels when they are also predicted by the Klein–Gordon (K–G) equation that is equivalent to the relativistic Schrodinger equation? In many of the early arguments against the K–G prediction of deep-orbits, the fact that it was valid for spinless particles was interpreted to mean that it was invalid for spin $^1/_2$ particles. These arguments ignored the fact that the Schrodinger equation (also valid for spinless particles) had been used for decades to describe electron orbitals. M&V buried this argument by showing that the relativistic Schrodinger equation also predicted the electron deep levels (EDLs). Furthermore, the effect of spin on the hydrogen atom calculation for the deep levels had an effect of only 2 keV out of over 500 keV. Thus:

Relativity introduces new electron binding energies at ~ 500 keV.

Spin affects these deep levels by only ~ 2 keV.

Second, a proton with a DDL electron is a neutral femto-atom. In matter, it acts like a 'fat' neutron.

2. What are Some Similarities between Atomic Orbitals and EDLs?

(1) Both sets of orbits are solutions of the same equations – they have identical starting assumptions and potentials.
(2) The solutions are a sequence that,
   (a) in the atomic case, depends on the electron orbital radius, energy, and angular momentum, along with the momentum, mass, and velocity (i.e., the deBroglie wavelength, $\lambda_c = h/mv$)
   (b) in the EDL cases, they also depend in a major way on relativity (in some form ?)
   (c) thus, while relativity could be involved in both atomic and DDL orbits, it is just not as noticeable in the low-energy atomic orbits.
(3) The sequences are related to the cyclical nature of something that relates to a wave, which periodically returns to identical conditions.
   (a) This depends on the conservation laws of energy and momentum (linear and angular).
   (b) Mathematically, integration about a closed path gives a zero result. Thus, no change occurs and a stable point is established. (See Appendix A.)
   (c) Closure depends on conditions being identical in all 'dimensions'. Orientation of a spin axis is one of those dimensions.
   (d) In an orbit, if the angular momentum vector remained 'fixed', it could not be a source of the periodicity.
      i. However, if the vector were to precess, then periodic motion, beyond that of the orbit, would be introduced.
      ii. It is assumed that the deBroglie wavelength, fundamental to atomic orbitals, is also based on some form of closed path. Is it based on vector precession? Precession of a spin vector would be a logical assumption for such paths, even for a linear trajectory.
      iii. The relativistic-Schrodinger equation predicts the discrete deep levels without explicitly resorting to spin. Nevertheless, the assumption of quantum mechanics is an inherent wave motion that, along with the deBroglie wavelength, could be related to a particle's spin.

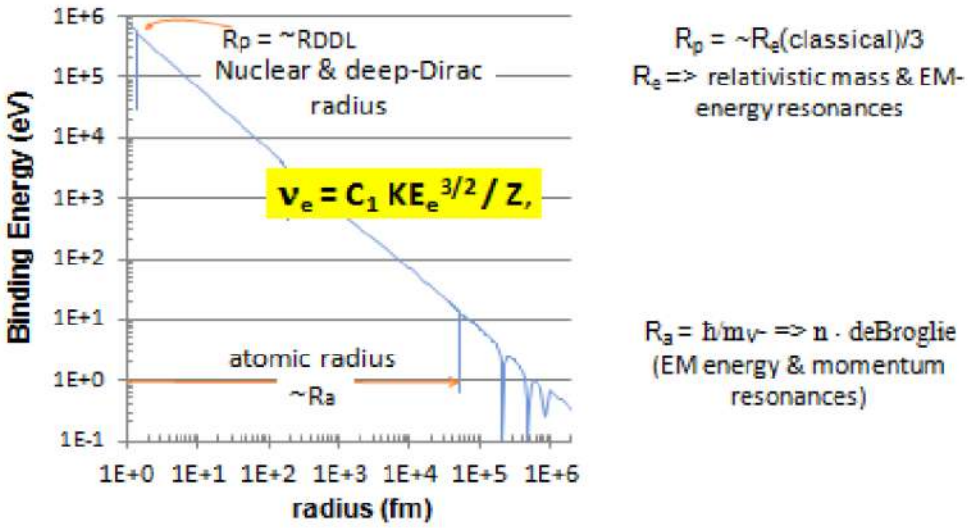

Figure 1. Binding energy vs. average electron-orbital radius. Both atomic-electron and deep-level electron orbital radii and energies are displayed as 'dips' in the Coulomb potential.

iv. Does the dependence on the deBroglie wavelength for atomic orbitals provide an equivalent to precession of a spin vector? (see Appendix A.) If so, can this concept be extended further to provide the relativity-induced deep orbits?

