

Sonofusion Compressibility of Liquid and Stability of Spherical Cavity

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Abstract

We proposed the sonofusion at ICCF3 and theoretically predicted by use of a simplified model that the temperature of gas within a supersonic cavity reaches more than $10^8 K$ if the initial radius of the cavity is less than $10\mu m$, that is, temperatures high enough for the occurrence of hot fusion.

In the present paper we consider a more realistic model by taking into account the compressibility of liquid and search for the optimum values of supersonic parameters for getting high gas-temperature. In addition the stability of a spherical cavity is examined.

1. Maximum Temperature of Hot Spot

As well known, a supersonic pressure field applied to a liquid induces the forced oscillation of cavities. In the contraction phase of the oscillation the gas content is greatly, adiabatically compressed and as a result a small region of high temperature and density is transiently created in the liquid. The direct evidence for the hot spot is provided through the sonoluminescence phenomenon.

Many experimentalists^{1,2)} succeeded in directly measuring the hot-spot temperature and nowadays it is widely believed that the temperature may reach one hundred thousand Kelvin. On the other hand, we had been interested in the possibility of other catalyzers of nuclear fusion than muon particles. After so-called cold fusion, we have focused on the question of what extreme states could be realized under ambient conditions and have engaged in the determination of the upper bound of hot-spot temperatures.³⁻⁵⁾

1.1 Zeroth Order Approximation

At first we assumed a gas-filled, spherical bubble and incompressible liquid. Moreover, the gas content was assumed to adiabatically respond to the motion of bubble wall. The time-evolution of the bubble radius R is, therefore,

subjected to the Rayleigh-Plesset equation⁶⁻⁸⁾

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}(P_L - P_\infty),$$

where

$$P_L = P_G - 2\sigma/R, \quad P_\infty = P_0 - P_A \sin(2\pi f_A t).$$

The ρ and σ are the density of liquid and surface tension. The P_0 is the ambient pressure of liquid, P_L , P_G and P_∞ being pressures of liquid and gas at the bubble wall and pressure of liquid at a remote point from the bubble center, respectively. The gas is assumed to obey van der Waals' equation of states. The second term on the right hand side of the second equation stands for an applied supersonic field with amplitude P_A and frequency f_A .

The initial condition is

$$R(0) = R_0, \quad \dot{R}(0) = 0.$$

That is, at $t = 0$, the bubble with radius R_0 is assumed to be in a dynamical equilibrium with the ambient pressure P_0 .

