J. Condensed Matter Nucl. Sci. 25 (2017) 76–99

Research Article

The Electron and Occam's Razor

Francesco Celani*

Istituto Nazionale di Fisica Nucleare (INFN-LNF), Via E. Fermi 40, 00044 Frascati, Roma, Italy

Antonino Oscar Di Tommaso†

Università degli Studi di Palermo – Department of Energy, Information Engineering and Mathematical Models (DEIM), viale delle Scienze, 90128 Palermo, Italy

Giorgio Vassallo‡

Università degli Studi di Palermo – Department of Industrial and Digital Innovation (DIID), viale delle Scienze, 90128 Palermo, Italy

Abstract

This paper introduces a *Zitterbewegung* (ZBW) model of the electron by applying the principle of Occam's razor to Maxwell's equations and by introducing a scalar component in the electromagnetic field. The aim is to explain, by using simple and intuitive concepts, the origin of the electric charge and the electromagnetic nature of mass and inertia. A ZBW model of the electron is also proposed as the best suited theoretical framework to study the structure of Ultra-Dense Deuterium (UDD), the origin of anomalous heat in metal–hydrogen systems and the possibility of existence of "super-chemical" aggregates at Compton scale.

Keywords: Compton scale aggregates, Electric charge, Elementary particles, Electron structure, LENR, Lorenz gauge, Occam's razor, Space–time algebra, Ultra-dense deuterium, Vector potential, Weyl equation, Zitterbewegung

Nomenclature **(see p. 77)**

1. Introduction

The application of Occam's razor principle to Maxwell's equations suggests a *Zitterbewegung*[a] (ZBW) interpretation of quantum mechanics [1] and a simple electromagnetic model for charge, mass and inertia. A new, particularly simple ZBW model of the electron is proposed as the best suited one to understand the structure of the Ultra-Dense Deuterium (UDD) [2,3] and the origin of Anomalous Heat in metal–hydrogen systems.

*Also at: International Society for Condensed Matter Nuclear Science (ISCMNS)-UK. E-mail: francesco.celani@lnf.infn.it.
†E-mail: antoninooscar.ditommaso@unipa.it.
‡Also at: International Society for Condensed Matter Nuclear Science (ISCMNS)-UK. E-mail: giorgio.vassallo@unipa.it.
[a] German word for "tremble" or "shaking motion".

Nomenclature

Symbol	Name	SI units	Natural units (NU)
$\boldsymbol{A}_{\square}$	Electromagnetic four-potential	V s m^{-1}	eV
$\boldsymbol{A}_{\triangle}$	Electromag. vector potential	V s m^{-1}	eV
$\boldsymbol{G}$	Electromagnetic field	V s m^{-2}	eV^2
$\boldsymbol{F}$	Electromagnetic field bivector	V s m^{-2}	eV^2
$\boldsymbol{B}$	Flux density field	V s m^{-2} =T	eV^2
$\boldsymbol{E}$	Electric field	V m^{-1}	eV^2
S	Scalar field	V s m^{-2}	eV^2
$\boldsymbol{J}_{\square e}$	Four-current density field	A m^{-2}	eV^3
$\boldsymbol{J}_{\triangle}$	Current density field	A m^{-2}	eV^3
$\boldsymbol{v}_{\square}$	Four-velocity vector	m s^{-1}	1
$\boldsymbol{v}_{\triangle}$	Velocity vector	m s^{-1}	1
ρ	Charge density	A s m^{-3} = C m^{-3}	eV^3
x, y, z	Space coordinates	m (1.97327×10^{-7} m $\approx$ 1 eV^{-1})	eV^{-1}
t	Time variable	s ($6.5821220 \times 10^{-16}$ s $\approx$ 1 eV^{-1})	eV^{-1}
c	Light speed in vacuum	2.99792458×10^8 m s^{-1}	1
$\hbar$	Reduced Planck constant	$1.054571726 \times 10^{-34}$ J s	1
μ_0	Permeability of vacuum	$4\pi \times 10^{-7}$ V s A^{-1} m^{-1}	4π
ϵ_0	Dielectric constant of vacuum	$8.854187817 \times 10^{-12}$ C (V m)$^{-1}$	$1/4\pi$
e	Electron charge	$1.602176565 \times 10^{-19}$ A s	0.085424546
m_e	Electron mass at rest	9.109384×10^{-31} kg	0.510998946×10^6 eV
λ_c	Electron Compton wavelength	$2.4263102389 \times 10^{-12}$ m	$1.229588259 \times 10^{-5}$ eV^{-1}
$\boldsymbol{P}_{\square}$	Energy-momentum four-vector	kg m s^{-1}	eV
$\boldsymbol{P}_{\triangle}$	Momentum vector	kg m s^{-1}	eV
U, W	Energy	J = kg m^2 s^{-2}	eV

One of the most detailed and interesting ZBW electron models has been proposed by David Hestenes, emeritus of Arizona State University. He rewrote the Dirac equation for the electron using the four dimensional real Clifford algebra $Cl_{1,3}(\mathbb{R})$ of space–time with Minkowski signature "$+ - - -$", eliminating unnecessary complexities and redundancies arising from the traditional use of matrices. The Dirac gamma matrices γ_μ and the associated algebra can be seen as an isomorphism of the four-basis vector of space–time geometric algebra. This simple isomorphism allows a full encoding of the geometric properties of the Dirac algebra, and a rewriting of Dirac equation that does not require complex numbers or matrix algebra. In this context the wave function ψ is characterized by the eight real values of the even grade multivectors of space–time algebra $Cl_{1,3}$ (STA). Even grade multivectors of STA can encode ordinary rotations as well as Lorentz transformations in the six planes of the space–time. Hestenes associates the rotations encoded by the wave function with an intrinsic very rapid rotation of the electron, the ZBW, that is considered at origin of the electron spin and magnetic moment. The word Zitterbewegung was originally used by Schrödinger to indicate a fast movement attributed to an hypothetical interference between "positive" and "negative" energy states. Kerson Huang later, more realistically, interpreted the ZBW as a circular motion [4].

In particular, B. Sidharth states that *"The well-known Zitterbewegung may be looked upon as a circular motion* *about the direction of the electron spin with radius equal to the Compton wavelength (divided by* 2π*) of the electron. The intrinsic spin of the electron may be looked upon as the orbital angular momentum of this motion. The current produced by the Zitterbewegung is seen to give rise to the intrinsic magnetic moment of the electron."* [5].

Hestenes considers the complex phase of the wave function solution of the traditional Dirac equation as the phase of the ZBW rotation, showing "*the inseparable connection between quantum mechanical phase and spin"* consequently rejecting the *"conventional wisdom that phase is an essential feature of quantum mechanics, while spin is a mere detail that can often be ignored"* [6]. Using the space–time algebra in [7] Hestenes defines the *"canonical form"* of *the real wave function* ψ:

$$\psi\left(x\right)=\left(\varrho\,\mathrm{e}^{i\beta}\right)^{1/2}R.$$

In the above equation x is a generic space–time point, $\varrho=\varrho\left(x\right)$ is a scalar function interpreted as a probability density proportional to charge density, i is the spatial bivector $i=\gamma_2\gamma_1$, $\beta=\beta\left(x\right)$ is a function representing the value of a rotation phase in the plane $\gamma_2\gamma_1$ and R is a rotor valued function that encodes a Lorentz transformation. In the STA canonical form for Dirac's wave function the imaginary unit i is replaced by a bivector that generates rotation in a well defined space-like plane and not in a generic undefined "complex plane". This simple approach clearly reveals the geometric meaning of the imaginary numbers in the wave functions of quantum mechanics [7]. In agreement with the most common interpretations of quantum mechanics Hestenes associates the probability density function with a point-like shaped charge. In Eq. (48) of [7], by applying the relativistic time dilation to the ZBW period, Hestenes predicts a ZBW angular frequency that slows as the electron speed increase.