(e) A consistent model results if precession of different angular momentum vectors (e.g., spin and rotational) is considered for electrons in linear and orbital motion. We assume that relativity-induced forces (torques on a body with angular momentum) are the source of such precession.

i. Relativity and deBroglie waves (based on a spin vector?)
ii. Relativity and angular velocity vectors.

(4) Relativistic effects, strong enough to significantly increase effective electron mass, would be the cause of the deep orbits in both the Schrodinger and Dirac models.

3. What are Some Differences between Atomic Orbitals and EDLs?

Comparisons are based mainly on the Maly and Va'vra results from the tables in Appendix B.

(1) The obvious differences are expressed in Fig. 1 as the binding energy and average orbital radii. The frequencies (dependent on kinetic energy) have a difference of nearly eight decades for the difference in binding energy of five decades.

(2) The EDL orbits are so close to the nucleus (see Appendix C) that the $1/r$ Coulomb potential, valid at the atomic-orbit levels (see Appendix D), is no longer a valid assumption. The KE = $|\mathrm{PE}|/2$ relation predicted by the virial theorem for a $1/r$ potential no longer holds here. A deep-orbit electron is very energetic and the relativistic virial theorem gives a different relationship between the kinetic and potential energies. Furthermore, because of proximity, the nucleus is so large relative to the electron's deep-orbit radius that only near-circular electron orbits stay in the $1/r$ potential region.

(3) For near-circular orbits (with a high angular momentum for its orbit), an electron only sees the $1/r$ potential, not the repulsive centrifugal core. This requirement is similar to that of the super-large Rydberg atomic orbitals that also avoid perturbations of the central regions by having high angular momenta and circular orbits. In the deep-electron orbitals, the central region must be avoided [6]. This is not because of the cloud of bound electrons in the center of a Rydberg orbit but because of the large centrifugal barrier and the non-$1/r$ potential inside the nucleus (e.g., $r_n =\sim 1$ fm), which is large relative to the electron deep orbit ($r_{\mathrm{EDL}} =\sim 2$ fm).

(4) The angular momentum of atomic-electron orbitals is sufficient to produce and receive photons as the energy-exchange medium. This is not the case for the EDLs where the angular momentum is on the order of $h/100$.[a] This is where Va'vra's EDL story goes wrong [7]. It is not his solutions of the Schrodinger or Dirac equations that are in error; it is in their misinterpretation (based on Va'vra's false assumption of the Heisenberg Uncertainty Principle being applicable to this region, see the paragraph beneath his Fig. 3) and then its extrapolation (his Figs. 5 and 6).

(5) The relativistic models predict many new and different energy levels:
   (a) Negative energy levels (positrons).
   (b) Electron deep levels that are not matched by positron deep levels.
   (c) Probably a symmetry-breaking effect.

(6) The relativistic mass increase, as a DDL electron gets closer to the nucleus and as its velocity approaches that of the speed of light, means that the stable orbits for increasing angular momentum must be closer to the nucleus than those of the low-l electrons (unlike Va'vra's predictions in his Figs. 5 and 6 of [7]), but more circular. This allows the electrons to move deeper into the Coulomb potential well and still avoid the centrifugal barrier in the center.

(7) Relativistic effects break the degeneracy in many of the atomic-electron levels. They are fundamental to all of the EDLs.

(8) The kinetic energy and mass (and our suggested angular-momentum increase in orbits nearer to the nucleus) are consistent with decreasing radii of more-nearly circular DDL orbits. We note that the low-angular momentum orbitals of high-n deep levels cannot exist since such highly elliptic paths would penetrate into the nucleus and encounter the altered Coulomb potential.
   (a) The selection rules differ for the EDLs and DDLs and the atomic-electron orbitals.
   (b) The atomic-electron levels exist for all positive integer *n* values.
   (c) The relativistic levels exist only for alternating integer *n* values.
      i. The relativistic Schrodinger levels exist only for odd integers.
      ii. The relativistic Dirac levels exist only for even integers.
   (d) The alternating levels is a relativistic effect.
   (e) The odd/even selection rules result from the spin effect.
   (f) The *multiple* levels predicted for the deep orbits are similar to those for the atomic orbits in some respects, not in others.
      i. They exist. There is not just a single deep level as predicted in some developments.
      ii. There is an integer change between energy levels in the solutions of the wave equations for both deep and atomic orbitals.

