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Research Article

Physical Model of Energy Fluctuation Divergence

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Abstract

In this paper, we propose a new classical model in which energy fluctuation diverges. In detail, for certain parameter ranges, kinetic energy diverges since the momentum obeys the Cauchy distribution. This phenomenon will be applied to the cold fusion since jumping over the potential wall is essential to cold fusion.

Keywords: Cauchy distribution, Chaos, Energy fluctuation divergence

1. Introduction

It is well known that the second moment of a probability variable x diverges such that

$$\left\langle x^2 \right\rangle = \int x^2 p(x)\, \mathrm{d}x = \infty,$$

when x obeys a power-law density function defined as

$$p\left(x\right) = O\left(\frac{1}{x^{1+\alpha}}\right), \qquad x \ll 1, \quad 0 < \alpha < 2.$$

Then, the fluctuation of the probability variable $\left\langle \left(x - \langle x \rangle\right)^2 \right\rangle$ also diverges. When momentum obeys a power-law density function, the fluctuation of kinetic energy diverges. Then energy fluctuation divergence will play an important role in jumping over a potential wall. One can read research about the divergence of fluctuation or large fluctuation of energy [1,2]. In this paper, we propose a classical chaotic model in which power-law density appears for certain parameter ranges and energy fluctuation can be observed.

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2. Classical Chaotic Model

We consider a classical dynamical system in which a power-law distribution can be observed owing to the chaotic structure. In general, chaos is characterized by initial condition sensitivity. That is, the distance between two orbits grows exponentially although the initial points of two orbits are very close. To realize a system in which power-law densities appear, we apply the fact that when a probability variable α obeys the uniform distribution on $\left(-\frac{1}{2},\ \frac{1}{2}\right)$, another probability variable β defined as $\beta = \gamma \tan(\pi\alpha)\,,\ \gamma > 0$ obeys the Cauchy distribution $f_\gamma(\beta)$ defined as

$$f_\gamma(\beta) = \frac{1}{\pi}\frac{\gamma}{\beta^2 + \gamma^2}.$$

Then, from a simple calculation, the variance of β, $\langle \beta^2 \rangle$ diverges. Then if momentum p obeys the Cauchy distribution, then the expectation value of kinetic energy $\left\langle \frac{p^2}{2} \right\rangle$ and its fluctuation

$$\left\langle \left(\frac{p^2}{2} - \left\langle \frac{p^2}{2} \right\rangle \right)^2 \right\rangle$$

diverge. From this, we propose a four dimensional classical Hamiltonian in order to realize the power-law distribution defined by

$$\begin{gathered} H(p_1, q_1, p_2, q_2) = \tfrac{1}{2}\left(p_1^2 + p_2^2\right) - \varepsilon \log\left|\cos\{\pi(q_1 - q_2)\}\right|, \\ V(q_1, q_2) = -\varepsilon \log\left|\cos\{\pi(q_1 - q_2)\}\right|, \end{gathered} \tag{1}$$

where ε is a perturbation parameter and

$$p_1, p_2 \in R, \quad q_1, q_2 \in I_{\delta_N}, \quad I_{\delta_N} = \left(-\frac{1}{2} + \frac{\delta_N}{\pi}, \frac{1}{2} - \frac{\delta_N}{\pi}\right).$$

δ_N satisfies the condition such that

$$\frac{2\left(1 - \frac{2\delta_N}{\pi}\right) - 2\varepsilon(\Delta\tau)\left[\tan\left(\frac{\pi}{2} - \delta_N\right) - \tan\left(-\frac{\pi}{2} + \delta_N\right)\right]}{1 - \frac{2\delta_N}{\pi}} = N,$$

where N is a natural number. Figures 1 and 2 show the shape of potential $V(q_1, q_2)$ for $\varepsilon = 2$ and $\varepsilon = -2$, respectively. Although, the potential $V(q_1, q_2)$ is artificially constructed in order to generate the power-law distribution, this potential is periodic and in condensed matter physics periodic potentials are often treated [3,4].