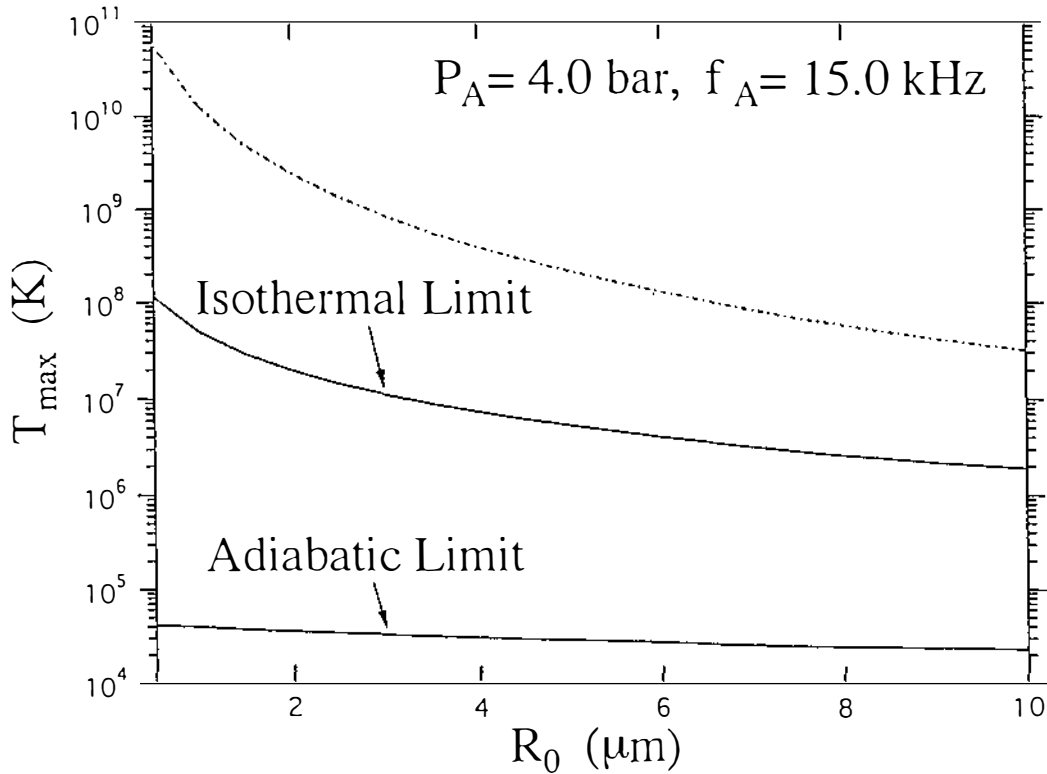


Fig. 1

We numerically solved the Rayleigh-Plesset equation and obtained the result depicted in Fig. 1 by a broken line. The values of supersonic parameters employed are also indicated. It was surprising that the maximum temperature T_{max} of hot spot exceeded 10^8 Kelvin when R_0 was less than 10 microns. This fact suggested the occurrence of hot fusion within a supersonic cavity if the gas content is D_2 . The preliminary calculation was so successful that we turned to a more realistic model.

1.2 Compressible Liquid

In the present paper we treat the compressible liquid, whose equation of states is assumed to take the form of

$$\frac{P + B}{P_0 + B} = \left(\frac{\rho}{\rho_0} \right)^n,$$

where

$$B = 3,000 \text{ atm}, \quad n \sim 7.0.$$

Namely, the liquid behaves like an incompressible one when $P \ll B$. The index n is chosen so as to reproduce the sound velocity of water, 1.45 km/s .

By use of the equations of continuity and of motion, and assuming the Kirkwood-Bethe hypothesis

$$\left(\frac{\partial}{\partial t} + (c + u) \frac{\partial}{\partial r} \right) \left(\frac{u^2}{2} + h \right) = 0,$$

where c and u are velocities of sound and liquid particle, respectively, and h is the enthalpy defined as

$$h = \int^P dP/\rho,$$

we can derive the Gilmore equation⁶⁻⁸⁾

$$R \left(1 - \frac{U}{C} \right) \dot{U} + \frac{3}{2} \left(1 - \frac{U}{3C} \right) U^2 = \left(1 + \frac{U}{C} + \frac{R}{C} \left(1 - \frac{U}{C} \right) \frac{d}{dt} \right) (H - H_\infty).$$

The C , U and H are, respectively, values at the bubble wall of corresponding quantities c , u and h . The H_∞ is

$$H_\infty = \int^{P_\infty} dP/\rho.$$

By numerically solving the above equation, we obtained two solid curves as shown in Fig. 1. In the contraction phase of bubble oscillation, the gas-content

may surely be subjected to adiabatic compression because of the swift contraction of the bubble. The expansion, on the other hand, may occur isothermally or adiabatically, depending on the rapidness of the process. The upper solid curve corresponds to the isothermal expansion, the lower one to the adiabatic process. The reality may lie between two curves.

One of the most striking features of Fig. 1 is that at the transition from incompressible liquid to compressible one T_{max} decreases by a few orders of magnitude. This fact unveils that our zeroth order calculation is meaningless. Secondly, there is a great discrepancy between the isothermal limit and adiabatic one, that is, the difference of two to three orders of magnitude over the indicated span of R_0 . It is noteworthy here that the isothermal curve still lies around 10^7 K when R_0 is a few microns. After all, the isothermal expansion is found to be indeed the one of necessary conditions for getting high enough T_{max} to provoke the hot fusion.

1.3 Optimization of Parameters

Remind that there exist three adjustable parameters in the present problem, that is, the initial radius of bubble, R_0 , and the amplitude and frequency of applied supersonic field, P_A and f_A . It is noteworthy here that the smaller the frequency f_A is, the more probable the occurrence of isothermal expansion is.