[a] In the solutions for the relativistic equations, the angular momentum quantum number of the electron is l. This value must work in both sets of solutions (the atomic and deep orbits), so it would appear that l should have the same value in both. This is not the case because the term that includes l has a multiplicative 'constant', which has different values for the two solutions. Consequently, when the binding energy changes, the angular momentum value does also, even though the meaning of the quantum number l does not.

- The difference between atomic levels is associated with h (the deBroglie wavelength).
- The difference in deep levels cannot be directly associated with h. (The angular momentum of any state below the atomic ground state is less than h.)

iii. Nevertheless, since the relativistic equations all include h and correctly predict the atomic-orbital angular momenta, there must be a relationship between the discrete deep levels and the discrete atomic-electron levels.

iv. **There must be an additional source of periodicity.**

v. We believe that the binding energy increases with angular momentum for the deep orbits (as indicated in Appendix B, but not in Fig. 6 of [7]–see Fig. C-1). This binding energy decreases with higher angular momentum in atomic orbits, but not in the deep orbits).

vi. The source of periodicity in the deep levels must be related to something beyond the deBroglie wavelength that dominates at atomic levels. There is insufficient electron angular momentum at these deep levels for any integer h change.

4. Comparisons between, and Questions about, Electron Deep Levels (EDLs) Predicted by the Relativistic Schrodinger Equation and the Deep Dirac Levels (DDLs)

(1) The EDLs and DDLs are not symmetric between the electrons and positrons.
(2) There are the same number of positron states and electron states within the Schrodinger results.
(3) There are more positron states than electron states in the Dirac results. Is this a result of spin? If so, why?
(4) Relativity has broken symmetry in both cases. Is it a valid conclusion, a real effect, or an artifact of the calculation? Would these same results appear if the calculation were instead carried out for positrons and anti-protons?
(5) Spin has little effect on the atomic-electron orbitals (beyond that of the Pauli Exclusion Principle).
(6) Spin has a much larger effect (keV) on the deep-levels.
(7) Spin introduces a new quantum number and, in the deep region, a reordering of the old.
(8) *The new quantum number (not found in the relativistic Schrodinger equation results) is related to spin, not to relativity.*
(9) *The source of the deep orbits is related to relativity, not to spin.*
(10) If the deep orbits are separated by even or odd integers, *what do the integers represent physically?*

5. Conclusions

Our analysis of the first papers [1,2] to both provide a non-singular potential for the relativistic Schrodinger and Dirac equations *and* then to find the solutions for the known and anomalous bound-electron levels has led to a number of surprises. These computational solutions include a coherent structure for an unusual family of electron deep levels (EDLs) with binding energy of their higher angular momentum orbitals asymptotically approaching that of the electron mass. While the papers have provided an immense amount of information, they have also left some unanswered questions.

Relativity has brought a solution to the central-body problem out of the center of the potential. We suggest that this and a definite, but low, angular momentum (i.e., within the limitations of $l = 0$ for the 'standard' centrifugal barrier of the atomic orbitals) provides an electron orbital close to the proton by using the new lower values of angular momentum (for deep orbits with its quantum numbers l).[b] The Coulomb potential is modified by the 'reduced' angular

[b]The possible confusion caused by the use of the same quantum numbers for different series of levels will have to be reduced in the future unless

momentum barrier, which introduces a series of deep levels available to electrons, as they, in turn, are modified by their increasing mass with kinetic energy. The deep orbits are 'quantized' with respect to an unknown source, apparently *not* one related to the deBroglie wavelength or the Heisenberg Uncertainty Relation and thus the Planck constant. Increased angular momentum within the deep-orbit framework (and at much less than h, therefore still with $l = 0$ in the atomic-electron framework) circularizes the deep-electron orbits and allows them to move closer to the proton.

