J. Condensed Matter Nucl. Sci. 24 (2017) 60–74

Research Article

LENR Theory Requires a Proper Understanding of Nuclear Structure

Norman D. Cook*

Department of Informatics, Kansai University, Osaka, Japan

Abstract

Deciphering the many puzzles of low-energy nuclear reactions (LENR) will require not only knowledge of the electromagnetic and chemical environment of the nuclear species that are susceptible to low-energy excitation, but also a detailed understanding of the many-body problem of the atomic nucleus itself. Today most LENR research focuses on the former issues, while nuclear structure theory remains the same conundrum that it has been for 80 years. The essence of the problem is that there has been no self-consistent *spatiotemporal* description of nuclear structure – and, as a consequence, no reliable framework within which to address the oldest and most outstanding problem in nuclear physics: the nuclear force. The unanswered question in traditional nuclear physics – and the new challenge in LENR – is how to reconcile the short-range nuclear force *known* from nucleon scattering experiments with the wealth of empirical data on nuclear structure. Despite problems due to the early rejection of spatiotemporal explanations in nuclear physics, 21st century computational power has opened the door to realistic "molecular dynamics"-style simulations of nuclear structure. Some implications both for LENR and for nuclear physics are discussed.
 ISSN 2227-3123

Keywords: Lattice simulations, Low-energy nuclear reactions, Nuclear structure theory, Realistic nuclear potentials

1. Introduction

The philosophical questions raised by quantum theory in the 1920s are issues that virtually nobody interested in empirical research wants to revisit. Various positions have been developed, debated, "proven" and "disproven" countless times – to the full satisfaction of no one (see the 16 "common" interpretations of quantum mechanics, as well as the 12 "minority" interpretations in Wikipedia). There is, consequently, little merit in discussing the phenomena of LENR on the basis of inherently controversial preconceptions about quantum philosophy.

It is nevertheless worth stating the historical fact that, *prior* to the beginnings of nuclear theory, uncompromising philosophical positions concerning "the correct" interpretation of quantum theory were already in place. For example, in 1925 Niels Bohr wrote that "In contrast to ordinary mechanics, the new *quantum mechanics does not deal with a space–time description* of the motion of atomic particles" (emphasis added) [1], see also [2]. Physics without space–time!?! It is no surprise that vociferous debates ensued – focusing on whether or not there is a meaningful distinction between classical mechanics and the newer quantum mechanics of the atom.

*E-mail: cook@res.kutc.kansai-u.ac.jp

 ISSN 2227-3123

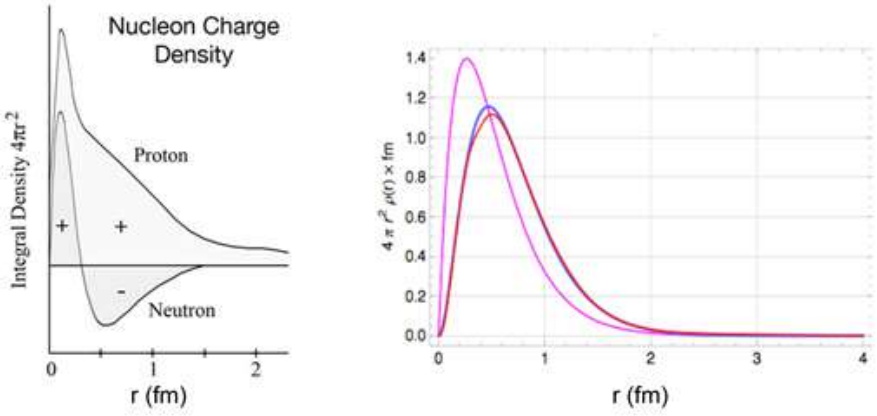

Figure 1. (*Left*) The charge densities of the proton and neutron [3]. Most of the proton charge is found within $r < 1.0$ fm; the neutron has a positively charged core ($r < 0.3$ fm) with a negatively charged surround ($r = 0.3$ to 1.0 fm). (*Right*) Various proton charge distributions obtained from multi-parameter models of the proton. Again, most of the proton charge is found at distances less than 1.0 fm from the nucleon center.

Bohr's intuitions have influenced many theorists over the past century, but it is relevant for modern researchers in LENR to realize that he reached his conclusions well *before* the discovery of the neutron (1932) and with *no knowledge* of the many developments in *nuclear* physics subsequent to 1935. Without casting doubts on Bohr's contributions to atomic physics (i.e., the *electron* structure of the atom), his deeply counter-intuitive philosophy of the quantum world was formulated prior to the advent of nuclear physics. Following Bohr, others soon added their own versions of non-spatiotemporal dynamics (Pauli's exclusion principle in 1925 and Heisenberg's uncertainty principle in 1927). Although Planck, Einstein, Schrödinger, and many others maintained that less radical interpretations were possible, today the rejection of *spatiotemporal* explanations in quantum physics, in general, and in nuclear structure theory, in particular, remains a popular, if dubious, stance.

2. The Spatiotemporal Dimensions of the Atomic Nucleus

2.1. Nucleons

On the basis of scattering experiments done at relatively low energies (<10 MeV), protons and neutrons are known to have properties indicative of particles of finite size. Their electrostatic and magnetic field radii have been measured many times since the 1950s, starting with the work of Hofstadter (Nobel Prize, 1961) and indicating an RMS charge radius for the proton of less than one fermi (Figs. 1 and 2, Tables 1 and 2). Today, the internal (quark? parton?) substructure of the lone nucleon is far from clear, but an electromagnetic radius of 0.8–0.9 fm for both protons and neutrons is well established. Note that the recent controversy concerning the 5% difference between the charge radius of the proton when measured using muons (0.84 fm) vs. electrons (0.88 fm) does not affect the conclusion that, at the relatively low energies of nuclear structure physics, the nucleon is a space-occupying particle with a rather large, impenetrable core ($R \sim 0.5$ fm) and a cut-off radius at about 1–2 fm.

2.2. Nuclear radii

Also dating from Hofstadter's work in the 1950s, the nuclear charge radii, the density of nucleons in the nuclear core, and the thickness of the so-called nuclear skin are well known [6]. Density values that are typically cited in nuclear

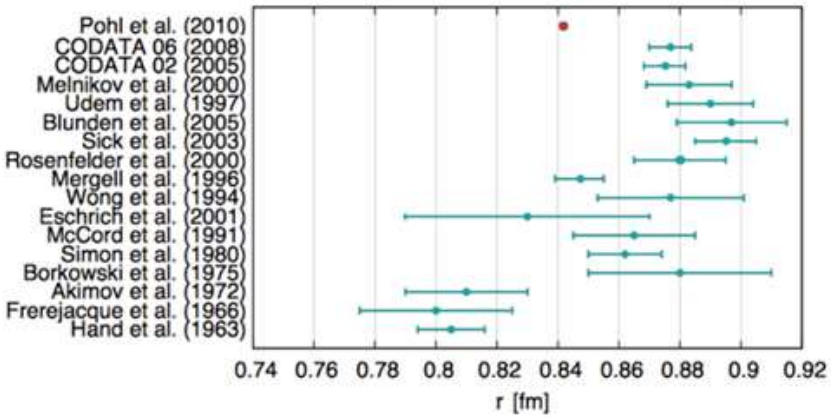

Figure 2. A summary of proton charge radii from muon (*red*) and electron (*blue*) scattering experiments [4].

