



Research Article

# Physical Reasons for Accepting the Deep-Dirac Levels– Physical Reality vs Mathematical Models in LENR

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## Abstract

Limitations to contemporary models of Coulomb and nuclear interactions have previously been identified in the development of low-energy nuclear reaction (LENR) theories based on experimental Cold Fusion (CF) results. However, relativistic quantum mechanics has already provided a means to overcome these limitations. Nevertheless, this ‘anomalous’ solution has been repeatedly rejected, based on the mathematical formalism of an assumed singular potential, simply because it has not been necessary heretofore in the description of available physical data. The physical realities of a non-singular potential and its interaction with a relativistic electron are not new physics and provide a solid theoretical basis for CF (as indicated in another paper in this conference, and the references therein) and for new fields in femto-physics and nuclear chemistry. This present paper is essentially a reasoned complaint against those critics who proclaim the limitations of the anomalous solution, yet ignore the inapplicability of their chosen potential to the real world where they claim the solution(s) fail. It also castigates those physicists who accept the mathematical limitations as applicable to the real world even when presented with the evidence of assumptions made where they are no longer valid. The paper picks a few of these alleged limitations and shows how relativity and near-field interactions alter predictions based on the mathematical models and approximations used to characterize the conventionally accepted observations. It identifies where the simple models fail, or violate physical reality, and points to the implications of extending the models to better fit the real world and to properly understand the physical mechanism(s) involved.

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## 1. Introduction

Few physicists today would question the inadequacy of 19th century physics to correctly calculate many effects when bodies approach the speed of light. And, when a bound s-orbit electron approaches its nucleus, the region of its highest

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probability density, most will legitimately ignore the effects of its obviously relativistic velocity in that region because it spends so little time there. Yet, when relativistic quantum mechanics predicts new effects, the basic effects are all accepted (even if perhaps misinterpreted) and some of the relativistic effects are ignored. The ‘selective acceptance’ of a theoretical model’s predictions is generally based on confirmation by experiment. However, the prediction of deep electron orbitals (with binding energies into the 0.5 MeV range) by the ‘anomalous’ solution of the relativistic Klein–Gordon (K–G) and Dirac equations [12] has been rejected in the past on theoretical and purely mathematical grounds. Thus, even if experimental evidence were now to be discovered for these orbits, it would be ignored, rejected, discounted, or attributed to other effects.

Rejection of these deep orbits, based on mathematical considerations, and recent arguments against such rejection are well documented in the present authors’ papers on the subject [3–5]. Even when not rejected by mathematical arguments, these deep electron orbits have been previously discounted by the predicted low-population density of such levels [6]. The authors of this early series of discounting papers accepted the anomalous solution, but rejected it as a source of energy for the observed cold fusion results. The rejection by physical consideration of the low population of such levels is revealing in that it shows that some physicists, beyond recent proponents of the deep levels [17] are at least willing to accept their existence and examine possible implications. This examination took place within the framework of the quantum mechanics that predict these levels. The results, while negative for cold fusion, are positive for the deep electron orbit solution. It will be argued below that the conclusions for their rejection of applicability to cold fusion, like that of the mathematical rejections, are based on incorrect assumptions and inadequate physical models.

The present paper seeks to completely eliminate the mathematical arguments against the deep levels by application of physical ‘reality checks’ that are ignored in the mathematical discussions. It will show that the arguments are based on mathematical models and approximations that do not reflect physical reality and therefore should not be applied to the real world. In particular, two commonly accepted models must be properly extended.

The ‘mathematically pure’  $1/r$  Coulomb potential gives a non-physical singularity when extended to  $r = 0$ . The failure of this potential, as it is expressed near to (or within) a distributed charge, is obvious and is described in most elementary college-physics courses. The possible correctness of the singular potential close to a point source and the resultant wave equations is vitiated by the required infinite energy and charge densities of such a source.

The mathematically pure classical and quantum mechanics must be extended into the realm of relativity [4] and to near-field interactions [8]. The mathematics of these extensions can also be pure. Is one model better than the other(s), which should therefore be considered invalid? The relativistic extension of the static Coulomb potential to a dynamic one introduces new ideas into the field of CF.