According to the model proposed in this paper, the electron characteristics may be explained by a massless charge distributed on the surface of sphere that rotates at the speed of light along a circumference with a radius equal to the reduced electron Compton wavelength (≈0.386159 pm), a value that is two times the one proposed by Hestenes in Eq. (33) of a relatively recent work [7]. The electron mass–energy, expressed in natural units, is equal to the angular speed of the ZBW rotation and to the inverse of the orbit radius (i.e. ≈511 keV), whereas the angular momentum is equal to the reduced Planck constant $\hbar$. Consequently, unlike the Hestenes prediction, our model proposes a relativistic contraction of the ZBW radius and a corresponding instantaneous ZBW angular speed that increases as the electron speed increases.

The inter-nuclear distance in UDD of ≈2.3 pm, found by Holmlid [2], seems to be compatible with proton–electron structures at the Compton scale [8,9] where the ZBW phases of neighbor electrons are correlated. These structures may generate unusual nuclear reactions and transmutations, considering the different sizes, time-scale and energies of these composites with respect to the dimension of the particles (such as neutrons) normally used in nuclear experiments.

By using the electromagnetic four-potential as a "*Materia Prima*" a natural connection between electromagnetic concepts and Newtonian and relativistic mechanics seems to be possible. The vector potential should not be viewed only as a pure mathematical tool to evaluate spatial electromagnetic field distributions but as a real physical entity, as suggested by the Aharonov–Bohm effect, a quantum mechanical phenomenon in which a charged particle is affected by the vector potential in regions in which the electromagnetic fields are null [10].

The present paper is structured in the following manner: Section 2 deals with a brief presentation of Maxwell's equations that does not use Lorenz gauge; Section 3 presents a new simple ZBW model of the electron with a list of the main parameters that can be deduced by applying this model; Section 4 describes an original method to easily derive the Lorentz force law from the electromagnetic field; Section 5 consists of a short introduction to the concept of "quanta current" and it also presents the relation between the ZBW modeling and Heisenberg's uncertainty principle; Section 6 summarizes other main models of the electron based on the concept of spinning charge distributions and, finally, Section 7 presents some preliminary hypotheses on UDD, Compton scale aggregates and the origin of anomalous heat in condensed matter.

In this paper all equations enclosed in square brackets with subscript "NU" have dimensions expressed in natural units.

2. Maxwell's Equations in $Cl_{3,1}$

The space–time algebra is a four dimensions Clifford algebra with *Minkowski signature* $Cl_{1,3}$ ("west coast metric") or $Cl_{3,1}$ ("east coast metric") [11,12].

In $Cl_{3,1}$ algebra, used in this work, calling $\{\gamma_x, \gamma_y, \gamma_z, \gamma_t\}$ the four unitary vectors of an orthonormal base the following rules apply:

$$\gamma_i\gamma_j = -\gamma_j\gamma_i \qquad \text{with} \quad i \neq j \quad \text{and} \quad i, j \in \{x, y, z, t\}, \tag{1}$$

$$\gamma_x^2 = \gamma_y^2 = \gamma_z^2 = -\gamma_t^2 = 1. \tag{2}$$

Maxwell's equations can be rewritten considering all the derivatives of the electromagnetic four-potential $\boldsymbol{A}_\square$:

$$\boldsymbol{A}_\square\,(x, y, z, t) = \gamma_x A_x + \gamma_y A_y + \gamma_z A_z + \gamma_t A_t. \tag{3}$$

Each of the vector potential components A_x, A_y, A_z and A_t is a function of space and time coordinates and has dimension in SI units equal to V s m^{-1}. $\boldsymbol{A}_\square$ is a *harmonic* function [13] that can be seen as the unique source of all concepts-entities in Maxwell's equations. Using the following definition of the operator $\boldsymbol{\partial}$ in space–time algebra, where

$$\nabla = \gamma_x\frac{\partial}{\partial x} + \gamma_y\frac{\partial}{\partial y} + \gamma_z\frac{\partial}{\partial z} \quad \text{and} \quad c = \frac{1}{\sqrt{\epsilon_0 u_0}},$$

$$\boldsymbol{\partial} = \gamma_x\frac{\partial}{\partial x} + \gamma_y\frac{\partial}{\partial y} + \gamma_z\frac{\partial}{\partial z} + \gamma_t\frac{1}{c}\frac{\partial}{\partial t} = \nabla + \gamma_t\frac{1}{c}\frac{\partial}{\partial t}, \tag{4}$$

the following expression can be written (see Table 1):

$$\boldsymbol{\partial}\boldsymbol{A}_\square = \boldsymbol{\partial}\cdot\boldsymbol{A}_\square + \boldsymbol{\partial}\wedge\boldsymbol{A}_\square = S + \boldsymbol{F} = \boldsymbol{G}, \tag{5}$$

where

$$S = \nabla\cdot\boldsymbol{A}_\triangle - \frac{1}{c}\frac{\partial A_t}{\partial t} \tag{6}$$

is the scalar field,

$$\boldsymbol{F} = \frac{1}{c}\boldsymbol{E}\gamma_t + I\boldsymbol{B}\gamma_t = \frac{1}{c}\left(\boldsymbol{E} + Ic\boldsymbol{B}\right)\gamma_t \tag{7}$$

the electromagnetic field and

$$I = \gamma_x\gamma_y\gamma_z\gamma_t \tag{8}$$

is the pseudoscalar unit.

Table 1. Relation between electromagnetic entities and the vector potential.

$\boldsymbol{\partial} \boldsymbol{A}_{\square}$	$\gamma_x A_x$	$\gamma_y A_y$	$\gamma_z A_z$	$\gamma_t A_t$
$\gamma_x \frac{\partial}{\partial x}$	S_1	B_{z1}	$-B_{y1}$	$\frac{1}{c} E_{x1}$
$\gamma_y \frac{\partial}{\partial y}$	B_{z2}	S_2	B_{x1}	$\frac{1}{c} E_{y1}$
$\gamma_z \frac{\partial}{\partial z}$	$-B_{y2}$	B_{x2}	S_3	$\frac{1}{c} E_{z1}$
$\gamma_t \frac{1}{c} \frac{\partial}{\partial t}$	$\frac{1}{c} E_{x2}$	$\frac{1}{c} E_{y2}$	$\frac{1}{c} E_{z2}$	S_4

The electromagnetic field $\boldsymbol{G}$ can be expressed in the following compact form

$$\boldsymbol{G}(x, y, z, t) = \nabla \cdot \boldsymbol{A}_{\vartriangle} - \frac{1}{c} \frac{\partial A_t}{\partial t} + \nabla A_t \gamma_t - \frac{1}{c} \frac{\partial \boldsymbol{A}_{\vartriangle}}{\partial t} \gamma_t + I \nabla \times \boldsymbol{A}_{\vartriangle} \gamma_t, \tag{9}$$

and the expression

$$\boldsymbol{\partial G} = \boldsymbol{\partial}^2 \boldsymbol{A}_{\square} = 0, \tag{10}$$

represents the four Maxwell's equations.