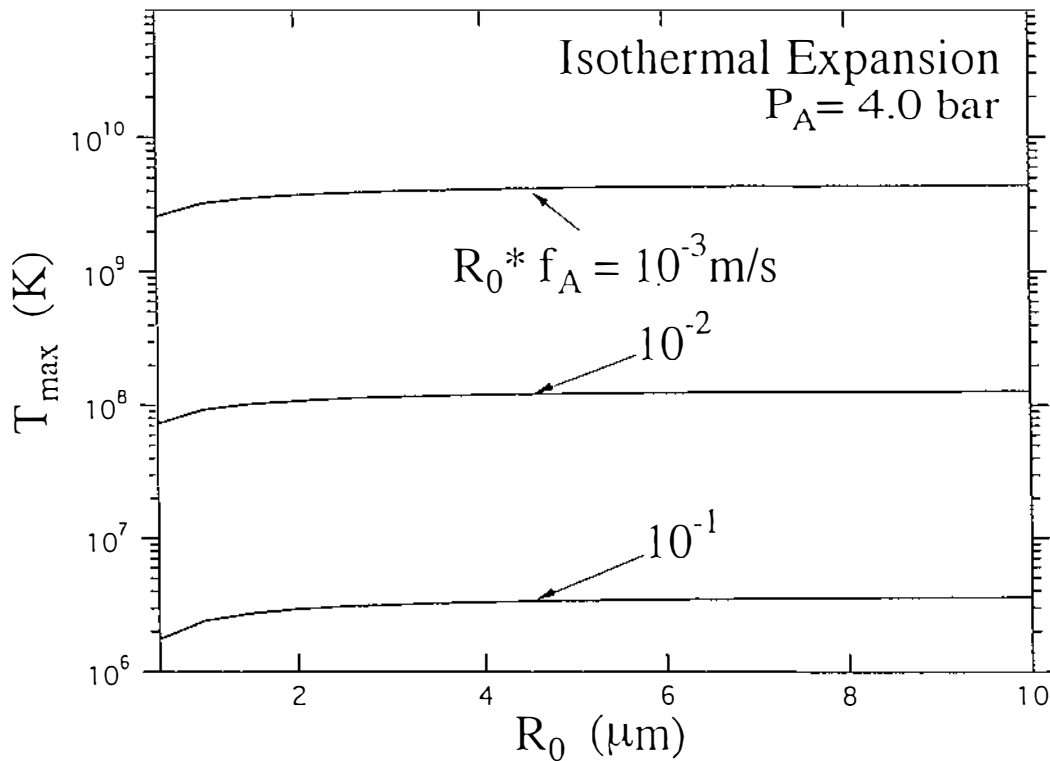


Fig. 2

Fig. 2 depicts T_{max} against R_0 for three fixed values of the product $f_A \times R_0$, where the isothermal expansion is postulated. Remarkable points of Fig. 2 are as follows: Three curves are almost parallel to the R_0 -axis, which means that T_{max} depends on f_A and R_0 only through the product $f_A \times R_0$. The second point is that, with decreasing product $f_A \times R_0$, T_{max} increases without the tendency of saturation. In short, the smaller frequency f_A which assures, on the one hand, the isothermal expansion of bubble supports the higher T_{max} simultaneously. That is unexpectedly fortunate situations for us.

The T_{max} also depends on the amplitude P_A . Fig. 3 depicts T_{max} versus R_0 for various values of P_A . With increasing P_A , T_{max} increases, but there can be seen the apparent tendency of saturation.