## 2. A Non-physical Singularity

When Coulomb proposed the  $1/r$  dependence of the electrostatic potential, it was based on macroscopic measurements. At the time, there was no concept of extending it to  $r = 0$ . The ‘requirement’ that Coulomb’s ‘law’ fulfill all of the mathematical niceties leads to physical absurdities and has obscured important details of the real potential between two charges. One physical absurdity is the resulting singularity of an infinite mass and charge density for every charged particle in the universe. This does not seem to bother the ‘opponents’ of the anomalous solution of both the relativistic Klein–Gordon and Dirac equations. They accept the singular potential and then reject the ‘anomalous’ solution, dependent on this potential, which they assume has a similar singularity, even for the defined states. For over 50 years the mathematical arguments against the anomalous solution have suppressed interest in its possible physical reality. Until recently, arguments against use of the singular potential appear to be ignored.

Strangely, the common practices in physics for handling singularities and other infinities, e.g. in [9] are also ignored in this case. Use of a non-singular nuclear Coulomb potential, by physical or mathematical modifications, is

a trivial change that has been published – and rejected or ignored by the critics. The standard practice of ‘cut-offs’, a mathematical artifice, to avoid these problems is, for some reason, ‘not allowed’ in the arguments against the possibility of the anomalous solution being valid. Of course, if incontrovertible proof of the predicted deep-electron orbits were to become available, these same critics would declare themselves experts in the field and using these very techniques would ‘prove’ the validity of the anomalous solutions (with ‘their’ modifications).

The problem with the mathematical rejection of the anomalous solution is not with the mathematics itself. It is with the approximations used, and the non-physical assumptions, interpretations and their implications for the solution. The  $1/r$  Coulomb potential is so imbedded within many physicists’ minds that deviation is unthinkable. We have been taught that it is valid - at least to the nucleus, e.g., [10]. However, the original ‘law’ must be corrected for source configurations and relativity in the nuclear range. When mathematical rigor, without physical consideration, is applied to the  $1/r$  problem at  $r = 0$ , these details are ignored. This allows a non-physical singularity and its consequences to be introduced into the mathematics. Such singularities are encountered routinely in physical models and ‘handled’ in various ways. Such treatments for the anomalous solution have been routinely ‘rejected’ over the decades and, without experimental observation of the deep-orbits predicted by the relativistic quantum mechanics, it is difficult to rationalize such measures. It is precisely under these conditions that near-field and far-field interactions become mixed (‘sequestration’ fails [11]), symmetry is broken, and ‘matter’ is created and destroyed. This is the basis for one model of cold fusion that uses the reduction of nuclear mass resulting from energy transfer to the deep-orbit electron to explain D–D fusion to  $^4\text{He}$  rather than to the normal ‘hot’ fusion paths of fragmentation into protons, neutrons, tritium, and  $^3\text{He}$  [12].

Modern quantum mechanics treats the ambiguities during transitions through these interaction regions by use of the Fock and Stueckelberg model and the uncertainty relation [13]. Energy, mass, and other constants need no longer be conserved during such transitions as long as they are conserved before and after the resultant interactions.

### 3. Solutions Avoiding the Singularity

Khelashvili and Nadareishvili [14] proved that the anomalous solution of the Klein–Gordon equation for the hydrogen atom with a singular  $1/r$  Coulomb potential is valid. However, in the same paper (and subsequently affirmed [15 16]), they cannot do the same for the anomalous solutions of the Dirac equations. In addition to the unaddressed higher order terms in  $V$  [17], what may be missing in the decades-old argument is that the actual anomalous solutions for the Dirac deep orbits have no electron probability distribution at the singularity (assuming that one physically exists). According to [1], there are no 1s-orbitals in the anomalous solution of the Dirac equations, furthermore, the deep-orbit ground state (2s in their nomenclature) has angular momentum as do all the deep-orbit s-orbital solutions. How can this happen and what does it mean?

In [1], Maly and Va’vra produced a detailed description and tables of the deep orbits, for both the relativistic K–G and the Dirac equations. In [2], the authors included figures of the electron densities for several representative orbits. In this later paper, they were estimating the size of the deep-orbit atoms and were not concerned about arguments against the existence of the deep orbits. However, the information provided in the two papers can be used to indicate that no anomalous solutions of the Dirac equations (for the hydrogen atom with a finite-sized nucleus, thus a non-singular Coulomb-type potential) have any value at  $r = 0$  (see next paragraph) If there is no electron density at  $r = 0$ , there is no singularity in the deep-orbit solution and then all arguments against the deep orbits (if based on a singularity) must be invalid. This result is a step beyond simply using a non-singular potential. As suggested by calculations, but not yet proven, it may be possible (even if unnecessary) to provide this same result with a singular potential.