By applying, now, the $\boldsymbol{\partial}$ operator to the scalar field S, we obtain the expression of the four-current as

$$\frac{1}{\mu_0} \boldsymbol{\partial} S = \frac{1}{\mu_0} \left(\gamma_x \frac{\partial S}{\partial x} + \gamma_y \frac{\partial S}{\partial y} + \gamma_z \frac{\partial S}{\partial z} + \gamma_t \frac{1}{c} \frac{\partial S}{\partial t} \right) = \boldsymbol{J}_{\square e}, \tag{11}$$

where $\boldsymbol{J}_{\square e} = \gamma_x J_{ex} + \gamma_y J_{ey} + \gamma_z J_{ez} - \gamma_t c\rho = \boldsymbol{J}_{\vartriangle} - \gamma_t c\rho = \rho(\boldsymbol{v} - \gamma_t c)$ is the four-current vector and $\boldsymbol{v}_{\square} = \gamma_x v_x + \gamma_y v_y + \gamma_z v_z - \gamma_t c = \boldsymbol{v} - \gamma_t c$ is a four-velocity vector. The $\boldsymbol{\partial}$ operator applied to the four-current gives the charge–current conservation law

$$\frac{1}{\mu_0} \boldsymbol{\partial} \cdot (\boldsymbol{\partial} S) = \boldsymbol{\partial} \cdot \boldsymbol{J}_{\square e} = \frac{\partial J_{ex}}{\partial x} + \frac{\partial J_{ey}}{\partial y} + \frac{\partial J_{ez}}{\partial z} + \frac{\partial \rho}{\partial t} = 0, \tag{12}$$

which can be written alternatively as

$$\boldsymbol{\partial} \cdot (\boldsymbol{\partial} S) = \boldsymbol{\partial}^2 S = \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 S}{\partial t^2} = \nabla^2 S - \frac{1}{c^2} \frac{\partial^2 S}{\partial t^2} = 0. \tag{13}$$

The charge is related to the scalar field according to

$$\frac{1}{c} \frac{\partial S}{\partial t} = \mu_0 J_{et} = -\mu_0 c \frac{\partial q}{\partial x \partial y \partial z} = -\mu_0 c\rho, \tag{14}$$

so that, by applying the time derivative to (13) and remembering (14), the wave equation of the charge density field $\rho(x, y, z, t)$ can be deduced:

$$\frac{\partial}{\partial t} \left(\boldsymbol{\partial}^2 S \right) = \boldsymbol{\partial}^2 \left(\frac{\partial S}{\partial t} \right) = \boldsymbol{\partial}^2 \left(-\mu_0 c^2 \rho \right) = -\mu_0 c^2 \boldsymbol{\partial}^2 \rho = 0, \tag{15}$$

whose last equality gives

$$\boldsymbol{\partial}^2 \rho = \nabla^2 \rho - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \rho = 0. \tag{16}$$

A more detailed development of Maxwell's equations in $Cl_{3,1}$ algebra, made by the authors, can be found in *J. Condensed Matter Nucl. Sci.* **25** (2017) entitled "Maxwell's Equations and Occam's Razor".

3. Electron *Zitterbewegung* Model

The concept of charge that emerges from this rewriting of Maxwell's equations has a non-trivial implication: the analysis of (13) and (16) shows that the time derivative of a field S which propagates at the speed of light, must necessarily represent charges that are also moving at the speed of light.

This observation advises a pure electromagnetic model of elementary particles based on the ZBW interpretation of quantum mechanics [1,14]. According to this interpretation, the electron structure consists of a massless charge that rotates at the speed of light along a circumference equal to electron Compton wavelength λ_c [15,16]. Calling r_e the ZBW radius, ω_e the angular speed and T its period we have:

$$r_e = \frac{\lambda_c}{2\pi} \approx 3.861593 \times 10^{-13} \text{ m}, \tag{17}$$

$$\omega_e = \frac{c}{r_e} = 2\pi \frac{c}{\lambda_c} \approx 7.763440 \times 10^{20} \text{ rad s}^{-1}, \tag{18}$$

$$T = \frac{2\pi}{\omega_e} = \frac{2\pi r_e}{c} \approx 8.093300 \times 10^{-21} \text{ s}. \tag{19}$$

The value of the electron mass, expressed in SI units, can be derived from the following energy equations [1]

$$W_{tot} = m_e c^2 = \hbar \omega_e = \frac{\hbar c}{r_e}, \tag{20}$$

from which

$$m_e = \frac{\hbar \omega_e}{c^2} = \frac{\hbar}{c r_e} = \frac{h}{c \lambda_c} \approx 9.109383 \times 10^{-31} \text{ kg} \tag{21}$$

is obtained. Using natural units with $\hbar = c = 1$ the electron mass (in eV) is equal to the angular speed ω_e and to the inverse of r_e:

$$\left[m_e = \omega_e = \frac{1}{r_e} \approx 0.511 \times 10^6 \text{ eV} \right]_{NU}.$$

Recently, a connection between frequency and mass, in agreement with De Broglie's formula $f = {}^{mc^2}\!/_h$, has been experimentally demonstrated [17].

3.1. Simple electron model

A charge rotating at speed of light generates a current I_e that is equal to the ratio of the elementary charge e and its rotation period T [18]:

$$I_\mathrm{e} = \frac{e}{T} = \frac{ec}{2\pi r_\mathrm{e}} = \frac{e\omega_\mathrm{e}}{2\pi} \approx 19.796331 \text{ A}. \tag{22}$$

The electron magnetic moment μ_B (Bohr magneton) is equal to the product between the current I_e and the enclosed area $\mathcal{A}_\mathrm{e}$

$$\mu_\mathrm{B} = I_\mathrm{e}\mathcal{A}_\mathrm{e} = \frac{e\omega_\mathrm{e}}{2\pi}\pi r_\mathrm{e}^2 = \frac{ec}{2} r_\mathrm{e} = \frac{ec^2}{2\omega_\mathrm{e}} = \frac{e\hbar}{2m_\mathrm{e}} \approx 9.274010 \times 10^{-24} \text{ A m}^2. \tag{23}$$

Occam's razor is an effective epistemological instrument that imposes to avoid as much as possible the introduction of exceptions. Following this rule a pure electromagnetic origin of the electron's "intrinsic" angular momentum should be found.

Consequently, the canonical momentum P_t of the rotating massless charge may be seen as the cause of the intrinsic angular momentum:

$$\Omega = P_t r_\mathrm{e},$$

where the canonical momentum P_t of e, in presence of a vector potential A, generated by the current I_e, is

$$P_t = eA.$$

Imposing the constraint that $\Omega = \hbar$ we can compute A as function of I_e

$$\Omega = eAr_\mathrm{e} = \frac{eAc}{\omega_\mathrm{e}} = \frac{e^2cA}{2\pi I_\mathrm{e}} = \hbar, \tag{24}$$

from which it is possible to derive the expression of the vector potential seen by the spinning charge

$$A = \frac{2\pi\hbar}{e^2c} I_\mathrm{e} = \frac{\hbar}{er_\mathrm{e}} = \frac{\hbar\omega_\mathrm{e}}{ec} = \frac{m_\mathrm{e}c}{e} \approx 1.704509 \times 10^{-3} \text{ V s m}^{-1}. \tag{25}$$

From (25) it is possible to derive the Fine Structure Constant (FSC)

$$\alpha = \frac{\mu_0}{4\pi}\frac{ce^2}{\hbar} = \frac{\mu_0}{4\pi}\frac{e\omega_\mathrm{e}}{A} \approx 7.297352 \times 10^{-3}. \tag{26}$$

Using natural units we get these simple relations:

$$\left[A = 2\pi\alpha^{-1} I_\mathrm{e}\right]_\mathrm{NU},$$

$$\left[eA = \omega_\mathrm{e} = r_\mathrm{e}^{-1} = m_\mathrm{e} = P_t\right]_\mathrm{NU},$$

where α^{-1}, the inverse of the FSC, is a pure number and e is the elementary charge expressed in natural units

$$\left[\alpha^{-1} = e^{-2} \approx 137.035989\right]_{\mathrm{NU}}.$$

3.2. Spin and intrinsic angular momentum

The intrinsic angular momentum $\hbar$ of the electron model (see (24)) is compatible with the spin value $\hbar/2$ if we consider the electron interaction with the external flux density field $\boldsymbol{B}_{\mathrm{E}}$, as in the Stern–Gerlach experiment. We can interpret the spin value $\pm\hbar/2$ as the component of the intrinsic angular momentum $\boldsymbol{\Omega} = \hbar$ aligned with the external flux density field $\boldsymbol{B}_{\mathrm{E}}$. In this case the angle between the $\boldsymbol{B}_{\mathrm{E}}$ vector and the angular momentum have only two possible values, namely $\pi/3$ and $2\pi/3$ while the electron is subjected to a Larmor precession with angular frequency $\omega_{\mathrm{p}} = {}^{\mathrm{d}\vartheta_{\mathrm{p}}}/_{\mathrm{d}t}$. The Larmor precession is generated by the mechanical momentum

$$\tau = |\boldsymbol{\mu}_{\mathrm{B}} \times \boldsymbol{B}_{\mathrm{E}}| = B_{\mathrm{E}}\mu_{\mathrm{B}} \sin\left(\frac{\pi}{3}\right). \tag{27}$$