Table 1. Summary of modern measurements of the proton electric and magnetic radii [4].

Electric and magnetic radii of protons	r_{E} (fm)	r_{M} (fm)
Mainz updated	0.8750(150)	0.799(28)
World updated	0.8810(110)	0.867(20)
Naïve global average	0.8790(90)	0.844(16)
Suggested global average	0.8790(110)	0.844(38)

textbooks range from 0.13 to 0.17 nucleons/fm^3, and are illustrated in Fig. 3. With the exception of ^{4}He, all nuclei exhibit a more-or-less constant core nucleon density, which is indication of the dense "packing" of nucleons into a small volume under the influence of the strong force. The slightly lower core density of the larger nuclei is presumably a consequence of the mutual repulsion of the many positive charges packed into the nuclear volume. However that may be interpreted, the mean charge (Z) density ($\sim$ 0.08 protons/fm^3) can be used to calculate the mean nucleon ($A = Z + N$) mass density of $\sim$ 0.16 nucleons/fm^3 under the assumption that most nuclei ($Z \sim N$) have similar distributions of protons and neutrons.

Two especially interesting aspects of nuclear density are illustrated in Fig. 3. The first is the anomalously high density of the alpha particle itself. It contains one spin-up and one spin-down nucleon for both the so-called isospin varieties, protons and neutrons − and, as such, should be a good representative of so-called "nuclear matter" ($Z = N$). Also yet, the nuclear matter of all other nuclei has a core density that is only half that of the alpha particle (Fig. 3a). This structural feature is precisely what is implied in a "close-packed" lattice model with an extremely high-density tetrahedron of nucleons forming the lowest energy ^{4}He isotope at the nuclear center (see below), but remains

Table 2. Summary of the electric and magnetic radii of both protons and neutrons [5].

Particle	Electric rms radii (fm)	Magnetic rms radii (fm)
Proton	0.895±0.018	1.086±0.012
Neutron	–0.113±0.003	0.873±0.015

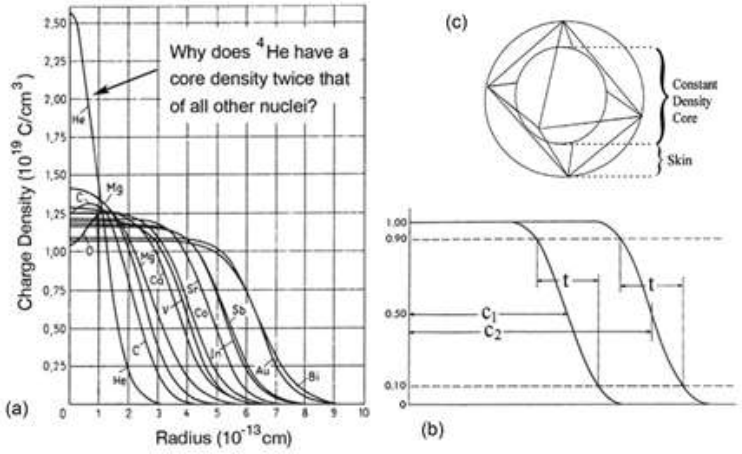

Figure 3. (a) The nuclear core density is more-or-less constant across the periodic chart. (b) The skin thickness (t) is also constant. (c) The approximately octahedral shape of most nuclei ($A > 16$) implies a "skin" region where the mean density gradually decreases from the core (data from Hofstadter [6]; reproduced from [7]).

anomalous in a simple-cubic packing (scp) lattice, and is totally unexplained in both liquid-phase and gaseous-phase nuclear models.

The second "anomaly" is the nuclear "skin" region (Fig. 3b) – which, regardless of nuclear radius, exhibits a gradual fall off of density from the core to the nuclear surface. Clearly, a billiard ball-like constant-density sphere (solid, liquid or gas) would exhibit a sharp drop in density from the core to the surface (i.e., no skin), but it is noteworthy that a constant-density, triaxially symmetrical nucleus (i.e., an octahedron with $x = y = z$) (Fig. 3c) would exhibit a gradual decrease from the constant-density core to the apices of the octahedron. In other words, the low-density nuclear skin is consistent with a solid-phase model of nuclear structure, but is anomalous in a spherical liquid-drop model and requires additional assumptions in a gaseous-phase model.

Of importance for understanding the spatiotemporal characteristics of nuclei are the experimental radial measurements that have been obtained from 908 isotopes [8]. As shown in Fig. 4, there is a linear increase in nuclear volume

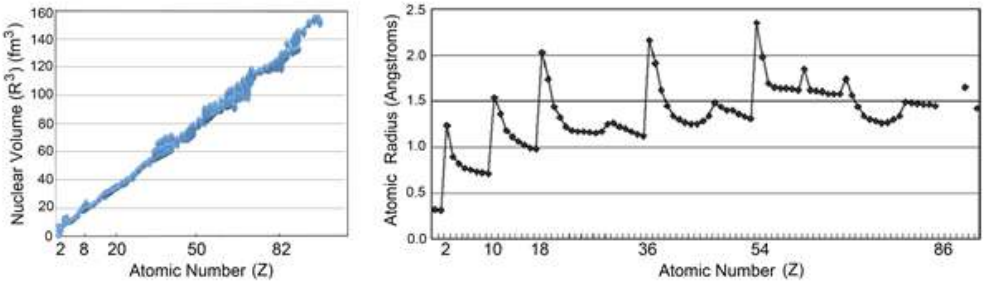

Figure 4. A comparison of nuclear and atomic radii. (*Left*) A linear relation is found when the nuclear volume is plotted against the number of protons (Z), with only minimal indication of shell structure. *Nuclear* size is strongly dependent on the number of nucleons present – a result to be expected from a constant-density substance (data on 908 isotopes from [8]). (*Right*) In contrast, the covalent radius of atoms shows marked changes – with minima evident at shell closures, and huge jumps when the external valence shell contains one or a few extra electrons [7].

that correlates strongly with the number of protons ($R = 0.995$). In detail, there are interesting isotopic effects that indicate slightly different contributions of protons and neutrons, as well as small shell and subshell effects. The linear trend for *nuclear* radii, however, contrasts sharply with the huge variation for covalent (or ionic) *atomic* radii when plotted against Z. That is, the linear increase in *nuclear* charge radii with increasing Z is precisely what would be expected from a constant-density nuclear liquid or solid, whereas the large variations in *atomic* radii are dependent on the closure of electron shells surrounded by external valence electrons – a rather malleable electron "gas" that determines the covalent (or ionic) radius. It is worth emphasizing that the simple fact that both systems are well described by quantum mechanics does *not* imply that nucleons "orbit" within the nuclear interior. On the contrary, nuclei are structurally quite different from atoms.