What is the basis of the non-population at the singular point? The deep level identified as 2s is perhaps better labeled as a 2p’ level (with angular momentum of  $\hbar?$ ). The reason for the new notation is related to the new quantum number,  $k$ , introduced in the Dirac equation solutions. The integer value of  $k$ , which corresponds to an angular momentum

resonance may be positive or negative. However, the deep-orbits all have positive integers,  $k$  (corresponding to a positive energy), and the deep-orbit ground state has a value of  $k = 1$ , which should correspond to a p-, not to an s-, orbital. With no  $k = 0$ , there is no solution with zero angular momentum. If there is any angular momentum at all, then the electron orbits cannot go to  $r = 0$  because the centrifugal barrier is too high. Use of the term “s-orbital,” which implies a maximum electron density at  $r = 0$  for the  $1/r$  potential, and to so identify the deep-orbit ground state is then misleading and could lead to one thinking of the solution as singular.

#### 4. Physical Size and Mass of the Electron and Proton

The unmentioned assumptions of the mathematical solution of the relativistic quantum equations leading to the deep orbits include the infinite mass of the point source of a Coulomb potential. We have already addressed the physical absurdity of the singular potential associated with a point source. Here we address the standard correction for the infinite-mass assumption. In elementary quantum mechanics courses, a correction for finite mass nucleus is made to the solution of the Schrodinger equation. This is done after the simplified model of infinite mass source is solved. This correction is based on recognition of the 2-body problem, as existing in the center-of-mass system, being reduced to a single-body problem by introduction of a ‘reduced’ mass.

Introduction of a finite nuclear mass immediately forces changes to many of the arguments against the deep orbit solutions. First off, the assumed singularity of a point source for the theoretical  $1/r$  Coulomb potential does not exist at  $r = 0$ , the center of mass. The proton is shifted away from  $r = 0$  and from the electron. In fact, the singular condition can only exist if the angular momentum of the system is exactly zero and the electron (also assumed to be a point source in the same approximation) is simultaneously at  $r = 0$  along with the point source. Despite the fact that the wave functions of both nucleus and electron may overlap at  $r = 0$ , the probability of both of these particles being together at the same time is zero. Thus, the potential is proportional to  $1/(r + \varepsilon)$  with  $\varepsilon > 0$ .

#### 5. Physical Approximations Used to Treat Infinities

Many models in physics lead to infinities in one limit or another. To get around this problem, it is common practice to introduce a ‘cut-off’ to prevent values from going to infinity. Such treatment must somehow be unacceptable to the mathematical physicists who proclaim the invalidity of the anomalous solutions. Perhaps, to be acceptable, such ‘fixes’ must only be validated by the proposed models better fitting the experimental data when the fixes are included. If so, cold fusion might provide those data. Unfortunately, such data are not yet universally accepted and therefore “any change is unnecessary.”

The key to the deep electron levels is relativity [17]. This is also a limit to some extent. Non-relativistic physics is an approximation that is valid under most conditions. In the nuclear range, electron velocities require inclusion of this effect. So, instead of imposing a cutoff such as that imposed by classical quantum mechanics (“no electron orbits can exist below the atomic levels”), it is possible to seek and find the physical limit imposed by, and the consequences of, relativity. Similarly, quantum mechanics has allowed the laws of physics to be ‘relaxed’ during certain interactions [13]. This limit is as crude as an arbitrary cutoff (and as uninformative). Relativity can provide a much more accurate and informative (physically reasonable) limitation to the classical interaction dynamics. Like quantum mechanics, it also provides a challenge to our perceptions of reality. Cold fusion provides a transition region between the classical atomic and nuclear regimes. It allows physically accessible information to be analyzed and compared with models.