But

$$\mathrm{d}\Omega = \Omega_{\perp}\mathrm{d}\vartheta_{\mathrm{p}} = \Omega \sin\left(\frac{\pi}{3}\right)\mathrm{d}\vartheta_{\mathrm{p}},$$

where $\Omega_{\perp}$ is the component of the intrinsic angular momentum orthogonal to $\boldsymbol{B}_{\mathrm{E}}$ and, therefore, it is possible to write

$$\tau = \frac{\mathrm{d}\Omega}{\mathrm{d}t} = \Omega \sin\left(\frac{\pi}{3}\right)\frac{\mathrm{d}\vartheta_{\mathrm{p}}}{\mathrm{d}t}. \tag{28}$$

By equating (27) and (28) we get

$$B_{\mathrm{E}}\mu_{\mathrm{B}} = \Omega\omega_{\mathrm{p}},$$

from which it is possible to determine the precession angular frequency

$$\omega_{\mathrm{p}} = \frac{B_{\mathrm{E}}\mu_{\mathrm{B}}}{\Omega} = \frac{B_{\mathrm{E}}\mu_{\mathrm{B}}}{\hbar}. \tag{29}$$

3.3. Value of the vector potential, cyclotron resonance and flux density field

The pure electromagnetic momentum eA of the spinning charge of an electron at rest can be seen as it were the momentum of a particle of mass m_{e} and speed c in classical Newtonian mechanics. Considering ω_{e} as the cyclotron angular frequency (which is coincident with the ZBW angular speed) given by the flux density field B generated by the current I_{e}

$$\omega_{\mathrm{e}} = \frac{eB}{m_{\mathrm{e}}} = \frac{eBc^2}{\hbar\omega_{\mathrm{e}}},$$

it is possible to deduce the magnetic flux density produced by the electron

$$B_{\mathrm{e}} = \frac{\hbar\omega_{\mathrm{e}}^2}{ec^2} \approx 4.414004 \times 10^9 \text{ V s m}^{-2}. \tag{30}$$

This very high flux density value (also known as *Landau critical value*) seems to be related to the physics of neutron stars and pulsars [19–21] or to that of superconductivity [22,23].

It is also possible to calculate the flux density at the center of the electron orbit by the following expression derived from the Biot–Savart law

$$B_0 = \frac{\mu_0}{2}\frac{I_{\mathrm{e}}}{r_{\mathrm{e}}} \approx 32.210548 \times 10^6 \text{ V s m}^{-2}. \tag{31}$$

Considering that

$$\mathrm{d}A = A\,\mathrm{d}\vartheta \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}t} = A\omega_{\mathrm{e}}, \tag{32}$$

where $\mathrm{d}\vartheta = \omega_{\mathrm{e}}\,\mathrm{d}t$ is the differential of the ZBW phase, and considering that the magnetic force F_B must be equal to the time derivative of the canonical momentum, it is possible to write

$$F_B = B_{\mathrm{e}}ec = e\frac{\mathrm{d}A}{\mathrm{d}t} = eA\frac{\mathrm{d}\vartheta}{\mathrm{d}t} = eA\omega_{\mathrm{e}} \approx 0.212014 \text{ N}. \tag{33}$$

Finally, by manipulating the previous equation, it is possible to recompute by another method the module of the vector potential

$$A = \frac{B_{\mathrm{e}}c}{\omega_{\mathrm{e}}} = \frac{\hbar\omega_{\mathrm{e}}}{ec} = \frac{\hbar}{er_{\mathrm{e}}}.$$

3.4. Value of magnetic and electrostatic energy, magnetic flux quantization and radius of the elementary charge

Once we obtaine the expression for the vector potential it is possible to determine the magnetic flux produced by the rotating elementary charge by applying the circulation of the vector potential *A*:

$$\phi_{\mathrm{e}} = \oint_{\lambda_{\mathrm{c}}} A\,\mathrm{d}\lambda = \int_0^{2\pi} \frac{\hbar}{er_{\mathrm{e}}} r_{\mathrm{e}}\,\mathrm{d}\vartheta = 2\pi\frac{\hbar}{e} = \frac{h}{e} \approx 4.135667 \times 10^{-15} \text{ V s}, \tag{34}$$

i.e., the magnetic flux crossing the surface described by the charge trajectory is quantized (*flux quantum*). This expression has been found with a different approach with respect to [24], i.e., with the application of the vector potential. This flux quantum, though different from the value given in CODATA 2014, is compatible with $h/2e$ for the same reasons explained in Section 3.2, with reference to $\hbar$. Now it is possible to calculate the magnetic energy stored in the field produced by the spinning charge

$$W_{\mathrm{m}} = \frac{1}{2}\phi_{\mathrm{e}}I_{\mathrm{e}} = \frac{1}{2}2\pi\frac{\hbar}{e}\frac{ec}{2\pi r_{\mathrm{e}}} = \frac{\hbar c}{2r_{\mathrm{e}}} \approx 4.093553 \times 10^{-14} \text{ J} \tag{35}$$

which is equal to half the electron rest energy W_{tot} as can be seen from (20). The other half part can be attributed to electrostatic energy, i.e.,

$$W_{\text{tot}} - W_{\text{m}} = W_{\text{e}} = \iiint_V w_{\text{e}}\, \text{d}V, \tag{36}$$

where w_{e} is the electrostatic energy per unit of volume and V is the volume in which the whole energy W_{e} is stored, and whose expression is given by

$$w_{\text{e}} = \frac{1}{2}\epsilon_0 E^2 = \frac{\epsilon_0}{2}\left(\frac{1}{4\pi\epsilon_0}\frac{e}{r^2}\right)^2 = \frac{1}{32\pi^2\epsilon_0}\frac{e^2}{r^4}. \tag{37}$$

By expanding and integrating (36), with $\text{d}V = 4\pi r^2 \text{d}r$ (the generic elementary volume of a spherical thin shell centered in the middle of the electron trajectory) we obtain

$$W_{\text{e}} = \frac{e^2}{32\pi^2\epsilon_0}\int_{r_0}^{\infty}\frac{1}{r^4}4\pi r^2\,\text{d}r = \frac{e^2}{8\pi\epsilon_0}\int_{r_0}^{\infty}\frac{1}{r^2}\,\text{d}r = -\frac{e^2}{8\pi\epsilon_0}\left.\frac{1}{r}\right|_{r_0}^{\infty} = \frac{e^2}{8\pi\epsilon_0 r_0}. \tag{38}$$

Now, by taking into account that $W_{\text{e}} = W_{\text{m}}$ we get the radius

$$r_0 = \frac{e^2}{8\pi\epsilon_0 W_{\text{e}}} = \frac{e^2}{8\pi\epsilon_0 W_{\text{m}}} = \frac{e^2}{8\pi\epsilon_0}\frac{2r_{\text{e}}}{\hbar c} = \frac{e^2 r_{\text{e}}}{4\pi\epsilon_0\hbar c} \approx 2.817940 \times 10^{-15}\ \text{m}, \tag{39}$$

whose value is coincident with the classical electron radius [25]. The upper equation states that the rotating charge must have a finite dimension, in particular it may be visualized as a sphere with charge equal to e uniformly distributed over its surface. The charge cannot be concentrated in a point in order to exhibit a finite electrostatic energy. It is interesting and at the same time very important to note that the ratio r_{e}/r_0 is exactly equal to the inverse of the FSC, i.e.,

$$\frac{r_{\text{e}}}{r_0} = \frac{4\pi\epsilon_0\hbar c}{e^2 r_{\text{e}}}r_{\text{e}} = \frac{4\pi\epsilon_0\hbar c}{e^2} = \alpha^{-1} \approx 137.035999. \tag{40}$$