While it would be nice to have experimental evidence directly supporting these deep electron levels, the low numbers and short lifetime of electrons in these levels prevent this. (This problem is suggested by the ease with which the resulting neutral femto-atoms can penetrate through an electron cloud and into atomic nuclei, thus producing the observed transmutation in cold fusion experiments.) If cold fusion, which depends on one or more electrons spending more time between protons or deuterons than possible in accepted physics, can provide enough used reactor material from its activity, then these levels might be indirectly, or even directly, observed. It is probable that sufficient used 'reactor' fuel material already exists (from 2013 or 2014 tests) for such additional analysis, if it were to be made available for that purpose. However, only a few facilities in the world may provide adequate analysis capabilities. Access to such facilities requires convincing their controllers and funding agencies that both cold fusion and the electron deep-level model are real and that such material with a long-enough life time for measurement could be produced in sufficient quantity for the tests. It is improbable that such tests will be made this year. However, we could work for that goal in 2016.

Appendix A. Integration along a Closed Path – the deBroglie Wavelength and Electron Orbitals

Integration along a 'closed' path gives a zero result (unless the path encircles a singularity). Extending this concept to wave motion along a linear path, x, allows a constant periodicity (i.e., to be a wave equation with $e^{-2\pi ni} = 0$, for integer values of n) with a repeat distance R of the deBroglie wavelength. ($R = nx/\lambda_{dB}$, where $\lambda_{dB} = h/p$ and p is the momentum of the object [8] and h being the Planck constant. Thus, we get the quantum mechanical wave equation of $e^{-2\pi ip\cdot x/h)}$). What does this signify? That will depend on some assumptions. Our assumption is that relativity-induced precession of a particle's spin-vector (from its velocity) is the basis of the wave motion and that the closed path is one traced out by a full cycle of the pointing angle of this vector. At a full cycle, the pointing angle of the vector is the same as at the beginning of the cycle and the conservation of momentum and energy says that they are independent of the position along the linear path. Thus, the integral over each cycle is zero.

In a non-relativistic bound orbit, the similar conditions hold; but now, the closed path must include both the orbital circumference and the deBroglie wavelength. The resonance of these two frequencies establishes the atomic-electron orbitals. In the relativistic case, additional closed orbits are predicted classically [9,10]. In relativistic quantum mechanics, these new deep orbits are also predicted [3], but cannot be associated with the Planck constant (since $l = 0$). Therefore, to be compatible with the closed-orbit model, there must be another wave mechanism. The orbital motion may provide the additional mechanism, via the relativity-induced precession (or nutation) from the Coulomb force, for the higher-frequency resonances needed to create the deep orbits.

Appendix B. Excerpts from Maly and Vavra's First Paper on Deep Dirac Levels

Published in *Fusion Technology*, Vol. 24, November 1993 (excerpts with permission, Copyright 11/93 by the American Nuclear Society, La Grange Park, Illinois [1])

people get used to the different values of angular momentum for different series. This is not confusing for the different values of energy (from the principal quantum numbers) for the atomic and deep orbits because these energies are both still related to hν. However, the fixation with only the Planck constant being associated with angular momentum and the Heisenberg uncertainty relation makes 'suspect' the new, lower, angular momenta values of the deep orbits.

Electron Transitions on Deep Dirac Levels I

J.A. Maly and J. Va'vra

Abstract

The original solutions of the Schroedinger relativistic equation and the Dirac equation for hydrogen-like atoms were analyzed for the possible existence of some other electron levels, which were not originally derived. It was found that besides the known atomic levels, each atom should also have the Deep Dirac Levels (DDL). The electron transition on such DDL would produce large amounts of atomic energy (400–510 keV per transition depending on the *Z* of the atom).

4. Calculations of New Energy Levels

A computer program was written which calculates atomic energy levels for the Relativistic Schroedinger and Dirac levels. For comparison, the non-relativistic Schroedinger levels given by a simple Bohr formula are also shown.

The Schroedinger levels are calculated in Table 1 with the plus sign inside the *s* ... in the E1S(+) column and with the minus sign inside the s in the E2S(-) column. The Dirac levels are calculated in Table 2 with the plus sign of *s* ... in the E1D(+) column and with the minus sign of s in the E2D(-) column. In describing Schroedinger levels, the notations $n = N =$ main quantum number, $l = L =$ angular quantum number, $n' = M =$ radial quantum number will be used. The nln' notation used in the equations of this paper is defined as the NLM level in Table 1 ($l = 0, 1, 2, 3, 4, 5, ...$ are also called s,p,d,f,g, ... levels in the spectroscopic notations). Similarly, we will use the notation $n = N =$ main quantum number, $n' = M =$ radial quantum number, $k = K = a(j + 1/2) =$ Dirac *k* number, $l = L =$ angular quantum number to describe the Dirac energy levels ($a = +$ or $-$ sign at *k*).

