Sonofusion
Compressibility of Liquid and Stability of Spherical Cavity

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Abstract
We proposed the sonofusion at ICCF3 and theoretically predicted by use
of a simplified model that the temperature of gas within a supersonic cavity
reaches more than $10^8 K$ if the initial radius of the cavity is less than $10 \mu m$, that
is, temperatures high enough for the occurrence of hot fusion.

In the present paper we consider a more realistic model by taking into
account the compressibility of liquid and search for the optimum values of su-
personic parameters for getting high gas-temperature. In addition the stability
of a spherical cavity is examined.

1. Maximum Temperature of Hot Spot
As well known, a supersonic pressure field applied to a liquid induces the
forced oscillation of cavities. In the contraction phase of the oscillation the gas
content is greatly, adiabatically compressed and as a result a small region of high
temperature and density is transiently created in the liquid. The direct evidence
for the hot spot is provided through the sonoluminescence phenomenon.

Many experimentalists\textsuperscript{1,2}) succeeded in directly measuring the hot-spot
temperature and nowadays it is widely believed that the temperature may reach
one hundred thousand Kelvin. On the other hand, we had been interested in the
possibility of other catalysts of nuclear fusion than muon particles. After so-
called cold fusion, we have focused on the question of what extreme states could
be realized under ambient conditions and have engaged in the determination of
the upper bound of hot-spot temperatures.\textsuperscript{3-5})

1.1 Zeroth Order Approximation
At first we assumed a gas-filled, spherical bubble and incompressible li-
quid. Moreover, the gas content was assumed to adiabatically respond to the
motion of bubble wall. The time-evolution of the bubble radius $R$ is, therefore,
subjected to the Rayleigh-Plesset equation

\[ R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{1}{\rho} (P_L - P_\infty), \]

where

\[ P_L = P_G - 2\sigma/R, \quad P_\infty = P_0 - P_A \sin(2\pi f_A t). \]

The \( \rho \) and \( \sigma \) are the density of liquid and surface tension. The \( P_0 \) is the ambient pressure of liquid, \( P_L \), \( P_G \) and \( P_\infty \) being pressures of liquid and gas at the bubble wall and pressure of liquid at a remote point from the bubble center, respectively. The gas is assumed to obey van der Waals' equation of states. The second term on the right hand side of the second equation stands for an applied supersonic field with amplitude \( P_A \) and frequency \( f_A \).

The initial condition is

\[ R(0) = R_0, \quad \dot{R}(0) = 0. \]

That is, at \( t = 0 \), the bubble with radius \( R_0 \) is assumed to be in a dynamical equilibrium with the ambient pressure \( P_0 \).

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**Fig. 1**

\[ P_A = 4.0 \text{ bar, } f_A = 15.0 \text{ kHz} \]

- **Isothermal Limit**
- **Adiabatic Limit**
We numerically solved the Rayleigh-Plesset equation and obtained the result depicted in Fig. 1 by a broken line. The values of supersonic parameters employed are also indicated. It was surprising that the maximum temperature $T_{max}$ of hot spot exceeded $10^8$ Kelvin when $R_0$ was less than 10 microns. This fact suggested the occurrence of hot fusion within a supersonic cavity if the gas content is $D_2$. The preliminary calculation was so successful that we turned to a more realistic model.

1.2 Compressible Liquid

In the present paper we treat the compressible liquid, whose equation of states is assumed to take the form of

$$\frac{P + B}{P_0 + B} = \left(\frac{\rho}{\rho_0}\right)^n,$$

where

$$B = 3,000 \text{ atm}, \quad n \sim 7.0.$$

Namely, the liquid behaves like an incompressible one when $P \ll B$. The index $n$ is chosen so as to reproduce the sound velocity of water, $1.45 \text{ km/s}$.