What do these numbers concerning the nucleon radius and the nuclear texture imply about the local environment of nucleons in the nuclear interior? The long-and-short conclusion is that nuclei are quite literally filled with nucleon matter. Unlike atomic structure – where electron matter occupies only a small percentage of the *atomic* volume, nucleon matter occupies well over 70% of the *nuclear* volume. In other words, unlike atoms, nuclei contain little unoccupied "free space" within which nucleons might randomly roam or "orbit". This empirical fact was already known at the advent of the so-called shell model in the early 1950s, but – in direct contradiction with the structural data – the shell model was built upon the assumption that nucleons are "point particles" that orbit for several intranuclear revolutions before they collide with other nucleons. Understandably, the debates on nuclear structure theory in the 1950s and 1960s were intense! The shell model was deeply unrealistic with regard to the spatiotemporal structure of nuclei, but it was also highly successful in predicting the J-values of virtually all ground-state isotopes and more than 500,000 excited states. As a consequence, even today textbooks struggle to explain the dilemma of "successful", but mutually contradictory, nuclear models.

Resolution of the low-density/high-density paradox requires a model that predicts the empirical J-values (like the shell model), but is based upon a realistic high-density nuclear texture (like the liquid-drop model) – and, moreover, can explain the alpha-particle substructure of many nuclei. It turns out that all of the n-shells and J-subshells of the shell model are reproduced in a very specific lattice (an antiferromagnetic face-centered-cubic [fcc] lattice with alternating proton and neutrons layers and built from a central tetrahedron). Two lattice configurations (fcc and hexagonal-close-packing, hcp) exhibit inherent tetrahedral subgrouping (alpha-particle substructure), but only the fcc lattice contains n-shells and J-subshells consistent with those of the shell model [7] (Section 5). The SCP lattice exhibits neither tetrahedral subgrouping nor any correspondence with the well-known quantum number symmetries of nucleons.

3. The Nuclear Force

The starting point for realistic "bottom-up" computations of nuclear structure is the nuclear potential obtained from nucleon–nucleon scattering experiments (Fig. 5). Of importance is the fact that the force is attractive at 0.7–3.0 fm, but strongly repulsive at 0.6 fm or less. The so-called "repulsive core" is a computational headache for theorists (and often unrealistically "softened" for computational convenience), but it bears emphasis that there is no empirical evidence suggesting that nucleons can spatially overlap with one another. Both protons and neutrons are hard-core particles (undoubtedly with internal quark substructure) whose interactions at separations of 1–3 fm (nucleon-center to nucleon-center) utterly dominate the structure of many-nucleon nuclei [9].

Theoretical work on the nuclear potential has been ongoing since the 1950s. As shown in Fig. 5, it consists of several independent components (corresponding to spin and isospin permutations), each of which requires ~ 10 variables for a total of 40–50 adjustable parameters [10] in order to reproduce "the" nuclear potential. Needless to say, a 40–50 parameter model allows for a huge range of possible nucleon-nucleon effects that cannot be decided *a priori*, but the overall shape of the nuclear potential (short-range repulsion, mid-range attraction and long-range absence of effect) must be maintained for agreement with experimental facts. Once a specific, multi-parameter model of the

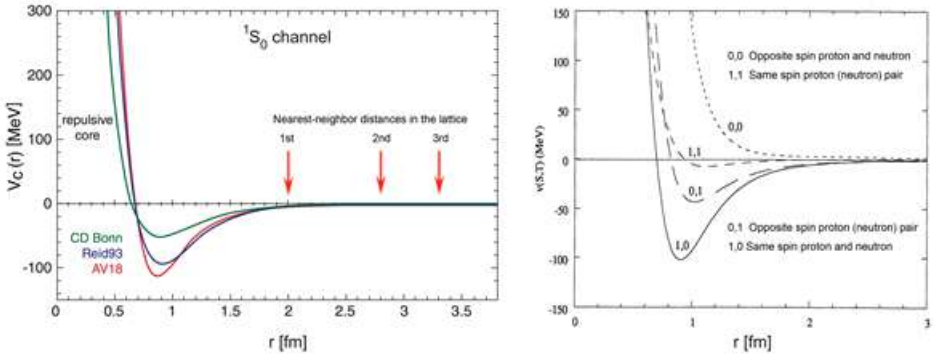

Figure 5. (*Left*) Three modern (2014) parametrizations of the nuclear potential well [10]. (*Right*) The spin and isospin components of the AV14 potential (1993) [9]. Most theoretical studies focus on the high-energy, short-range effects (>100 MeV) at distances less than 1.0 fm, but the *known* nuclear force at work in *stable* nuclei produces notably low-energy effects ($< \pm 5$ MeV) at the realistic nucleon–nucleon distances of 2.0–3.0 fm.

nuclear force has been decided on, its application to multinucleon nuclei in so-called "no-core", *ab initio* calculations becomes feasible.

Also herein lies the computational revolution of 21st century nuclear structure theory. Although the nuclear force itself remains an analytically complex mystery, the overall effect of the nuclear potential can be simulated with great precision and implemented within various nuclear models. The possibility for numerical study of nuclear binding energies and excited states, even without a rigorous analytical solution to the nuclear force itself, was of course already discussed in the early days of nuclear structure theory, but the computational reality has remained difficult for all but the smallest $(A \leq 4)$ nuclei due to the combinatorial explosion involved in calculating nucleon interactions for larger nuclei.

What now promises to revolutionize nuclear structure theory is the introduction of femto-level nuclear substructure to computational techniques. Ground-breaking work was done by Bauer [11,12] in the 1980s and subsequently by Lee [13] and others, but brought to a new level of rigor by Epelbaum, Meißner and colleagues [10,14–24] with the introduction of what they refer to as "nuclear lattice effective field theory". In the case of both liquid- and gaseous-phase models, the computational bottleneck remains insurmountable insofar as nucleon interactions must be calculated for all nucleon–nucleon permutations in all possible spatiotemporal configurations (A^{A-1}) for whatever level of sub-fermi resolution that is deemed necessary. Fortunately, that bottleneck is greatly alleviated when using a solid-phase lattice because (i) the lattice provides a finite number of first-order nuclear configurations (at the $\sim$2 fm lattice-spacing needed to reproduce nuclear core densities), and (ii) the local environment of each nucleon is necessarily determined by the lattice geometry. By restricting nuclear force calculations to nearest and next-nearest neighbors (neglecting more distant neighbors whose influence is, on theoretical grounds, thought to be minimal), the lattice provides a computationally simplified, but rigorous version of the many-body problem of nuclear structure theory.

For those skeptical about the significance of computer simulations, it is worth recalling that molecular dynamics simulations at the atomic/molecular level have already attained a practical level of sophistication in computational chemistry. For example, starting with the coordinates of the atoms contained within large membrane proteins (and the membrane lipids themselves), the mechanisms underlying ligand binding, pore opening/closing, and the flux of ions into and out of the cell has been demonstrated with atomic resolution. Such simulations are fully realistic in terms of the electromagnetic interactions among the atomic constituents. In spite of the fact that direct observation of those molecular events remains impossible (due to the constraints of the uncertainty principle), the simulated spatiotemporal dynamics accurately predict measurable properties (of, for example, ion flow through membrane channels). Note that

the distinct advantage of molecular dynamics simulations in computational chemistry, relative to the nuclear case, is *not* due to the scale (dimensions in angstroms vs. femtometers), but simply due to the fact that the *known* electromagnetic force is implemented in the molecular case, whereas the parametrization of the nuclear force is still a contentious issue.