The expression of the ratio ϕ_{e}/T has in SI the dimension of a voltage:

$$V_{\text{e}} = \frac{\phi_{\text{e}}}{T} = \frac{h}{e}\frac{c}{2\pi r_{\text{e}}} = \frac{\hbar c}{e r_{\text{e}}} \approx 5.109989 \times 10^5\ \text{V}, \tag{41}$$

where T is defined by (19). Now, dividing the above voltage by the current generated by the rotating charge expressed by means of (22), we find the von Klitzing constant or *quantum of resistivity*, related to the quantum Hall effect [24]

$$R_{\text{K}} = \frac{V_{\text{e}}}{I_{\text{e}}} = \frac{h}{e}\frac{c}{2\pi r_{\text{e}}}\frac{2\pi r_{\text{e}}}{ec} = \frac{h}{e^2} = \frac{2\pi\hbar}{e^2} \approx 25812.807\ \Omega. \tag{42}$$

An alternative expression of the von Klitzing constant can be derived from the electrostatic potential φ_{e} and the current I_{e}

$$R_{\text{K}} = \frac{\varphi_{\text{e}}}{I_{\text{e}}} = \frac{1}{4\pi\epsilon_0}\frac{e}{r_0}\frac{2\pi r_{\text{e}}}{ec} = \frac{1}{2c\epsilon_0}\frac{r_{\text{e}}}{r_0} = \frac{\mu_0 c}{2\alpha} \approx 25812.807\ \Omega. \tag{43}$$

Finally, it is possible to deduce the values of two interesting electrical parameters, namely the inductance $L_{\rm e}$, the capacitance $C_{\rm e}$ of the electron and the frequency $f_{\rm e}$. In fact

$$L_{\rm e} = \frac{\phi_{\rm e}}{I_{\rm e}} = 4\pi^2 \frac{\hbar r_{\rm e}}{e^2 c} \approx 2.089108 \times 10^{-16}\ \Omega\ {\rm s}, \tag{44}$$

$$C_{\rm e} = \frac{e}{\varphi_{\rm e}} = 4\pi\epsilon_0 r_0 \approx 3.135381 \times 10^{-25}\ {\rm F} \tag{45}$$

and

$$f_{\rm e} = \frac{1}{\sqrt{L_{\rm e} C_{\rm e}}} \approx 1.235590 \times 10^{20}\ {\rm Hz}. \tag{46}$$

3.5. Electron kinematics

The finite dimension of the elementary charge imposes the constraint that all points of the surface of the spinning charged sphere must have the same instantaneous speed of light c (see Eq. (16)) and the same angular speed. In a frame rotating with the ZBW frequency the spinning charged sphere rotates around its center with opposite speed with respect to the ZBW angular frequency:

$$\omega_0 = -\omega_{\rm e}. \tag{47}$$

The new (not point-like) electron model and the speed diagrams are shown in Fig. 1a. Here the charge rotates with angular speed ω_0 around the axis passing through the center of the sphere and, therefore, all points of the sphere have the same absolute speed c.

During the revolution around the origin C the charge describes a torus whose cross section is equal to πr_0^2 and having a volume equal to $2\pi^2 r_{\rm e} r_0^2$. In Fig. 1b, the elementary charge is represented as a charged sphere.

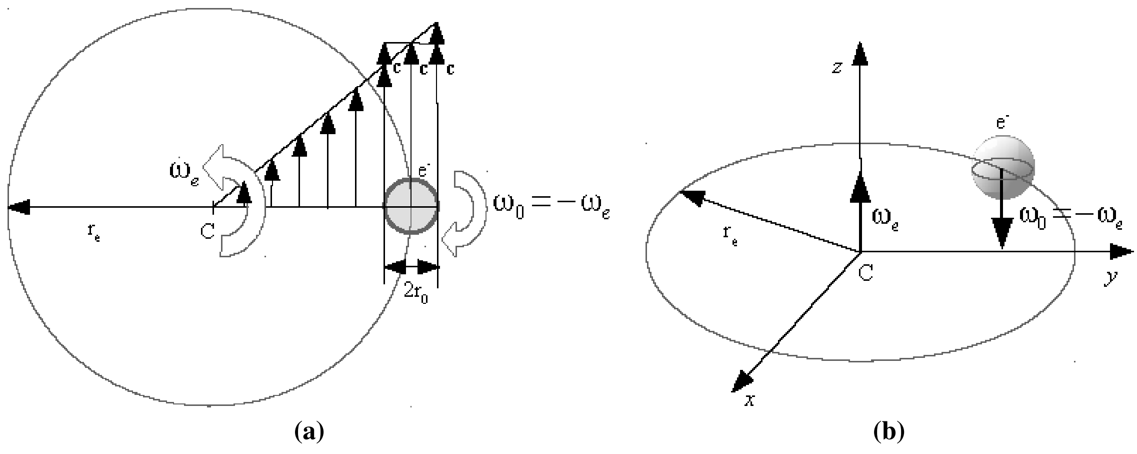

Figure 1. (a) ZBW model and speed diagrams of the electron charge (e^-). All points of the sphere have an absolute speed equal to c. (b) 3D representation. The charged sphere is rotating with the relative angular speed $\omega_0 = -\omega_{\rm e}$ on the trajectory having radius $r_{\rm e}$ around the vertical axis passing through the center of the sphere.

3.6. Electron and electromagnetic Lagrangian density

From (25), (22) and (39), with the hypothesis that the electron is characterized by a uniform current density, we get the first term of the interaction part of the Lagrangian density

$$L_{\text{int1}} = \boldsymbol{J}_{\triangle}\boldsymbol{A}_{\triangle} = JA = \frac{I_{\text{e}}}{\pi r_0^2}\frac{m_{\text{e}}c}{e} \approx 1.352604 \times 10^{27}\ \text{J m}^{-3}. \tag{48}$$

By integration over the volume described by the electron toroidal trajectory, it is possible to recompute its rest energy:

$$W_{\text{tot}} = \iiint_V JA\,\text{d}V = \frac{I_{\text{e}}}{\pi r_0^2}\frac{m_{\text{e}}c}{e}2\pi^2 r_{\text{e}} r_0^2 \approx 8.187106 \times 10^{-14}\ \text{J} = 510.998946\ \text{keV}, \tag{49}$$

which gives the same result calculated by means of (36). The same results are obtained, apart from a sign –, by the following relations

$$L_{\text{int2}} = -\rho\varphi_{\text{e}} = \frac{e}{2\pi^2 r_{\text{e}} r_0^2}\frac{e}{4\pi\epsilon_0 r_{\text{e}}} = -\frac{e^2}{8\pi^3\epsilon_0 (r_{\text{e}} r_0)^2} \approx -1.352604 \times 10^{27}\ \text{J m}^{-3}, \tag{50}$$

$$W_{\text{tot}} = \iiint_V |-\rho\varphi_{\text{e}}|\,\text{d}V = \frac{e^2 2\pi^2 r_{\text{e}} r_0^2}{8\pi^3\epsilon_0 (r_{\text{e}} r_0)^2} = \frac{e^2}{4\pi\epsilon_0 r_{\text{e}}} \approx 8.187106 \times 10^{-14}\ \text{J} = 510.998946\ \text{keV}. \tag{51}$$

All parameters that can be deduced by the application of the present ZBW model are resumed in Table 2, where the first three rows are referred to the model's input parameters.