From this Hamiltonian, one obtains the canonical equation such that

$$\begin{aligned} \dot{p}_1 &= -\tfrac{\partial H}{\partial q_1} = \pi\varepsilon \tan[\pi\{q_1 - q_2\}], \quad & \dot{q}_1 &= \tfrac{\partial H}{\partial p_1} = p_1, \\ \dot{p}_2 &= -\tfrac{\partial H}{\partial q_2} = -\pi\varepsilon \tan[\pi\{q_1 - q_2\}], \quad & \dot{q}_2 &= \tfrac{\partial H}{\partial p_2} = p_2. \end{aligned} \tag{2}$$

By using first order symplectic integrator [5] defined as

$$\begin{aligned} p_i(n+1) &= p_i(n) - \tfrac{\partial H}{\partial q_i}\left(p_1(n),\, q_1(n+1), p_2(n),\, q_2(n+1)\right), \quad i = 1, 2, \\ q_i(n+1) &= q_i(n) + \tfrac{\partial H}{\partial p_i}\left(p_1(n),\, q_1(n), p_2(n),\, q_2(n)\right), \quad i = 1, 2, \end{aligned} \tag{3}$$

one obtains four dimensional time discrete model T_ε as follows.

$$\begin{pmatrix} p_1(n+1) \\ q_1(n+1) \\ p_2(n+1) \\ q_2(n+1) \end{pmatrix} = T_\varepsilon \begin{pmatrix} p_1(n) \\ q_1(n) \\ p_2(n) \\ q_2(n) \end{pmatrix} = \begin{pmatrix} p_1(n) - \varepsilon \tan[\pi\{q_1(n+1) - q_2(n+1)\}] \\ p_1(n) + q_1(n) \mod I_{\delta_N} \\ p_2(n) + \varepsilon \tan[\pi\{q_1(n+1) - q_2(n+1)\}] \\ p_2(n) + q_2(n) \mod I_{\delta_N} \end{pmatrix}, \tag{4}$$

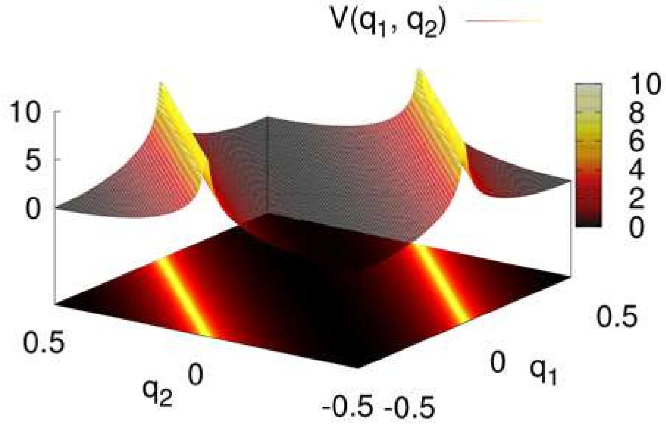

Figure 1. The shape of potential $V(q_1, q_2)$ for $\varepsilon = 2$. $V(q_1, q_2)$ diverges where $\cos\{\pi(q_1 - q_2)\} = 0$.

where the operation $\bmod\ I_{\delta_N}$ is defined such that

$$\begin{array}{l} x \quad \bmod\ I_{\delta_N} = x - n\left(1 - \frac{2\delta_N}{\pi}\right), \\ -\frac{1}{2} + \frac{\delta_N}{\pi} + n\left(1 - \frac{2\delta_N}{\pi}\right) < x \le \frac{1}{2} - \frac{\delta_N}{\pi} + n\left(1 - \frac{2\delta_N}{\pi}\right), \quad n \in Z. \end{array} \tag{5}$$

This map preserves the sum of moment such as $p_1(n) + p_2(n) = \cdots = p_1(0) + p_2(0)$. Then, the probabilistic property for p_2 is the same as that of p_1. One obtains chaotic orbits for almost all initial points for $\varepsilon < 0$, $\frac{2}{\pi} < \varepsilon$.

According to [6], one can prove that $\{\ q_1(n) - q_2(n)\ \}$ obeys the uniform distribution on I_{δ_N} and have mixing property owing to the chaotic structure when the parameter satisfies the condition $\varepsilon < 0$, $\frac{2}{\pi} < \varepsilon$. When we set N as

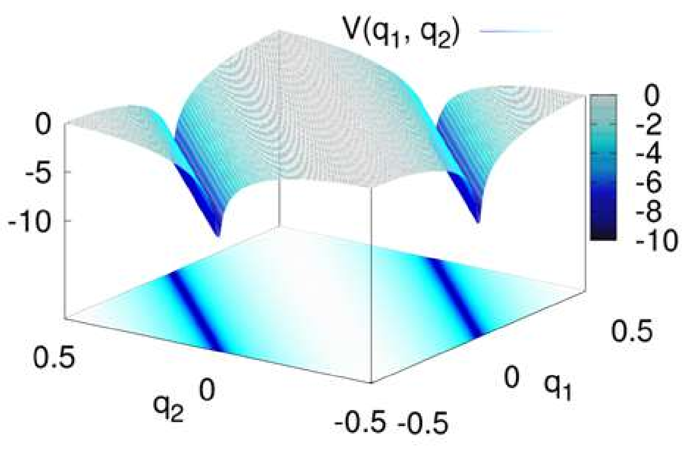

Figure 2. The shape of potential $V(q_1, q_2)$ for $\varepsilon = -2$. $V(q_1, q_2)$ diverges where $\cos\{\pi(q_1 - q_2)\} = 0$.