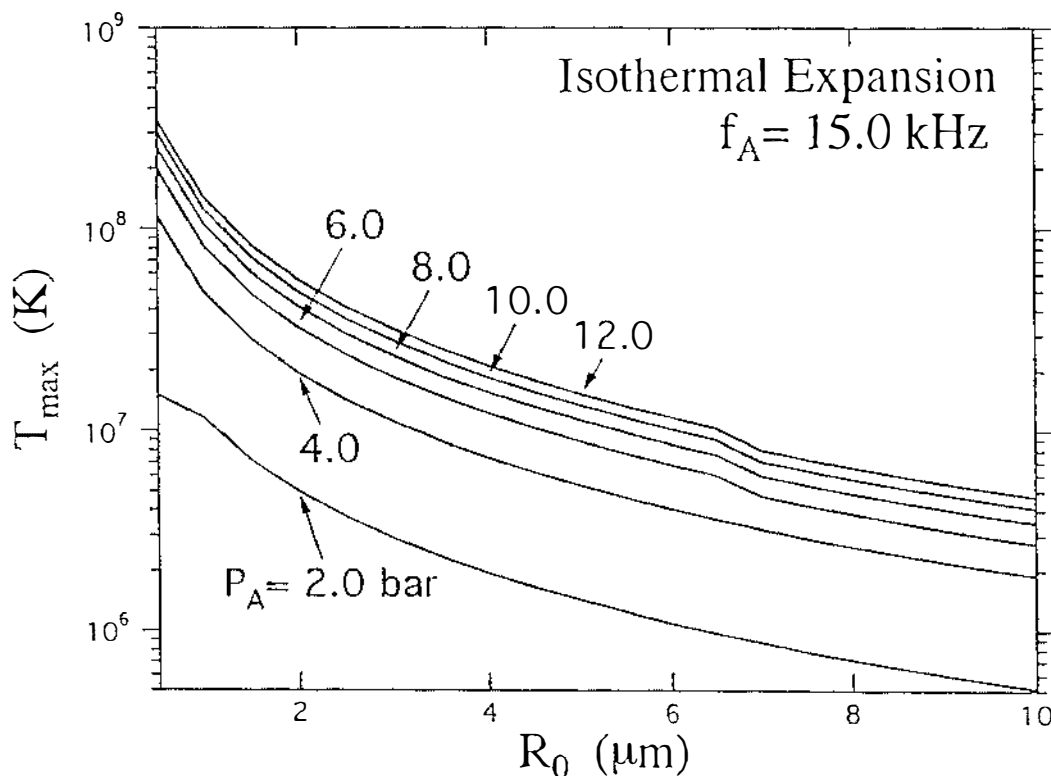


Fig. 3

2. Stability of Spherical Cavity

In this section we examine the time-evolution of a small deviation from the spherical symmetry of the bubble wall. This problem was investigated in detail by Plesset⁸⁾. Let the distance between the bubble center and the bubble wall be $R_s(\theta, \phi, t)$, which can be written as

$$R_s = R + \sum_{l>1} a_{lm} Y_{lm},$$

where R is the radius of a spherical bubble and the last term on the right hand side describes a small deviation from the spherical symmetry. The Y_{lm} is the spherical harmonics. Up to the first order of the deviation parameter a_{lm} , we can get the Gilmore equation for R and so-called Plesset equation

$$\ddot{a} + 3\frac{\dot{R}}{R}\dot{a} = \frac{l-1}{R} \left(\ddot{R} - (l+1)(l+2)\frac{\sigma}{\rho R^2} \right) a$$