3.7. ZBW and a simple derivation of the relativistic mass

With the ZBW model it is possible to show a simple, original and intuitive explanation of the relativistic mass concept. For an electron moving at constant speed v_z along the z-axis orthogonal to the rotation plane, calling $v_\perp$ the component of the velocity of the rotating charge in the $\gamma_x\gamma_y$ plane we can find the value of the ZBW radius r of the moving electron. In fact, assuming a constant value of ω_{e}, we have

$$v_z^2 + v_\perp^2 = c^2, \tag{52}$$

that can be written as

$$v_z^2 + \omega_{\text{e}}^2 r^2 = \omega_{\text{e}}^2 r_{\text{e}}^2 = c^2.$$

Therefore

$$\frac{r^2}{r_{\text{e}}^2} = 1 - \frac{v_z^2}{c^2}$$

or

$$r = r_{\mathrm{e}}\sqrt{1-\frac{v_z^2}{c^2}}. \tag{53}$$

Finally, by considering that the mass is inversely proportional to r, it is possible to write the relativistic expression of the mass as

$$m = \frac{m_{\mathrm{e}}}{\sqrt{1-\frac{v_z^2}{c^2}}}, \tag{54}$$

where m_{e} is the electron mass at rest. Fig. 2 represents the ZBW trajectory of the spinning charge of an electron subjected to an acceleration directed along the positive z-axis. Due to the acceleration the radius reduces itself according to (53).

3.8. Dirac equation and spinor representation of motion

By using space–time algebra and following the idea of Hestenes–Dirac equation

$$i\not{\partial}\psi - m\psi = 0 \tag{55}$$

becomes the Hestenes–Dirac equation [26]

Table 2. Parameters of the *Zitterbewegung* model.

Item	Symbol	Value (SI)	Unit (SI)
Charge	e	$1.602176565\times10^{-19}$	C = A s
ZBW orbit radius	$r_{\mathrm{e}}=\lambda_{\mathrm{c}}/2\pi$	3.861593×10^{-13}	m
Intrinsic angular momentum	$\Omega=\hbar=h/2\pi$	$1.054571726\times10^{-34}$	J s
Spin[1]	$\hbar/2$	$0.527285863\times10^{-34}$	J s
Angular speed	ω_e	7.763440×10^{20}	rad s^{-1}
Mass	m_{e}	9.109384×10^{-31}	kg
Current	I_{e}	19.796331	A
Magnetic moment (Bohr magneton)	μ_{B}	9.274010×10^{-24}	A m^2
Vector potential	A	1.704509×10^{-3}	V s m^{-1}
Magnetic flux density	B_{e}	4.414004×10^{9}	V s m^{-2}
Magnetic flux	$\phi_{\mathrm{e}}=h/e$	4.135667×10^{-15}	V s
Magnetic energy	W_{m}	4.093553×10^{-14}	J
Electrostatic energy	W_{e}	4.093553×10^{-14}	J
Electron energy at rest	$W_{\mathrm{tot}}=m_{\mathrm{e}}c^2$	8.187106×10^{-14}	J
Charge radius	r_0	2.817940×10^{-15}	m
Inverse of the FSC	$\alpha^{-1}=r_{\mathrm{e}}/r_0$	137.035999	1
Von Klitzing constant	$R_{\mathrm{K}}=h/e^2=\mu_0c/2\alpha$	25812.807	Ω
Inductance	$L_{\mathrm{e}}=4\pi^2\hbar r_{\mathrm{e}}/e^2c$	2.089108×10^{-16}	Ω s
Capacitance	$C_{\mathrm{e}}=4\pi\epsilon_0r_0$	3.135381×10^{-25}	F
1-st part of L_{int}	JA	1.352604×10^{27}	J m^{-3}
Electron energy at rest	$\iiint_V JA\,\mathrm{d}V$	510.998946	keV
Electron energy at rest	$\iiint_V \rho\varphi_{\mathrm{e}}\,\mathrm{d}V$	510.998946	keV

[1]Component of the angular momentum due to Larmor precession along the external magnetic field B_{E} (see (27)).

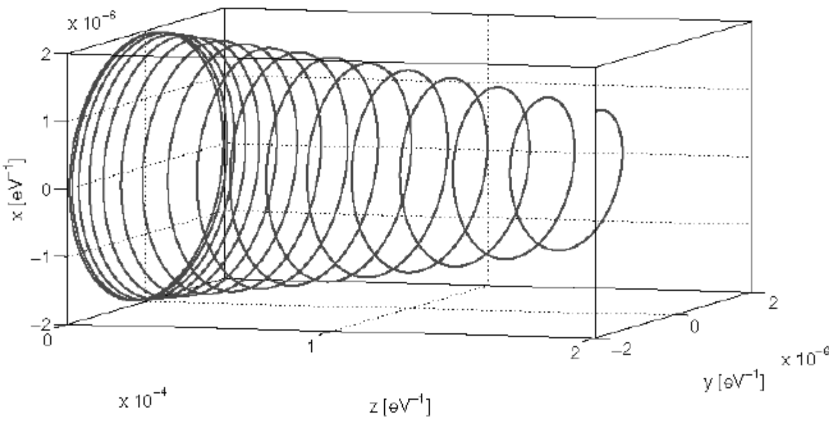

Figure 2. *Zitterbewegung* trajectory during an acceleration of the electron in the z-direction.

$$\boldsymbol{\partial}\boldsymbol{\psi} - m\boldsymbol{\psi}\gamma_t\gamma_x\gamma_y = 0, \tag{56}$$

where $\boldsymbol{\partial}$ is the same operator used in Maxwell's equations $\boldsymbol{\partial G} = 0$ (see (4)), but using the space–time Minkowski signature of $Cl_{1,3}$ "$+ - - -$". For a massless particle, where $m = 0$, (56) becomes the Weyl equation

$$\boldsymbol{\partial}\boldsymbol{\psi} = 0, \tag{57}$$

which is formally identical to (10). In all cases the solution is a *spinor* field. A spinor is a mathematical object that in space–time algebra is simply a multivector with only even grade components. The geometric product of an even number of vectors is always a spinor. A spinor that is the geometric product of two unitary vectors is a unitary rotor. The movement of a point charge that rotates in the plane $\gamma_x\gamma_y$ and at the same time moves up along the γ_z axis can be seen as the composition of an ordinary rotation in the plane $\gamma_x\gamma_y$ followed by a scaled hyperbolic rotation in the plane $\gamma_z\gamma_t$. The composition of these two rotations can be encoded with a single spinor of $Cl_{3,1}$. Therefore, if $\mathbf{r}_{\mathrm{e}\square 0}$ is the coordinate of the center of the charged sphere at $t = t_0$, r_e the Compton radius we have

$$\mathbf{r}_{\mathrm{e}\square 0} = \gamma_x r_\mathrm{e} + \gamma_t c t_0,$$

$$\left[\mathbf{r}_{\mathrm{e}\square 0} = \gamma_x r_\mathrm{e} + \gamma_t t_0\right]_{\mathrm{NU}}.$$

By introducing the rotor R_{xy}, that generates ordinary rotation in the $\gamma_x\gamma_y$ plane, and remembering that ω_e is the ZBW angular frequency, with

$$R_{xy}(t) = \cos\left(\frac{\omega_\mathrm{e} t}{2}\right) + \gamma_x\gamma_y \sin\left(\frac{\omega_\mathrm{e} t}{2}\right) = \exp\left(\gamma_x\gamma_y\frac{\omega_\mathrm{e} t}{2}\right),$$

we obtain the instantaneous position of the center of the charged sphere:

$$\mathbf{r}_{\mathrm{e}\square}\left(t\right)=R_{xy}\left(\mathbf{r}_{\mathrm{e}\square 0}+\gamma_t ct\right)\widetilde{R_{xy}}.$$