$N \gg 1$, variable $\{\ q_1\left(n\right) - q_2\left(n\right)\ \}$ are considered to distribute uniformly on $\left(-\frac{1}{2},\ \frac{1}{2}\right)$. Then the Cauchy distribution appears. That is, the time series $\{x_n = -\varepsilon \tan\left[\pi\left\{q_1\left(n\right) - q_2\left(n\right)\right\}\right]\}$ obey the Cauchy distribution $f\left(x\right)$ denoted as

$$f\left(x\right) = \frac{1}{\pi}\frac{|\varepsilon|}{x^2 + |\varepsilon|^2}, \tag{6}$$

when the parameter satisfies the condition $\varepsilon < 0,\ \frac{2}{\pi} < \varepsilon$. The momentum p_1 and p_2 are denoted by sum of variables $\{x_n\}_{n\geq 0}$ such that

$$p_1\left(n\right) = p_1\left(0\right) + \sum_{i=1}^{n-1} x_i,\ p_2\left(n\right) = p_2\left(0\right) - \sum_{i=1}^{n-1} x_i. \tag{7}$$

In this system, time series $\{x_n\}_{n\geq 0}$ have mixing property when the parameter satisfies the condition $\varepsilon < 0,\ \frac{2}{\pi} < \varepsilon$. Then, according to [7], momentum p_1 and p_2 obey the stable distribution in the condition. If each element of $\{x_n\}_{n\geq 0}$ are independent of the other elements, then p_1 and p_2 obey certain Cauchy distribution $\bar{f}\left(x\right)$ and the expectation value of kinetic energy $\langle K_1\rangle$ and $\langle K_2\rangle$ defined as

$$\langle K_1\rangle = \left\langle \frac{p_1^2}{2}\right\rangle = \int_{-\infty}^{\infty}\frac{p_1^2}{2}\bar{f}\left(x\right)\mathrm{d}x, \qquad \langle K_2\rangle = \left\langle \frac{p_2^2}{2}\right\rangle = \int_{-\infty}^{\infty}\frac{p_2^2}{2}\bar{f}\left(x\right)\mathrm{d}x, \tag{8}$$

diverges even though an initial condition of the kinetic energy $K_1\left(0\right)$ and $K_2\left(0\right)$ are finite.

We investigate whether $\{p_1\left(n\right)\}$ obey the Cauchy distribution at a finite time n. Figure 3 shows the result of numerical simulation for a distribution with $\{p_1\left(100\right)\}$ for $\varepsilon = 1$ where the number of ensemble M is as $M = 10^6$.

When the momentum $\{p_1\left(100\right)\}$ obey the Cauchy distribution whose scale parameter is γ, a new probabilistic variable $\{y\}$ defined as

$$y \equiv \frac{p_1\left(100\right) - \mu}{\sqrt{\sigma^2}}$$

obey $g(y)$ such that

$$g(y) = \frac{1}{\pi}\frac{\gamma\sqrt{\sigma^2}}{\left(\sqrt{\sigma^2}y\right)^2 + \gamma^2}, \tag{9}$$

where for finite number of ensemble of $\{p_1\left(100\right)\}$, μ and σ^2 correspond to their average and variance, respectively. Figure 3 shows the density function with $\{y\}$ and $g(y)$ where $g(y)$ is fitted by least squares method.

Then from the fitted density function $g(y)$, one obtains the estimated scale parameter $\hat{\gamma}$ such as $\hat{\gamma} = 98.1 \approx 1.00 \times 100$. If the time series $\{x_n\}_{n\geq 0}^{99}$ are independent, the estimated scale parameter $\hat{\gamma}$ satisfies the relation as $\hat{\gamma} = 100$. Then the result as $\hat{\gamma} = 98.1 \approx 1.00 \times 100$ means that the times series $\{x_n\}$ are almost independent. Therefore, this result suggests that not only $\{x_n\}$ but also $\{p_{1,2}\left(n\right)\}$ obey the Cauchy distribution.