The upshot of the basic structural findings on both nucleons and nuclei (Section 2), and the unambiguous findings on the nuclear force (Section 3) is that the overwhelmingly dominant realm of nuclear force effects in stable nuclei occurs at distances of 1–3 fermi. Again, these short distances from nucleon-center to nucleon-center are the experimental values. Based on theoretical assumptions concerning the much larger distances over which nucleons might "effectively" interact (despite empirical indications to the contrary), a huge "effective field theory" literature has come to dominate nuclear structure theory since the 1950s. Most nuclear theorists would defend effective field theory as a reasonable approach to the difficult many-body problem of the nucleus, but there is a self-fulfilling prophecy at work here. If one assumes that long-range and weak "effective" nucleon-nucleon interactions are important, then a nuclear "gas" (as assumed in the nuclear shell model, ca. 1950) is justified. If, however, one assumes a nuclear liquid (Bohr, 1935) or solid (Wigner, 1937), then the experimentally *known* short-range and strong nuclear force is justified. Both approaches appear to be justifiable, but clearly both cannot be simultaneously correct.

For connoisseurs of the history of nuclear theory, it is worth pointing out that the acceptance of the gaseous texture of the nuclear interior (seemingly an inevitable implication of the shell model) in the early 1950s was the beginning of unbridled theoretical bizarreness in microphysics – first, regarding nucleons (hard-core particles, but transparent to one another, later justified by the invention of a previously unknown "force" implied by the Pauli exclusion principle), secondly, regarding the nuclear force (experimentally short-range, but theoretically long-range, later justified by separating physical parameters into "real" and "imaginary" values), followed by the paradox of quarks (particles that cannot, in principle, be isolated) and onward into string theory and beyond. The well-established and once indubitable facts of nucleon/nuclear size and the nuclear force (Sections 2 and 3) were re-parameterized to allow for a theoretically gaseous nuclear interior, leading to a free-for-all that continues unabated into the 21st century. In contrast, as discussed in Section 5, by deriving the entire shell/subshell structure of the shell model from a particular solid-phase lattice, the necessity of unrealistic, counter-intuitive nuclear modeling disappears. Questions concerning quark substructure remain unanswered, but are safely ignored at the energy levels typical of nuclear structure theory (several MeV per nucleon).

Note that, (i) if nucleons were "point" particles – as assumed in the early shell model; or (ii) if nucleons were somehow transparent to one another – in contradiction to the extremely high-energy "impenetrable core" region of both protons and neutrons; or (iii) if the size of *nucleons* were small relative to the size of *nuclei* (in analogy with atomic physics); or (iv) if the nuclear force acted weakly up to relatively large distances ($\sim$10 fm), then nuclear matter might be accurately characterized as a low-density gaseous substance, with nucleons roaming freely with lengthy "mean free paths" within the nucleus. Molecular dynamics-style simulations would then remain computationally infeasible. The high-density structural reality of the nucleus is, however, quite the converse of a "nucleon gas", and high-resolution, computationally-intensive, spatiotemporal simulations of nuclear structure have become possible.

4. Which Lattice?

The one remaining roadblock to realistic nuclear-level simulations concerns the choice of the type of nuclear lattice. The chronic pessimist might argue that there are simply too many possibilities to embark on a coherent research project, but the results of theoretical studies in the 1960s and 1970s on the "condensation" of nuclear matter in neutron stars paint a much more optimistic picture. Specifically, under the assumption of "infinite" nuclear matter, analytical conclusions based on quantum mechanical effects have been obtained for a variety of lattice types with varying spin- and isospin-layering topologies. Those studies have examined the binding properties of the unit structures of the most common of the cube-based lattices known from crystallography, as shown in Fig. 6 and summarized Table 3.

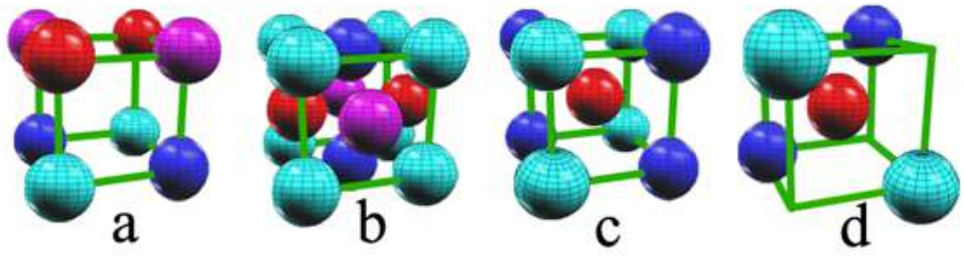

Figure 6. The geometry of four of the most common crystal lattices found in nature: (a) simple cubic packing (scp), (b) face-centered cubic (fcc) close-packing, (c) body-centered cubic (bcc) packing, and (d) diamond packing. The two varieties of nucleon, protons and neutrons, are depicted in red and blue, and the spin-up and spin-down properties are distinguished by the different shades of red and blue.

Most importantly, differences among the lattice substructures imply between 4 and 12 nearest-neighbors (i.e., the number of strong nucleon-nucleon interactions per nucleon). Insofar as nuclear force effects are already negligible at 3.0 fermi, the different numbers of nearest and second-nearest neighbors will be crucial in determining nuclear binding effects, while more distant neighbors can be ignored in first approximation. Depending upon assumptions concerning spin and isospin symmetries within the lattice, several candidate models can be enumerated. Comprehensive comparisons among all theoretical possibilities would be useful, but neutron star research has already provided rough guidelines. That is, consideration of both the Coulomb repulsion among protons and the magnetic dipole interactions between nucleons indicates that lattices with (i) isospin layering, and (ii) within each isospin layer, antiferromagnetic alignment of magnetic dipoles will produce greater nuclear binding [26]. Configurations with randomized spin and isospin (etc.) configurations may also be of interest, but whatever tentative answers are obtained from theoretical analysis, they can then be rigorously tested within the framework of "experimental theory", i.e., molecular dynamics simulations.

In conclusion, although various nuclear force parametrizations can be implemented for simulating nuclear dynamics, the remaining unresolved issue at the nuclear level is the purely spatiotemporal question of lattice geometry (lattice type and intranuclear spin/isospin symmetries). Answers are not yet in hand, but theoretical considerations dating from 1937 [27], rediscovered in 1958 [28] and developed in the 1970s–1990s [7,29–33] indicate the importance of the antiferromagnetic fcc lattice with isospin layering (Fig. 7).

Table 3. The characteristics of five of the most common crystal lattices [25].