We introduce now the rotor R_{zt}, that generates a hyperbolic rotation with rapidity φ in the $\gamma_z\gamma_t$ plane, in order to encode the motion at speed v_z along the γ_z direction

$$R_{zt}=\cosh\left(\varphi\right)^{-1/2}\left[\cosh\left(\frac{\varphi}{2}\right)-\gamma_z\gamma_t\sinh\left(\frac{\varphi}{2}\right)\right]=\cosh\left(\varphi\right)^{-1/2}\exp\left(-\gamma_z\gamma_t\frac{\varphi}{2}\right),$$

where

$$\varphi=\tanh^{-1}\left(\frac{v_z}{c}\right).$$

The instantaneous position $\mathbf{r}'_{\mathrm{e}\square}\left(t\right)$ of the center of the rotating and translating sphere can be obtained by applying R_{zt}:

$$\mathbf{r}'_{\mathrm{e}\square}\left(t\right)=R_{zt}\mathbf{r}_{\mathrm{e}\square}\widetilde{R_{zt}}.$$

With the definition of spinor R

$$R=R_{zt}R_{xy}=\cosh\left(\varphi\right)^{-\frac{1}{2}}\exp\left(-\gamma_z\gamma_t\frac{\varphi}{2}\right)\exp\left(\gamma_x\gamma_y\frac{\omega_{\mathrm{e}}t}{2}\right),$$

we can rewrite the instantaneous position in a compact form as

$$\mathbf{r}'_{\mathrm{e}\square}\left(t\right)=R\left(\mathbf{r}_{\mathrm{e}\square 0}+\gamma_t ct\right)\widetilde{R}.$$

The instantaneous coordinate of a generic point on the surface of the charged sphere can be obtained adding a specific fixed vector $\mathbf{r}_{o\triangle}$

$$\mathbf{r}_{\mathrm{surf}\square}\left(t\right)=\mathbf{r}'_{\mathrm{e}\square}\left(t\right)+\mathbf{r}_{\mathbf{o}\triangle}.$$

The module of vector $\mathbf{r}_{o\triangle}$ is equal to $r_{o\triangle}=\alpha r'_{\mathrm{e}\triangle}$, a value equal to the classical radius of the electron for non relativistic speeds.

It is important to note that, according to Hestenes, the ZBW angular frequency is two times the De Broglie value $m_{\mathrm{e}}c^2/\hbar$ used in our model: *"The diameter of the helix is the electron Compton wavelength* $2\lambda_0={}^{2c}/_{\omega_0}={}^{\hbar}/_{mc}$*"* [27]. The value 1.93079×10^{-13} m of the "zitter-radius" of Hestenes' electron model is confirmed in Eq. (33) of a more recent work [7].

4. Electromagnetism, Mechanics and Lorentz force

The "pure electromagnetic" vector $e\boldsymbol{A}_\square$ may be interpreted as the momentum–energy $\boldsymbol{P}_\square$ of a particle with electric charge e, momentum $\boldsymbol{P}_\triangle$ and energy $U=P_tc$:

$$\boldsymbol{P}_\square=e\boldsymbol{A}_\square,\tag{58}$$

$$\boldsymbol{P}_{\square} = \gamma_x P_x + \gamma_y P_y + \gamma_z P_z + \gamma_t \frac{U}{c} = \boldsymbol{P}_{\triangle} + \gamma_t \frac{U}{c}. \tag{59}$$

For a particle that moves with speed $\boldsymbol{v}$ along a direction *z* orthogonal to the ZBW rotation plane, the momentum $\boldsymbol{P}_{\triangle}$ can be decomposed in two vectors, one parallel and one orthogonal to $\boldsymbol{v}$. The orthogonal component is a rotating vector, that indicates the component of the momentum due to the angular frequency ω_{e} in the spatial plane *xy* orthogonal to *z*

$$\boldsymbol{P}_{\triangle} = \boldsymbol{P}_{\parallel} + \boldsymbol{P}_{\perp}, \tag{60}$$

where

$$|\boldsymbol{P}_{\perp}| = \frac{\hbar\omega_{\mathrm{e}}}{c} = m_{\mathrm{e}}\omega_{\mathrm{e}}r_{\mathrm{e}} = m_{\mathrm{e}}c,$$

$$\left[|\boldsymbol{P}_{\perp}| = \frac{1}{r_{\mathrm{e}}} = \omega_{\mathrm{e}} = m_{\mathrm{e}}\right]_{\mathrm{NU}}.$$

The $\boldsymbol{P}_{\parallel}$ component can be seen as the usual three components momentum of a particle with mass at rest m_{e}. For simplicity of notation, from now, we will call $\boldsymbol{P}$ this component, so that

$$P_{\square}^2 = e^2 A_{\square}^2 = P_{\triangle}^2 - \frac{U}{c^2}^2 = P^2 + m_{\mathrm{e}}^2 c^2 - \frac{U}{c^2}^2.$$

The relativistic mass m can be derived directly by the application of the Pythagorean theorem

$$m^2c^2 = m_{\mathrm{e}}^2c^2 + P^2 = P_{\perp}^2 + P^2 = m_{\mathrm{e}}^2c^2 + m^2v^2.$$

Consequently this electromagnetic four-momentum $\boldsymbol{P}_{\square}$, for electrons moving with uniform velocity, is a light-like vector:

$$P_{\square}^2 = m^2c^2 - \frac{U}{c^2}^2 = 0.$$

An electron that moves with velocity $v \ll c$ has an approximate momentum P given by

$$P = eA_{\parallel} \simeq P_{\perp}\frac{v}{c} = m_{\mathrm{e}}v,$$

and a variation of speed

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \text{implies a force} \quad f = \frac{\mathrm{d}P}{\mathrm{d}t} = e\frac{\mathrm{d}A_{\parallel}}{\mathrm{d}t} = m\frac{\mathrm{d}v}{\mathrm{d}t}.$$

Now recalling that the bivector part of (5) is

$$\boldsymbol{\partial} \wedge \boldsymbol{A}_{\square} = \boldsymbol{F}$$

after multiplying both sides by the charge e, it becomes

$$e\boldsymbol{\partial} \wedge \boldsymbol{A}_{\square} = \boldsymbol{\partial} \wedge e\boldsymbol{A}_{\square} = e\boldsymbol{F}. \tag{61}$$

By considering (58) and (59) this equation can be rewritten as

$$\boldsymbol{\partial} \wedge \left(\boldsymbol{P}_{\vartriangle} + \gamma_t \frac{U}{c}\right) = e\boldsymbol{F}, \tag{62}$$

or, by means of (60), as

$$\boldsymbol{\partial} \wedge \left[(\boldsymbol{P} + \boldsymbol{P}_{\perp}) + \gamma_t \frac{U}{c}\right] = e\boldsymbol{F}. \tag{63}$$

The term $\boldsymbol{\partial} \wedge \boldsymbol{P}_{\perp}$ can be carried out because the average value of $\boldsymbol{P}_{\perp}$, in a scale time much larger than the ZBW period, is zero:

$$\boldsymbol{\partial} \wedge \left(\boldsymbol{P} + \gamma_t \frac{U}{c}\right) = e\boldsymbol{F}, \tag{64}$$

$$\boldsymbol{\partial} \wedge \left(\boldsymbol{P} + \gamma_t \frac{U}{c}\right) = \frac{e}{c}\boldsymbol{E}\gamma_t + eI\boldsymbol{B}\gamma_t.$$

Equating only the components that contain bivectors with γ_t terms we obtain

$$\left(\frac{\partial \boldsymbol{P}}{\partial t}\right)_{\text{EU}} \gamma_t + \nabla U \gamma_t = e\boldsymbol{E}\gamma_t$$

or

$$\left(\frac{\partial \boldsymbol{P}}{\partial t}\right)_{\text{EU}} = e\boldsymbol{E} - \nabla U. \tag{65}$$

In (65) $(\partial \boldsymbol{P}/\partial t)_{\text{EU}}$ is the force acting on the charge e due both to the electric field $\boldsymbol{E}$ (Coulomb force) and to the gradient of the "potential energy" U. Instead, by equating only the components that contain pure spatial bivectors we get

$$\nabla \wedge \boldsymbol{P} = eI\boldsymbol{B}\gamma_t = -eI\gamma_t\boldsymbol{B} = eI_{\vartriangle}\boldsymbol{B}, \tag{66}$$

where the term $-I\gamma_t = \gamma_x\gamma_y\gamma_z = I_{\vartriangle}$ is the unitary volume of the three dimensional space. Left-multiplying both sides of (66) by $I_{\vartriangle}$ gives

$$I_{\vartriangle}\nabla \wedge \boldsymbol{P} = -e\boldsymbol{B}, \tag{67}$$

which is equivalent to the two following equations in the ordinary algebra

$$\nabla \times \boldsymbol{P} = e\boldsymbol{B}. \tag{68}$$

Table 3. Products $\boldsymbol{v}\times(\nabla\times\boldsymbol{P}-e\boldsymbol{B})$.