By use of the equations of continuity and of motion, and assuming the Kirkwood-Bethe hypothesis

$$\left(\frac{\partial}{\partial t} + (c + u) \frac{\partial}{\partial r}\right) \left(\frac{u^2}{2} + h\right) = 0,$$

where $c$ and $u$ are velocities of sound and liquid particle, respectively, and $h$ is the enthalpy defined as

$$h = \int dP/\rho,$$

we can derive the Gilmore equation$^{6-8}$

$$R\left(1 - \frac{U}{C}\right) \dot{U} + \frac{3}{2} \left(1 - \frac{U}{3C}\right) U^2 = \left(1 + \frac{U}{C} + \frac{R}{C}\left(1 - \frac{U}{C}\right) \frac{d}{dt}\right) \left(H - H_\infty\right).$$

The $C$, $U$ and $H$ are, respectively, values at the bubble wall of corresponding quantities $c$, $u$ and $h$. The $H_\infty$ is

$$H_\infty = \int dP/\rho.$$

By numerically solving the above equation, we obtained two solid curves as shown in Fig. 1. In the contraction phase of bubble oscillation, the gas-content
may surely be subjected to adiabatic compression because of the swift contraction of the bubble. The expansion, on the other hand, may occur isothermally or adiabatically, depending on the rapidness of the process. The upper solid curve corresponds to the isothermal expansion, the lower one to the adiabatic process. The reality may lie between two curves.

One of the most striking features of Fig. 1 is that at the transition from incompressible liquid to compressible one $T_{\text{max}}$ decreases by a few orders of magnitude. This fact unveils that our zeroth order calculation is meaningless. Secondly, there is a great discrepancy between the isothermal limit and adiabatic one, that is, the difference of two to three orders of magnitude over the indicated span of $R_0$. It is noteworthy here that the isothermal curve still lies around $10^7$ K when $R_0$ is a few microns. After all, the isothermal expansion is found to be indeed the one of necessary conditions for getting high enough $T_{\text{max}}$ to provoke the hot fusion.

### 1.3 Optimization of Parameters

Remind that there exist three adjustable parameters in the present problem, that is, the initial radius of bubble, $R_0$, and the amplitude and frequency of applied supersonic field, $P_A$ and $f_A$. It is noteworthy here that the smaller the frequency $f_A$ is, the more probable the occurrence of isothermal expansion is.

![Figure 2: Isothermal Expansion](image)
Fig. 2 depicts $T_{\text{max}}$ against $R_0$ for three fixed values of the product $f_A \times R_0$, where the isothermal expansion is postulated. Remarkable points of Fig. 2 are as follows: Three curves are almost parallel to the $R_0$-axis, which means that $T_{\text{max}}$ depends on $f_A$ and $R_0$ only through the product $f_A \times R_0$. The second point is that, with decreasing product $f_A \times R_0$, $T_{\text{max}}$ increases without the tendency of saturation. In short, the smaller frequency $f_A$ which assures, on the one hand, the isothermal expansion of bubble supports the higher $T_{\text{max}}$ simultaneously. That is unexpectedly fortunate situations for us.

The $T_{\text{max}}$ also depends on the amplitude $P_A$. Fig. 3 depicts $T_{\text{max}}$ versus $R_0$ for various values of $P_A$. With increasing $P_A$, $T_{\text{max}}$ increases, but there can be seen the apparent tendency of saturation.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Isothermal Expansion $f_A = 15.0$ kHz}
\end{figure}

2. Stability of Spherical Cavity

In this section we examine the time-evolution of a small deviation from the spherical symmetry of the bubble wall. This problem was investigated in detail by Plesset\(^8\). Let the distance between the bubble center and the bubble wall be $R_s(\theta, \phi, t)$, which can be written as

$$R_s = R + \sum_{l>1} a_{lm} Y_{lm}$$
where \( R \) is the radius of a spherical bubble and the last term on the right hand side describes a small deviation from the spherical symmetry. The \( Y_{lm} \) is the spherical harmonics. Up to the first order of the deviation parameter \( a_{lm} \), we can get the Gilmore equation for \( R \) and so-called Plesset equation

\[
\ddot{a} + 3 \frac{\dot{R}}{R} \dot{a} = \frac{l - 1}{R} \left( \frac{\ddot{R} - (l + 1)(l + 2) \frac{\sigma}{\rho R^2}}{R} \right) a
\]

\[ R_0 = 10.0 \ \mu m \]

\[ P_A = 4.0 \ \text{bar}, \ f_A = 15.0 \ \text{kHz} \]