Lattice type	fcc/bcc	bcc	scp	Diamond
Number of nearest neighbours	12	8	6	4
Nearest-neighbour distance (fm)[a]	2.0262	1.9697	1.8052	1.7896
Number of second-nearest neighbour	6	6	8	8
Second-nearest neighbor distance (fm)	2.8655	2.2744	2.5529	2.9223
Coulomb force between neighbouring protons (MeV)[b]	0.7107	0.6331	0.7977	0.4927

[a]Nearest-neighbor distance that implies a density equivalent to that of the nuclear core (0.17 nucleons fm^{-3}).
[b]Assuming structures with alternating isospin layers, the Coulomb force is that at the nearest-neighbor distance in the fcc/hcp and scp lattices, with a larger Coulomb effect in the scp lattice because of the shorter nearest-neighbor distance. In the bcc and diamond lattices, the Coulomb force is that between second-nearest neighbors (which is as close as protons come to one another in these lattices), with a stronger effect in the more compact bcc lattice.

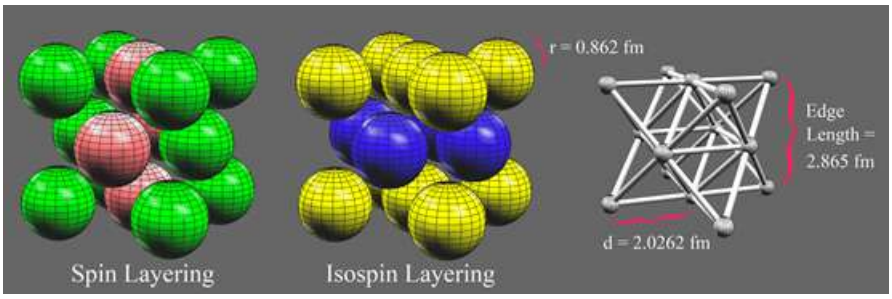

Figure 7. Illustration of the *realistic* spatial dimensions of the nuclear core. While the colors and the spherical structure of nucleons are fanciful "artistic renditions", the size of the nucleons and the density of their packing in the nuclear core are drawn using the empirical values. In the antiferromagnetic fcc nuclear lattice model, the spin and isospin layering implies attractive nearest-neighbor magnetic dipole interactions within each layer and a means for reducing Coulomb repulsion between the proton layers, respectively. The inherent tetrahedral (alpha particle) and octahedral regions in the close-packed lattice are apparent on the right. Note that the fcc "unit cube" contains eight tetrahedral spaces and a total of four octahedral spaces (one at the center of the cube and 12 octahedral quadrants on the 12 edges of the cube). This geometry gives the unit cube an overall density approximately one half that of the tetrahedron. A nearest-neighbor distance (d) of 2.026 fm implies a cube edge distance of 2.865 fm. Since each unit cube contains a total of four nucleons (six hemispheres on the cube faces and eight octants at the corners), the density of the nuclear interior (shown left and center) is $4/(2.865)^3$ nucleons per fm^3 or 0.170 n/fm^3, which is the known core density [6].

5. Nuclear Structure Theory

The foundations of the independent-particle model (IPM) were laid by Eugene Wigner [27] and later exploited in the development of the shell model by Mayer and Jensen – all three receiving Nobel Prizes for nuclear structure theory in 1963. Although Wigner clearly stated that the symmetries of the quantum numbers in the nuclear Hamiltonian had an inherent fcc structure, the spatiotemporal geometrical implications were generally neglected in the development of "non-spatiotemporal" nuclear structure theory through the 1950s and 1960s. In the recent development of computational nuclear lattice theory, spatiotemporal considerations have become inevitable. Unfortunately, most researchers have resorted to the simple cubic packing scheme (for its obvious algorithmic simplicity) and not made comparisons among different lattice configurations. Here I show that, despite the slightly more complex lattice geometry of the close-packing schemes (see the Appendix), the fcc lattice exhibits a remarkably easy-to-understand isomorphism with the conventional IPM – a geometrical fact first illustrated with a pen-and-ink freehand drawing by Wigner in *Physical Review* [27] 80 years ago!

Inevitably, the first impression of such visual depictions of the nuclear texture is one of classical mechanics, but Wigner's main argument concerned the quantal symmetries inherent to the nuclear Hamiltonian. Whatever may be the correct physical interpretation of those symmetries, they are fundamentally a consequence of the well- established quantum texture (noted by subscripts *n, l, j, m, s, i* and parity for each nucleon) that is the essence of the Schrödinger wave equation used in nuclear quantum mechanics. In many publications (e.g., [7,25,29–35]), we have shown that all of the nucleon quantum numbers have unambiguous geometrical definitions within the framework of the fcc lattice (Eqs. (1)–(7)):

$$\text{principal}, n = (|x| + |y| + |z| - 3)/2, \tag{1}$$

$$\text{orbital angular momentum}, l = (|x| + |y|)/2, \tag{2}$$

$$\text{total angular momentum}, j = (|x| + |y| - 1)/2, \tag{3}$$

$$\text{azimuthal}, m = (|x|\ (-1)^{(x-1)/2})/2, \tag{4}$$

$$\text{spin}, s = \left((-1)^{(x-1)/2}/\,2, \right. \tag{5}$$

$$\text{isospin}, i = \left((-1)^{(z\text{-}1)/2}/\,2, \right. \tag{6}$$

$$\text{parity}, \ \pi = \text{sign}(xyz), \tag{7}$$

where all quantum numbers are defined in terms of each nucleon's unique set of x, y, z coordinates in Cartesian space. *All* of the nucleon shells/subshells and their occupancies are thereby reproduced. That fact indicates that the IPM and the lattice model are fundamentally isomorphic, but they clearly differ in implying: a diffuse, gaseous nuclear interior, on the one hand, or a high-density nuclear interior where nucleon–nucleon interactions are local, on the other.

In fact, the physical interpretation of the mathematical correspondence between the symmetries of the nuclear Hamiltonian and the fcc lattice is far from obvious. Both the "orbital angular momentum", l, and the "total angular momentum, j, cannot be realistically interpreted as indicating the "orbiting" of nucleons within the extremely dense nuclear interior around a central potential-well of the nuclear force. Both angular momentum values for nucleons must therefore be understood as indicative of the orbital motion of charge within the individual nucleon itself. In fact, that assumption leads to realistic magnetic dipole moments that are dependent on nucleon j-values, as summarized by the so-called Schmidt lines of conventional nuclear structure theory, but leaves questions unanswered concerning the internal structure of the nucleons. The non-orbiting of nucleons within the nuclear volume is of course the reason why all of the details of nuclear structure physics cannot be explained simply in terms of a highly-miniaturized version of the s-, p, d-, f-orbitals for electrons in the theory of atomic structure.

Suffice it to say that, based on Eqs. (1)–(7), there is a deep correspondence between the fcc lattice and the known symmetries of the nucleus [7]. Its physical interpretation still needs the attention of nuclear structure theorists, but it is worth reiterating that nothing similar is found with other nucleon lattices. The known symmetries of nucleon quantum states – as successfully summarized in the shell model from 1950 – is reproduced in its entirety uniquely in the fcc lattice. It is for this reason that the rather sophisticated computational work currently being done in "nuclear lattice effective field theory" needs to be expanded to include a comparison of, at the very least, both scp and fcc lattices, in order to make a fair-minded, objective evaluation of candidate lattice configurations.