#### 6. Relativistic Coulomb Potential and Heisenberg Uncertainty Relation (HUR)

For some people, the fatal flaw in the concept of deep orbits is their failure to obey the HUR. At the predicted KE  $< 1$  MeV level of kinetic energies of an electron bound in the low femtometer radius orbits of the near-nuclear environ-

ment, the angular momentum is  $\sim \hbar/100$ . Thus, the HUR is violated by two orders of magnitude, unless the kinetic electron energy is somehow raised to the  $\sim 100$  MeV level [17]. An electron with this energy cannot be bound in the *static* Coulomb potential of a single proton. This argument is hard to contest without going against what has become a fundamental tenet of quantum mechanics. One of the present authors (JLP) has found a possible answer in effects such as the near-field magnetic interactions between the nucleus and the electron, that raise the kinetic energy of a tight-bound electron into the 100 MeV range [17,18].

It is known that the deep orbits predicted by the Klein–Gordon and Dirac equations are a result of relativity [5]. We have been seeking the physical mechanism that satisfies both the singularity and the HUR limitations of the anomalous solutions. Early-on in the study, it was recognized that there was a correction needed to the virial theorem as it is applied to deep orbits [19]. Orbital requirements in a central potential are defined by the virial theorem. For non-relativistic orbits in a  $1/r$  Coulomb potential, the virial theorem requires that the kinetic energy of the bound particle be one-half of the magnitude of its potential energy. The relativistic correction to the virial theorem predicts that as the particle velocity approaches that of light, this ratio of the energies approaches unity (i.e.,  $KE \Rightarrow |PE|$ ). While this correction has major implications for the creation of a deep-orbit-electron population, it resolves neither the singularity nor the HUR problem of the solution.

The most recent contributions [5] to resolving the HUR problem are also presented at this conference [17]. Relativity not only introduces mass energy into the Hamiltonian, thus providing the deep orbits, it also affects (greatly increasing) the attractive forces between the proton and electron as the average electron velocity approaches a significant fraction of  $c$ . In the low-femtometer range of orbits, the Coulomb potential is no longer a  $1/r$  potential and, with magnetic interactions, bound-electron kinetic energies can exceed 100 MeV. This unexpected turn of events resolves the HUR issue very nicely. However, it introduces some other questions, such as “where do the  $>100$  MeV energies come from?” and “how do these relativistic electromagnetic energies compare with the nuclear energies (fractional quark size) that are so much greater than the static Coulomb energies normally considered?”

## 7. Conclusion

The use of mathematics in physics is essential. However, it is a tool. Any dogmatic adherence to its ‘rules’ and predictions, which may be based on improper or inadequate formulation and approximations should be avoided. The mathematics needs to be guided by the physics, before it can properly guide physics into (or out of) new regions of interest. Neither physics nor mathematics should be considered sacrosanct. This paper has addressed a few of these issues associated with the deep electron orbits predicted by relativistic quantum mechanics.

Physics has said that the Coulomb potential varies as  $1/r$ . Unguided mathematics then states, categorically, that any solution of an equation using this potential must be rejected unless it is zero at  $r = 0$  because it is otherwise singular at that point. Both are correct in their frame of reference; but, they are both wrong when applied to the special case near  $r = 0$ . We have shown that the singularity in the  $1/r$  potential is non-physical and therefore mathematical solutions that include the singularity must be ‘adjusted’ so that they are no longer singular and rejected. The means of such adjustment are well known in physics and some have even been applied to the case of interest here.

We believe that the dilemma of cold fusion theory is resolved in the acceptance of the deep-orbit electrons. Because of the uncertainties in the influence of some of the physical interactions (e.g., spin–spin) at these femtometer dimensions, actual values for the deep-orbit radii are still under investigation. Nevertheless, within the wide range of approximations and assumptions made in the calculations, most variations are not great and the implications for CF are unchanged.

In the process of exploring (with an open mind) the mathematics and physics of this near-field region, important things beyond CF are being discovered. The most recent of these is the effect of relativity on the Coulomb potential energy for deep-orbit electrons. With energies greatly in excess of the known nuclear binding energies and with

the greatly enhanced interaction between the bound electron and the charged-quarks of a nucleon, the theoretical relationship between the strong-nuclear force, the electromagnetic force, and the weak interaction may be altered. It would appear that cold fusion is operating in the transition region between the atomic and the nuclear forces and is providing experimental data that will illuminate many aspects of physics today.

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