3. Time Evolution of Energy

In Section 2, we introduced the property of this chaotic model, in which it was proven that the expectation value of kinetic energy diverges in the conditions described. In this section we show the time evolution of total energy in this system defined as follows.

$$E\left(t\right) = \frac{p_1^2\left(t\right)}{2} + \frac{p_2^2\left(t\right)}{2} - \varepsilon\log\left|\cos\left\{\pi\left(q_1\left(t\right) - q_2\left(t\right)\right)\right\}\right|. \tag{10}$$

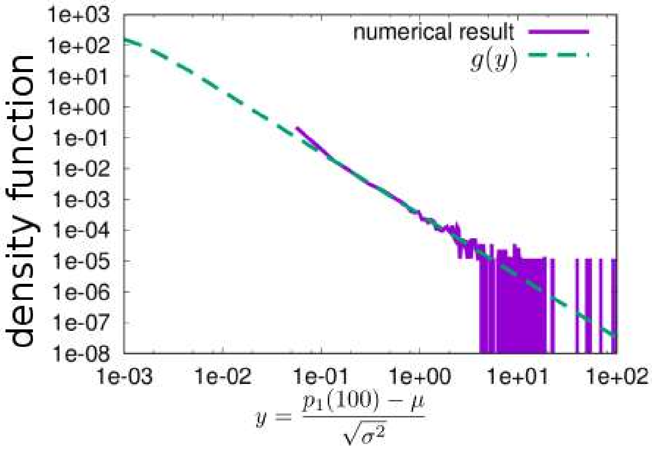

Figure 3. The density function of y composed of p_1 (100) for $\varepsilon = 1$. A solid line shows the result of the numerical simulation and a broken line shows a density function $g(y)$ fitted by least squares method. The value of average and variance for finite number of data are about $\mu \approx -7.13 \times 10^5$ and $\sqrt{\sigma^2} \approx 7.01 \times 10^5$ points M is $M = 10^6$. Estimated parameter value is $\hat{\gamma} = 98.1 \approx 1.00 \times 100$.

According to [8], by discretization, a system does not conserve energy although the Hamiltonian (1) is an autonomous system. According to [9], the energy fluctuates focusing on the initial value. From Fig. 4, the energy is not so large until $n \approx 2 \times 10^2$ and the intermittent burst occurs around $n \approx 3 \times 10^2$. Thereafter the energy begins to fluctuate wildly.

4. Conclusion

In this paper, a classical chaotic model has been proposed. It has been proven that this model has mixing property and momentum $\{p_1, p_2\}$ obey the stable distribution for $\varepsilon < 0, \frac{2}{\pi} < \varepsilon$ [1]. By numerical simulation, one can see

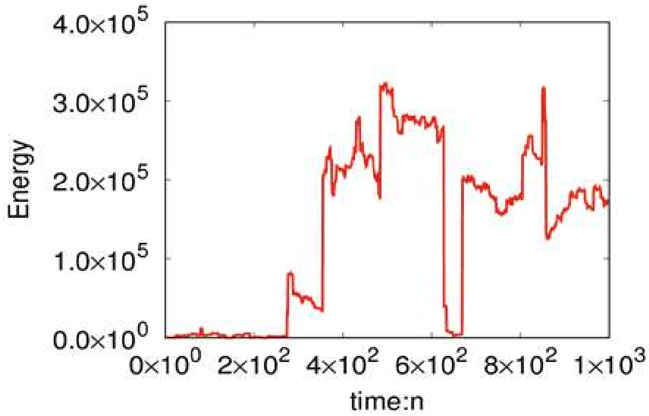

Figure 4. The time behavior of the total energy for $\varepsilon = 1$. Although the initial total energy is about 12.1, it fluctuates wildly and one can observe intermittency. Initial condition is $(p_1, q_1, p_2, q_2) = (0.3,\ 0,\ 4.91,\ 0)$.

$\{p_1, p_2\}$ obey the Cauchy distribution. From this result, we have confirmed that the variance of $\{p_1, p_2\}$ diverges and the expectation value of the kinetic energy $\langle K_1 \rangle$ and $\langle K_2 \rangle$ also diverges. Thus, this model can simply explain the phenomena in which the energy fluctuation divergence occurs although the initial energy is finite. This model can simply explain the phenomenon in which jumping over the potential wall occurs and can be a toy model which can explain cold fusion.

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