The dangers inherent to proceeding on the basis of unanalyzed starting assumptions is fully apparent in the nuclear lattice simulations already published by Epelbaum, Meißner and colleagues [10, 13–24]. On the one hand, they have implemented a modern version of the nuclear potential that is far more sophisticated than the nuclear force effects in earlier lattice model simulations of multi-fragmentation effects or nuclear binding energies [1,12]. On the other hand, they have examined nuclear binding effects only in the scp lattice and, moreover, using an unusual parametrization of the nuclear potential that leads to counterintuitive results (Fig. 8).

The nuclear potential used in obtaining the results shown in Fig. 8 allows multiple nucleons to reside at any given lattice site [10,13–24]. Indeed, experiments at much higher energies (>300 MeV) than are likely to occur in stable nuclei suggest that there is some degree of nucleon "transparency", but, at the low energies of nucleon–nucleon binding in nuclear structure physics (<10 MeV), nucleons typically exhibit an impenetrable "hard core" ($Q > 300$ MeV) of 0.6 fm (Fig. 5). Clearly, simulation results very different from those illustrated in Fig. 8 would be obtained from a parametrization that forbids overlap of nucleons. As Meißner and colleagues themselves have commented, "A concerted effort should be made to improve the current computational algorithms to handle interactions with more short-range repulsion" [21].

The assumption of an scp lattice is also deeply suspect. In the alpha-clustering studies by the Meißner group, they report alpha clusters (four nucleons on a given lattice site) for ^{16}O, with again counter-intuitive geometries. The lowest-energy 4-cluster configuration was reported as "tetrahedral" (Fig. 8), but is actually a quarter-pyramid, not a regular tetrahedron. The difference between these two geometries is that the quarter -pyramid has three alpha clusters

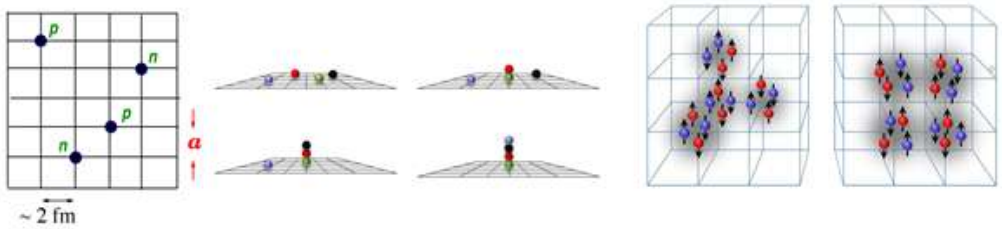

Figure 8. The "nuclear lattice effective field theory" advocated by Meißner and colleagues. (*Left and Center*) The basic dimensions of the 3D scp lattice and the occupancy of lattice sites with up to four nucleons (spin up/down protons and neutrons). A huge number of configurations of 16 nucleons distributed at random within a $6 \times 6 \times 6$ lattice volume (216 grid sites) was generated and total nuclear binding energies calculated [18]. (*Right*) The lowest-energy alpha particle structure obtained for ^{16}O was a quarter-pyramid, but was described as "tetrahedral" and the second-lowest-energy structure was square planar. Neither result requires supercomputer calculations, and both are a simple consequence of the high binding energy of four nucleons located on the same lattice site (due to the a priori assumption of a "soft" nuclear potential with no short-range repulsion)..

bound to a common vertex, but without nearest-neighbor bonds among the three remaining alphas, whereas a regular tetrahedron of alphas would have fully six nearest-neighbor alpha-alpha bonds. A regular tetrahedral configuration was not obtained in the Meißner simulation for the simple reason that an scp lattice constrains all nucleons to sites on the cubic coordinates of the lattice; there are no tetrahedral substructures. As such, the reported "reproduction" of the traditional tetrahedral alpha structure of the conventional alpha-cluster model, is utterly false. Their alpha structure is "tetrahedral" only in the sense that any non-planar configuration of four points gives a 4-vertex, 6-edge, 4-sided structure can be described as tetrahedral, but the claim for a *regular* tetrahedron requires that all six edges are of the same length.

For anyone interested in the possibility of publishing in the mainstream nuclear physics literature, it is relevant to point out that the Meißner paper [18] reporting a "tetrahedral" alpha structure for ^{16}O was published in one of the world's leading physics journals, *Physical Review Letters*. The computational result indicating a "tetrahedral" geometry is clearly misleading, but in fact most of the methodological detail has been published elsewhere – notably, in 76 (!) papers by Meißner in the *European Physical Journal G: Hadrons and Nuclei*. By chance, this is the same journal where Meißner has been the Editor responsible for "theoretical reviews" for the past 13 years [36].

Again, the counterintuitive results are a direct result of the choice of (i) an scp lattice, and (ii) a parametrization of the nuclear force that does *not* reproduce the known nucleon hard-core. Such assumptions about the lattice type may fall within the realm of plausible hypotheses, but the results obtained through computer simulations then need to be compared between (i) an fcc lattice, and (ii) an scp lattice, and most importantly (iii) utilizing parametrizations of the nuclear force that realistically reproduce the nucleon hard-core radius of 0.6 fm. Without the realism of a nuclear force that is deduced from low-energy experimental results, it is questionable why one should bother with the lattice simulations in the first place.

6. LENR

Finally, let us consider the conundrum of LENR research. The intellectual challenge of low-energy nuclear reactions has many aspects, but the clearest indications that LENR is truly nuclear come from the published works on nuclear transmutations. Notable are the papers by Tadahiko Mizuno [37], Yasuhiro Iwamura [38], George Egely [39], and the group that examined the ash obtained from Andrea Rossi's E-Cat [40]. Many other, unreplicated reports of transmutation are to be found in the literature, but they are, without exception, highly suspect from the perspective of conventional nuclear physics. The same, of course, could be said about the early reports of the fission of Uranium isotopes.

Only once the phenomenon was shown to be replicable – with reliable, repeated detection of neutron radioactivity, heat, and fission transmutation products – did there emerge a consensus among scientists that fission was real. Technological applications followed immediately. The much lower energies involved in LENR and the apparently much more delicate chemistry that produces nuclear active environments have made the phenomena harder to reproduce. When experimental LENR systems have become established, it will then be possible to focus on the low-energy dynamics of the nucleus – i.e., traditional nuclear structure physics – rather than the extremely high-energy phenomena that continue to fascinate most theorists.

Many constructive suggestions about how to demonstrate LENR phenomena and proceed to applications have been made. The most promising theoretical developments concern the production of neutrons from low-lying electron states [41,42] – leading ultimately to a diversity of low-energy nuclear reactions. Unfortunately, most proposals for experimental research imply massive funding that is unlikely to be realized until the fundamental phenomena themselves are shown to be replicable and open to detailed analysis. In contrast, a rather modest theoretical research project could be undertaken strictly within the framework of molecular dynamics-like simulations at the nuclear level. While experimentalists deal with the issues of the mechanisms for inducing LENR, quantitative simulations could, in principle, predict the likely nuclear reaction products for comparison against experimental findings.