Table 1. Relativistic Schroedinger levels for H ($Z = 1$) in eV

	$E(N, Z)$	N	M	L	E1S	E2S	
1s	−13.605826	1	0	0	−13.606597	−507171.937500	
2p	−3.501457	2	0	1	−3.401449	−13.605632	*
2s	−3.401457	2	1	0	−3.401570	−13.603699	
3d	−1.511759	3	0	2	−1.511747	−3.401425	*
3p	−1.511759	3	1	1	−1.511755	−509755.250000	
3s	−1.511759	3	2	0	−1.511790	−3.401207	
4f	−1.511764	4	0	3	−0.850357	−1.511744	*
4d	−0.850364	4	1	2	−0.850358	−13.605434	*
4p	−0.850364	4	2	1	−0.850361	−13.604666	
4s	−0.850364	4	3	0	−0.850376	−1.511683	
5g	−0.850364	5	0	4	−0.544228	−0.850356	*
5f	−0.544233	5	1	3	−0.544228	−3.401415	*
5d	−0.544233	5	2	2	−0.544229	−510264.468750	

*Negative energy states, not observable.

Appendix C. Excerpts from Vavra's Presentation at Siegen University, Germany, Nov. 25, 1998 (excerpts with permission of J. Va'vra)

On a possibility of existence of new atomic levels, which were neglected theoretically and not measured experimentally.

Table 2. Dirac levels of hydrogen-like atoms for H ($Z = 1$) in eV.

	$E(N,Z)$	N	M	K	L1	E1D	L2	E2D	
1s	−13.605826	1	0	1	0	−13.605873	1	−13.605873	*
2p	−3.401457	2	0	2	1	−3.401434	2	−3.401434	*
2s	−3.401457	2	1	1	0	−3.401479	1	−509133.375000	
3d	−1.511759	3	0	3	2	−1.511746	3	−1.511746	*
3p	−1.511759	3	1	2	1	−1.511750	2	−13.605512	*
3s	−0.850364	3	2	1	0	−1.711764	1	−13.604422	
4f	−0.850364	4	0	4	3	−0.850356	4	−0.850356	*
4d	−0.850364	4	1	3	2	−0.850357	3	−3.401419	*
4p	−0.850364	4	2	2	1	−0.850359	2	−510064.125000	
4s	−0.850364	4	3	1	0	−0.850365	1	−3.401298	
5g	−0.544233	5	0	5	4	−0.544228	5	−0.544228	*
5f	−0.544233	5	1	4	3	−0.544228	4	−1.511744	*
5d	−0.544233	5	2	3	2	−0.544229	3	−13.605389	*
5p	−0.544233	5	3	2	1	−0.544230	2	−13.604785	
5s	−0.544233	5	4	1	0	−0.544233	1	−1.511710	
6h	−0.377940	6	0	6	5	−0.377936	6	−0.377936	*
6g	−0.377940	6	1	5	4	−0.377936	5	−0.850356	*
6f	−0.377940	6	2	4	3	−0.377936	4	−3.401412	*
6d	−0.377940	6	3	3	2	−0.377937	3	−510381.343750	

*Negative energy states, not observable.

The electron density distribution is calculated as follows and as pictured in Fig. C-1.

$$\text{Eld} = 4\pi r^2 R^2(r) = 4\pi r^2 \mathrm{e}^{-\rho}\rho^{2s}L^2(\rho).$$

The radial distributions for the deep-Dirac levels have different quantum numbers, but nearly identical shapes for

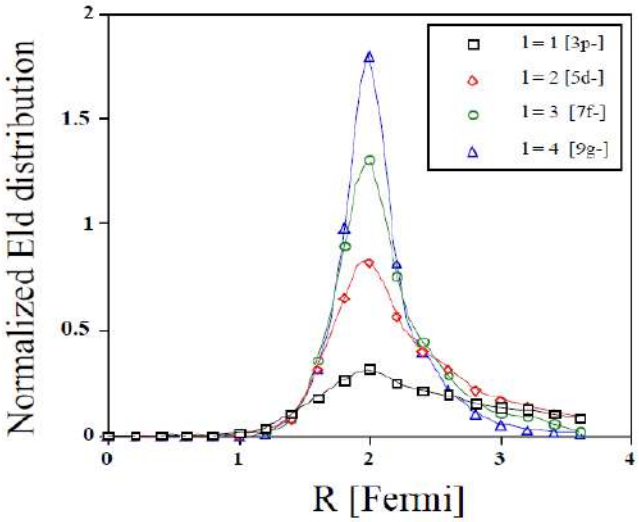

Figure C-1. Normalized radial distribution of the first four deep-electron levels based on the relativistic Schroedinger equation with Nix potential.