Fig. 4
Fig. 4 illustrates the time-evolution of $R$ and $a_{lin}$ over a period of supersonic field. That is, $R$ starts from $R_0$ and goes up to the maximum value of about 40 times $R_0$. It then rapidly goes down to the minimum of about 0.1 times $R_0$. On the other hand, the deviation starts with its initial value $a_0$, oscillates for a while and then reduces to very small values. At the final stage of the contraction, it suddenly increases up to about 2.4 times $a_0$. In any way, the deviation remains finite and therefore the bubble is proven to be stable against a small deviation from the spherical symmetry.

In Fig. 4, only the $l = 2$ case is depicted. General case will be given in another place. In addition the spacial uniformity of the pressure field is assumed in our calculation. On the other hand, it is well-known experimentally that the cavity is unstable near solid surface, that is, in the presence of spatial non-uniformity of pressure field. It is interesting, therefore, to examine the stability of cavities against a small spatial variation of the pressure field.

3. Conclusion and Discussions

We calculated the maximum temperature $T_{max}$ of a hot spot which is induced by an applied supersonic field. It is proven that the smaller the frequency $f_A$ of the supersonic field is, the higher $T_{max}$ can be obtained, which is indeed high enough for the occurrence of hot fusion. Especially without apparent tendency of saturation. Furthermore, with increasing amplitude $P_A$ of the supersonic field the $T_{max}$ increases with apparent saturation tendency. Finally, the stability of spherical cavity is verified.

In our calculation, however, a lot of simplifications are made and thereby our conclusion is still tentative and should not be laterally accepted. The following is the list of simplifying assumptions we have made: "We considered a gas-filled cavity but completely neglected (1) the evaporation and condensation of vapor. Next in spite of the formation of a hot spot of high temperature and density, (2) the thermal irradiation, (3) heat conduction and (4) mass diffusion from the hot spot to surrounding liquid are not taken into account. Furthermore, the gas-content of a supersonic cavity should undergo (5) the phase transition and execute (6) the nonadiabatic response such as the formation of shock wave.

In order for getting quantitatively acceptable results, we should engage ourself in the calculation of $T_{max}$ by taking account of the above effects and then we should identify the optimum experimental conditions for realizing the upper bound of $T_{max}$. 
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List of Symbols

- \(a_{lm}, a\) = Deviation from Sphere
- \(c\) = Sound Velocity of Liquid
- \(f_A\) = Frequency of ultrasound
- \(H\) = Value of \(h\) at B.W.
- \(n\) = Index in EOS of Liquid
- \(P_A\) = Amplitude of ultrasound
- \(P_L\) = Pressure of Liquid at B.W.
- \(R\) = Radius of Bubble
- \(\rho\) = Density of Liquid
- \(\sigma\) = Surface Tension
- \(T\) = Temperature
- \(u\) = Velocity of Liquid Particle

\(B = 3,000\ atm\)
\(C = \text{Value of } c \text{ at Bubble Wall (B.W.)}\)
\(h = \text{Enthalpy of Liquid}\)
\(H_\infty = \text{Value of } h \text{ at Remote Point (R.P.)}\)
\(P = \text{Pressure of Liquid}\)
\(P_0 = \text{Ambient Pressure of Liquid}\)
\(P_G = \text{Pressure of Gas at B.W.}\)
\(r = \text{Distance from Origin}\)
\(R_0 = \text{Initial value of } R\)
\(\rho_0 = \text{Value of } \rho \text{ at R.P.}\)
\(t = \text{Time}\)
\(T_{\text{max}} = \text{Max. Temp. of Hot Spot}\)
\(U = \text{Value of } u \text{ at B.W.}\)

References

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