7. Conclusions

A "return" to Wigner's ideas from 1937 [27] may appear to be "regressive" to some theorists, but it is relevant to note that theoretical progress in clarifying the so-called nuclear force has been remarkably slow since the 1930s. Authors of modern textbooks on nuclear physics are still obliged to note that admittedly imperfect, contradictory nuclear models are employed specifically *because* fundamental questions concerning the nuclear force remain unanswered. The theoretical stalemate concerning the nuclear models contrasts sharply with the remarkable advances in both experiment and practical applications of all things nuclear. Although theoretical nuclear physics is *not* totally unhinged pseudoscience, its lack of spatiotemporal grounding and the absence of a unified understanding of the nuclear force are noteworthy deficiencies. Nevertheless, despite those shortcomings, computational power has grown significantly in the past two decades and, indeed, the empirical landscape of nuclear stability, instability and excitability has been mapped out in impressive detail. As a consequence, prospects for achieving a self-consistent understanding of nuclear structure – even while using a make-shift, imperfect model of the nuclear force – have improved remarkably.

Contrary to first impressions, recasting nuclear structure theory within the lattice representation of quantal symmetries is a less radical renovation than is currently appreciated by theorists still wedded to the weak analogies and unrealistic models of the 1950s and 1960s. Principally because the fcc lattice symmetries map directly onto the known quantal symmetries of the nucleus, there is reason to believe that the present generation of computationally intensive nuclear structure simulations will produce coherent answers that finally put to rest the "many mutually contradictory models" of traditional nuclear structure theory. At the very least, it can be said today that the properties of the nuclear force *known* from nucleon-nucleon scattering experiments are consistent with the lattice perspective on nuclear structure. A return to spatiotemporal nuclear theory would be welcomed.

Finally, we can look forward to applications of lattice simulations to the problems in LENR. The transmutation data from many groups are some of the most tantalizing new findings in nuclear physics since the discovery of fission in 1938. In parallel with continuing efforts by the experimentalists, attempts at sorting out the wheat from the chaff using fermi-level molecular dynamics simulations would be useful.

Appendix. An Algorithm in C for Generating Sequential Shells of fcc Coordinates

Computational "nuclear lattice effective field theory" is arguably on track for providing numerically satisfying answers to many of the old puzzles in nuclear structure theory. The chore of comparing various lattice configurations has just

begun, but, in whatever way that such work may proceed, it will be necessary to use parametrizations of the nuclear force that reproduce the known hard-core repulsion of the nuclear force. From there, it will be of interest to make comparisons between, at the very least, various spin-, and isospin-configurations within scp and fcc lattices. Because of the algorithmic simplicity of simple-cubic packing, most previous studies have examined exclusively the scp lattice, but it is, on the contrary, uniquely the fcc lattice that exhibits interesting symmetries related to the symmetries known in traditional nuclear structure theory [7]. For that reason, the algorithm below may be useful for producing isospin-layered, antiferromagnetic fcc lattice coordinates in sequential shells containing 4,16,40,80,140,224, and 336 nucleons ($Z = N$). An Excel spreadsheet with similar results [34] and a Fortran program which includes bcc and diamond lattices can be found elsewhere [35]. See the following program.

```
#include <stdio.h>
#include "math.h"
int x, y, z, N, L, JJ, M, i, K=1, f, parity, nuc[241][3][2];
int Nvalue[241][2], Lvalue[241][2], Svalue[241][2], Jvalue[241][2], Mvalue[241][2], Pvalue[241][2];

int main(int argc, char** argv){
   printf(" N(x  y  z) N L  J   M   S  P  P(x  y  z) N L  J   M   S  P ¥n");
   for(N=0;N<=6;N=N+1){            /* N-shells up to N=6 to give 168 protons and 168 neutrons */
      for(L=N;L>=0;L=L-1){         /* L-subshells, higher L-values come first: L = N,..,2,1,0 */
         JJ=(2.0*((float)L+.5));   /* JJ is 2*j-value, J = |L+spin| = 1, 3, 5, ...            */
         for(M=-JJ;M<=JJ;M=M+2){   /* M is 2*m-value.  M = -JJ, ..., -3, -1, 1, 3, ..., JJ    */
            for(i=-1;i<=1;i=i+2){ /* isospin: neutrons = -1, protons = 1                      */
               f=((i+1)/2);       /* 0 and 1 for neutrons and protons                         */
               x=(abs)(M)*(pow(-1.0,(float)M/2.0+.5));
               y=(JJ+1.0-abs((int)x))*pow(-1.0,-i/2.0+JJ/2.0+M/2.0+.5);
               z=(2.0*N+3.0-abs((int)x)-abs((int)y))*pow(-1.0,-i/2.0+N-JJ/2.0-1.0);
               nuc[K][0][f]=(int)x;
               nuc[K][1][f]=(int)y;
               nuc[K][2][f]=(int)z;
               Nvalue[K][f]=(abs((int)x)+abs((int)y)+abs((int)z)-3)/2;     /* N is an integer */
               Lvalue[K][f]=L;                                             /* L is an integer */
               Jvalue[K][f]=(abs(nuc[K][0][f])+abs(nuc[K][1][f]))-1;       /* J is 2*j        */
               if(fmod((float)nuc[K][0][f]+1.0,4.0)==0.0) Svalue[K][f]=-1; /* -1 "spin down"  */
               else Svalue[K][f]=1;                                        /* 1 "spin up"     */
               Mvalue[K][f]=(abs(nuc[K][0][f]))*Svalue[K][f];              /* M is 2*m        */
               parity=nuc[K][0][f]*nuc[K][1][f]*nuc[K][2][f];              /* P is +1 or -1   */
               if(parity>=0) Pvalue[K][f]=1; else Pvalue[K][f]=-1;
            } /* end of isospin i-loop */

printf("%d%3d %3d %3d  %2.f %2.f %2d/2 %2d/2 %2d/2 %2d   %3d %3d %3d  %2.f %2.f %2d/2 %2d/2 %2d/2 %2d ¥n",
K, nuc[K][0][0], nuc[K][1][0], nuc[K][2][0], (float)Nvalue[K][0], (float)Lvalue[K][0], Jvalue[K][0],
Mvalue[K][0], Svalue[K][0], Pvalue[K][0], nuc[K][0][1], nuc[K][1][1], nuc[K][2][1], (float)Nvalue[K][1],
(float)Lvalue[K][1], Jvalue[K][1], Mvalue[K][1], Svalue[K][1], Pvalue[K][1]);
               K=K+1;
         } /* end of M-loop */
      } /* end of L-loop */
   } /* end of N-loop */
} /* end of main function */
```

Sample output where fcc lattice coordinates [$n(xyz)$ and $p(xyz)$] are followed by their quantum numbers.