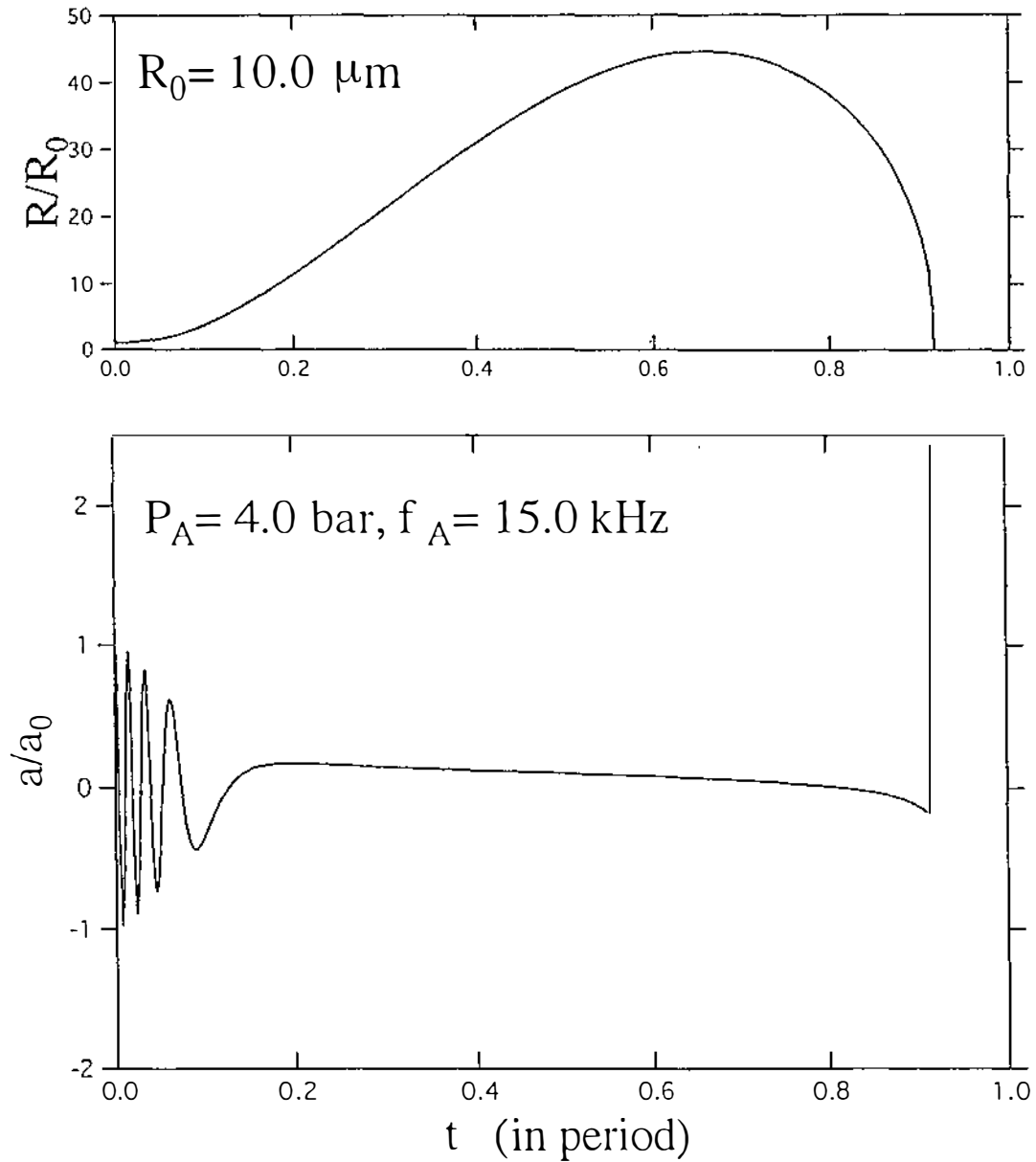


Fig. 4

Fig. 4 illustrates the time-evolution of R and a_{lm} over a period of supersonic field. That is, R starts from R_0 and goes up to the maximum value of about 40 times R_0 . It then rapidly goes down to the minimum of about 0.1 times R_0 . On the other hand, the deviation starts with its initial value a_0 , oscillates for a while and then reduces to very small values. At the final stage of the contraction, it suddenly increases up to about 2.4 times a_0 . In any way, the deviation remains finite and therefore the bubble is proven to be stable against a small deviation from the spherical symmetry.

In Fig. 4, only the $l = 2$ case is depicted. General case will be given in another place. In addition the spacial uniformity of the pressure field is assumed in our calculation. On the other hand, it is well-known experimentally that the cavity is unstable near solid surface, that is, in the presence of spatial non-uniformity of pressure field. It is interesting, therefore, to examine the stability of cavities against a small spatial variation of the pressure field.

3. Conclusion and Discussions

We calculated the maximum temperature T_{max} of a hot spot which is induced by an applied supersonic field. It is proven that the smaller the frequency f_A of the supersonic field is, the higher T_{max} can be obtained, which is indeed high enough for the occurrence of hot fusion. Especially without apparent tendency of saturation. Furthermore, with increasing amplitude P_A of the supersonic field the T_{max} increases with apparent saturation tendency. Finally, the stability of spherical cavity is verified.

In our calculation, however, a lot of simplifications are made and thereby our conclusion is still tentative and should not be laterally accepted. The following is the list of simplifying assumptions we have made: "We considered a gas-filled cavity but completely neglected (1) the evaporation and condensation of vapor. Next in spite of the formation of a hot spot of high temperature and density, (2) the thermal irradiation, (3) heat conduction and (4) mass diffusion from the hot spot to surrounding liquid are not taken into account. Furthermore, the gas-content of a supersonic cavity should undergo (5) the phase transition and execute (6) the nonadiabatic response such as the formation of shock wave.

In order for getting quantitatively acceptable results, we should engage ourself in the calculation of T_{max} by taking account of the above effects and then we should identify the optimum experimental conditions for realizing the upper bound of T_{max} .

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List of Symbols

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|-------------------------------------|---|
| a_{lm}, a = Deviation from Sphere | $B = 3,000 \text{ atm}$ |
| c = Sound Velocity of Liquid | C = Value of c at Bubble Wall(B.W.) |
| f_A = Frequency of ultrasound | h = Enthalpy of Liquid |
| H = Value of h at B.W. | H_∞ = Value of h at Remote Point(R.P.) |
| n = Index in EOS of Liquid | P = Pressure of Liquid |
| P_A = Amplitude of ultrasound | P_0 = Ambient Pressure of Liquid |
| P_∞ = Pressure at R.P. | P_G = Pressure of Gas at B.W. |
| P_L = Pressure of Liquid at B.W. | r = Distance from Origin |
| R = Radius of Bubble | R_0 = initial value of R |
| ρ = Density of Liquid | ρ_0 = Value of ρ at R.P. |
| σ = Surface Tension | t = Time |
| T = Temperature | T_{max} = Max. Temp. of Hot Spot |
| u = Velocity of Liquid Particle | U = Value of u at B.W. |

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