the different orbitals. Note that the first level is extended to higher and lower radii (i.e., it is composed of more elliptical orbits). Sequential orbitals (with higher angular momenta) become more confined into a spherical shell as the distribution settles deeper into the potential well.

Appendix D. Deviation from the $1/r$ Coulomb potential

The Hamiltonian in all of the models, non-relativistic and relativistic, used the $1/r$ Coulomb potential. Feynman states that this potential is valid, at least up to the nucleus. Nevertheless, he introduces the angular momentum, in the form of a centrifugal force as a pseudo-potential, into the solution for the hydrogen atom [11]. Thus, unless the angular momentum contribution is removed (leaving $l = 0$), the virial theorem must be determined for the $1/r$ Coulomb potential V_C as modified by the addition of significant angular momentum with quantization, e.g., $l = 1, 2, ..., n - 1$):

$$|V_C| = |ee'|/r = e^2/r \Rightarrow |V_C'| = e^2/r - l(l+1)h^2/2mr^2(D-1). \tag{D.1}$$

When these two potentials are equal in magnitude (and opposite in direction) the attractive potential of the nucleus for an electron becomes repulsive.

At the Bohr radius, $a_0 = 4\pi\varepsilon_0 h^2/me^2 = h/mc\alpha$, and the turnover potential is found at:

$$e^2/a_0 = l(l+1)h^2/2ma_0^2 = l(l+1)h^2/2m(h/\alpha * mc)^2 = \alpha^2 l(l+1)mc^2/2(D-2). \tag{D.2}$$

Since the attractive Coulomb potential energy at a_0 is 27.2 eV (= $\alpha^2 mc^2$), this is equal to the repulsive energy at $l(l+1) = 2$. This allows $l = 0$ to be the only possibility for a ground state, since the $l = 1$ possibility would require an exactly circular (a classical) orbit with a nearly exact energy rather than the spread in orbital parameters as required by the Heisenberg relation. However, this quantization of angular momentum does not prevent all non-integer angular momenta (for $0 \leq l < 1$). The uncertainty relation permits some variation, despite the $l = 0$ restriction. Again, *any* angular momentum prevents the singularity at $r = 0$. However, for angular momentum approaching *h*, e.g., $l = 1$, the effective central potential is no longer well approximated by the $1/r$ Coulomb potential for the $n = 1$ orbital. For the $n = 2$ levels (or above) and $l = 1$, the approximation is still adequate. Thus, there is a physical basis for the normal selection rules (integer values $0 < l < n - 1$).

At the classical electron radius, $r_c = \alpha * h/mc$ (and $e^2/r_c = e^2/\alpha * h/mc$, and the turning point is at:

$$e^2/r_c = l(l+1)h^2/2mr_c^2 = l(l+1)h^2/2m(\alpha * h/mc)^2 = l(l+1)mc^2/2\alpha^2(D-3). \tag{D.3}$$

The point where Coulomb potential and centrifugal barrier are equal in magnitude gives $l(l+1) = 2\alpha^2 = 2/137^2$. It is clear that any orbits based on the centrifugal barrier, for angular momentum quantized in units of the Planck constant, is forbidden except for l very near to zero. This same restriction also applies to any such localization where the Heisenberg Uncertainty Relation holds. How can the Dirac anomalous solution predict orbits that oppose these restrictions? The condition for stable orbits in the sub-atomic regime must result from a momentum operator that is not based on the Planck constant. An understanding and description of this requirement is the basis of another paper. However, in the M&V papers, their analysis included a finite-sized nucleus with a non-singular potential for electrons. In the atomic-electron case, this has little effect since it is nearly 5 orders of magnitude smaller than the orbitals. In the deep-orbit electron case, the nucleus is nearly one-half the orbital radius and this severely restricts the possible orbits. At the femtometer orbits, the nuclear size, relativity, quantized angular momentum, and their effects on the electron must be added to the centrifugal potential that alters the virial theorem results for the Coulomb potential.

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