	$N(x,y,z)$			N	L	J	M	S	P	$p(xyz)$			N	L	J	M	S	P
1	1	–1	–1	0	0	1/2	1/2	1/2	1	1	1	1	0	0	1/2	1/2	1/2	1
2	–1	1	–1	0	0	1/2	–1/2	–1/2	1	–1	–1	1	0	0	1/2	–1/2	–1/2	1
3	–3	–1	–1	1	1	3/2	3/2	1/2	–1	–3	1	1	1	1	3/2	3/2	1/2 1/2	–1
4	1	3	–1	1	1	3/2	1/2	1/2	–1	1	–3	1	1	1	3/2	1/2	1/2	–1
5	–1	–3	–1	1	1	3/2	–1/2	–1/2	–1	–1	3	1	1	1	3/2	–1/2	–1/2	–1
6	3	1	–1	1	1	3/2	–3/2	–1/2	–1	3	–1	1	1	1	3/2	–3/2	–1/2	–1
7	1	–1	3	1	0	1/2	1/2	1/2	–1	1	1	–3	1	0	1/2	1/2	1/2	–1
8	–1	1	3	1	0	1/2	–1/2	–1/2	–1	–1	–1	–3	1	0	1/2	–1/2	–1/2	–1

References

[1] N. Bohr, Atomic theory and mechanics, In *Philosophical Writings of Niels Bohr,* Vol. 1, 1925, p. 48, reprinted by Woodbridge, CN: Ox Bow Press, 1987.

[2] A. Plotnitsky, *Niels Bohr and Complementary*, Chapter 3, Springer, New York, 2013.

[3] R.M. Littauer, H.F. Schopper and R.R. Wilson, *Phy. Rev. Lett.* **7** (1961) 144.

[4] I. Sick and D. Trautmann, Proton root-mean-square radii and electron scattering, *Phy. Rev. C* **89** (2014) 012201.

[5] I. Sick, *Prog. Particle Nucl. Phys.* **55** (2005) 440.

[6] R. Hofstadter, *Rev. Modern Phys.* **28** (1956) 214.

[7] N.D. Cook, *Models of the Atomic Nucleus*, 2nd Edn., Springer, New York, 2010, freely available at: www.res.kutc.kansai-u.ac.jp/~cook/PDFs/ man2.pdf.

[8] I. Angeli and K.P. Marinova, Table of experimental nuclear ground state charge radii: An update, *Atomic Data and Nuclear Data Tables* **99** (2013) 1

[9] R.B. Wiringa, *Rev. Modern Phys.* **65** (1993) 231.

[10] U.-G. Meißner, A new tool in nuclear physics: Nuclear lattice simulations, *Nucl. Phys. News* **24** (2014) 11.

[11] W. Bauer, D.R. Dean, U. Mosel and U. Post, New approach to fragmentation reactions – The nuclear lattice model, *Phys Lett B* **150** (1985) 53

[12] W. Bauer, U. Post, D.R. Dean and U. Mosel, The nuclear lattice model of proton-induced multifragmentation, *Nucl. Phys A* **452** (1986) 699

[13] D. Lee, Lattice simulations for few- and many-body systems, arXiv:0804.3501v2 (2008).

[14] B. Borasoy, E. Epelbaum, H. Krebs, D. Lee and U.-G. Meißner, *Euro. Phys. J. A* **31** (2007) 105.

[15] E. Epelbaum, H.-W. Hammer and U.-G. Meißner, *Rev. Modern Phys.* **81** (2009) 1773.

[16] E. Epelbaum, H. Krebs, T.A. Lahde, D. Lee,U.-G. Meißner and G. Rupak, *Phy. Rev. Lett.* **112** (2014) 10.

[17] E. Epelbaum, H. Krebs, A. Lahde, D. Lee and U.-G. Meißner, *Euro. Phys. J. A* **49** (2013) 82.

[18] S. Elhatisari, N. Li, A. Rokash, J. M. Alarcon, D. Du, N. Klein, B.-N. Lu, U.-G. Meißner, E. Epelbaum, H. Krebs, T. A. Lahde, D. Lee and G. Rupak, Nuclear binding near a quantum phase transition, *Phy. Rev. Lett.* **117** (2016) 132501.

[19] E. Epelbaum H. Krebs, T.A. Lähde, D. Lee, U.-G. Meißner and G. Rupak, *Phy. Rev. Lett.* **112** (2014) 102501.

[20] U.-G. Meißner, *EPJ Web of Conferences* **66** (2014) 01012.

[21] T.A. Lähde, E. Epelbaum, H. Krebs, D. Lee, U.-G. Meißner and G. Rupak, Lattice effective field theory for medium-mass nuclei, *Phys. Lett. B* **732** (2014) 110.

[22] T.A. Lähde, T. Luu, D. Lee, U.-G. Meißner, E. Epelbaum, H. Krebs and G. Rupak, *Euro. Phys. J. A* **51** (2015) 92.

[23] S. Elhatisari, D. Lee, G. Rupak, E. Epelbaum, H. Krebs, T.A. Lähde, T. Luu and U.-G. Meißner, *Nature* **528** (2015) 111.

[24] U.-G. Meißner, T.A. Lähde and T. Luu, Pushing the boundaries of nuclear physics with lattice simulations, in, *NIC Symposium 2016*, K. Binder, M. Mueller, M. Kremer and A. Schnurpfeil (Eds.), NIC Series, Vol. 48, 2016.

[25] N.D. Cook, *J. Phys. G: Nucl. Particle Phys.* **20** (1994) 1907.
[26] V. Canuto and S.M. Chitre, *Nature Phys. Sci.* **243** (1973) 63.
[27] E. Wigner, *Phy. Rev.* **51** (1937) 106.
[28] F. Everling, Notarized document (1958) and many references at www.everling-nuclear-lattice-model.com/ (2016).
[29] K.J. Lezuo, *Atomkernenergie* **23** (1974) 285.
[30] N.D. Cook, *Atomkernenergie* **28** (1976) 195.
[31] V. Dallacasa, *Atomkernenergie* **31** (1981) 143.
[32] N.D. Cook and V. Dallacasa, *Phy. Rev. C* **36** (1987) 1883.
[33] N.D. Cook, Is the lattice gas model a unified model of nuclear structure? *J. Phys. G* **25** (1999) 1213.
[34] N.D. Cook, available at: http://www.res.kutc.kansai-u.ac.jp/~cook/QuantumNumbers.xls.
[35] N.D. Cook, Computing nuclear properties in the FCC model, *Computers Phys.* **3** (1989) 73, available at: http://www.res.kutc.kansai-u.ac.jp/~cook/PDFs/CompPhys.pdf.
[36] U-.G. Meißner, https://www.itkp.uni-bonn.de/~meissner/cvr.pdf.
[37] T. Ohmori and T. Mizuno, Nuclear transmutation reaction caused by light water electrolysis on tungsten cathode under incandescent conditions, *Infinite Energy* **5** (1999) 27.
[38] Y. Iwamura, T. Itoh and S. Tsuruga, Transmutation reactions induced by deuterium permeation through nano-structured Pd multilayer thin film, *Current Sci.* **108** (2015) 628.
[39] G. Egely, Nanodust fusion, *Infinite Energy* **11** (2012) 102.
[40] G. Levi, E. Foschi, T. Hartman, B. Höistad, R. Pettersson, L. Tegnér and H. Essén, Indication of anomalous heat energy production in a reactor device, arXiv:1305.3913v3 (2013).
[41] J.L.Paillet and A. Meulenberg, Special relativity: the source of electron deep orbits, *Found. Phys.,* online 09/02/2017.
[42] R. Godes, http://brillouinenergy.com/science/the-quantum-reaction-hypothesis/.