$\boldsymbol{v}\times(\nabla\times\boldsymbol{P}-e\boldsymbol{B})$	$\gamma_x\left(\frac{\partial P_z}{\partial y}-\frac{\partial P_y}{\partial z}-eB_x\right)$	$\gamma_y\left(\frac{\partial P_x}{\partial z}-\frac{\partial P_z}{\partial x}-eB_y\right)$	$\gamma_z\left(\frac{\partial P_y}{\partial x}-\frac{\partial P_x}{\partial y}-eB_z\right)$
$\gamma_x v_x$	0	$\gamma_z\left(-\left.\frac{\partial P_z}{\partial t}\right\|_{xy}-ev_xB_y\right)$	$\gamma_y\left(\left.\frac{\partial P_y}{\partial t}\right\|_{xz}-ev_xB_z\right)$
$\gamma_y v_y$	$-\gamma_z\left(\left.\frac{\partial P_z}{\partial t}\right\|_{yx}-ev_yB_x\right)$	0	$\gamma_x\left(-\left.\frac{\partial P_x}{\partial t}\right\|_{yz}-ev_yB_z\right)$
$\gamma_z v_z$	$-\gamma_y\left(-\left.\frac{\partial P_y}{\partial t}\right\|_{zx}-ev_zB_x\right)$	$-\gamma_x\left(\left.\frac{\partial P_x}{\partial t}\right\|_{zy}-ev_zB_y\right)$	0

As an example, the component of the above equation along the x axis is

$$\gamma_x\left(\frac{\partial P_z}{\partial y}-\frac{\partial P_y}{\partial z}\right)=\gamma_x eB_x.$$

Now, by applying the cross product of the velocity $\boldsymbol{v}$ of charge e to both terms in (68) we obtain

$$\boldsymbol{v}\times(\nabla\times\boldsymbol{P}-e\boldsymbol{B})=0. \tag{69}$$

The components of (69) are represented in Table 3 considering that

$$v_i\frac{\partial P_j}{\partial i}=\frac{\partial i}{\partial t}\frac{\partial P_j}{\partial i}=\frac{\partial P_j}{\partial t},$$

$$v_j\frac{\partial P_j}{\partial i}=\frac{\partial j}{\partial t}\frac{\partial P_j}{\partial i}=\frac{\partial j}{\partial i}\frac{\partial P_j}{\partial t}=0\quad \text{for} i\neq j,\quad \text{where } i,j\in\{x,y,z\}.$$

For these reasons (69) leads to the usual form of the force contribution due to the magnetic flux density field $\boldsymbol{B}$

$$\left(\frac{\partial\boldsymbol{P}}{\partial t}\right)_B=e\boldsymbol{v}\times\boldsymbol{B}. \tag{70}$$

Finally, we get the whole force contribution by summing up the forces

$$\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t}=\left(\frac{\partial\boldsymbol{P}}{\partial t}\right)_{\mathrm{EU}}+\left(\frac{\partial\boldsymbol{P}}{\partial t}\right)_B$$

given respectively by (65) and (70)

$$\frac{d\boldsymbol{P}}{dt}=e\left(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}\right)-\nabla U. \tag{71}$$

5. Energy, Momentum and Quanta Current

The nature of energy and momentum can be understood if we *consider the quantum of action $\hbar$ as a "physical object" that moves in space–time*. The Planck relation $W_\varphi=\hbar\omega={}^{2\pi\hbar}/_{T_\varphi}$ tells us that photons with energy W_φ can transmit quanta of actions with a quanta current Q_{c} (number of quanta of actions per time unit)

$$Q_\mathrm{c} = n\frac{\hbar}{T_\varphi} = n\frac{W_\varphi}{2\pi}. \tag{72}$$

The same quanta current can be obtained in a time unit by a large number of low energy photons or by a small number of high energy photons.

Calling W_H the energy of high energy photons and W_L the energy of low energy ones, and n_H and n_L their number, we observe that the same Q_c can be obtained with the same total energy following two different ways if $n_\mathrm{H}W_\mathrm{H} = n_\mathrm{L}W_\mathrm{L}$:

$$\frac{n\hbar}{T_\varphi} = \frac{n_\mathrm{H}W_\mathrm{H}}{2\pi}, \tag{73}$$

$$\frac{n\hbar}{T_\varphi} = \frac{n_\mathrm{L}W_\mathrm{L}}{2\pi}. \tag{74}$$

In the first case we have few "high speed" – high energy photons that guarantee the required current. In the latter case the same result is obtained by many "low speed" – low energy photons. An information technology analogy can be given if we consider a bus with few wires driven by a high frequency clock compared to a large bus with many wires driven by a low frequency clock. The information per unit time is the same in both case. Now, following the above considerations the equation $P_\varphi = \hbar k = 2\pi\hbar/\lambda_\varphi$, where $k = 2\pi/\lambda_\varphi$ is the wave number, should be viewed as the direction of quanta current in space. For photons the four-momentum $\boldsymbol{P}_\square$ is a light-like vector

$$P_\square^2 = 0. \tag{75}$$

In this case the momentum is a vector that gives the direction of quantum of action in space and the module of momentum P_φ is the energy W_φ:

$$\left[P_\varphi^2 - W_\varphi^2 = 0\right]_\mathrm{NU}.$$

5.1. ZBW and Heisenberg's uncertainty principle

The concept of "measure" is strictly related to the quantum of action as the concept of information is related to the binary digit (bit). In natural units the quantum of action is a dimensionless (i.e., scalar) value, as it is always the ratio (measure) of two values of the same nature. For this reason the concepts of "energy", "momentum", "space" and "time" cannot be separated and space and time can be measured, using natural units, in eV^{-1}.

The product of the momentum P of the rotating charge and the radius r of the orbit in the proposed ZBW model is always equal to $\hbar$:

$$Pr = \hbar. \tag{76}$$

This expressions points out that the intrinsic momentum of a particle confined in a spherical space of radius r cannot be less than $\hbar/r$. Calling $t_r = T/2\pi = \omega^{-1}$ the inverse of the ZBW angular frequency, and remembering that $mc^2 = \hbar\omega$, we can observe that

$$mc^2 t_r = \hbar\omega t_r = \hbar = Pr. \quad (77)$$

This formula just states that the energy of a particle at rest (such as an electron) confined in a spherical space of radius r cannot be less than $\hbar/t_r = \hbar c/r$. Both (76) and (77) can be viewed as a particular reformulation of Heisenberg's uncertainty principle. From this point of view, energy can be seen as strictly related to the concept of quantum of action density in space–time. We remember that in quantum mechanics the concept of "particle at rest" cannot be considered as realistic, because the assumed point-like model of elementary particles implies that, due to the Heisenberg's principle, the momentum is not determined as the precision in position tends to zero.

6. Some other Spinning Charge Models

In 1915 Alfred Lauck Parson published "A Magneton Theory of the Structure of the Atom" in the Smithsonian Miscellaneous Collection, Pub 2371 [28], where he proposed a spinning ring model of the electron. Various forms of the spinning charge model of electrons have been rediscovered by many authors. However, the incompatibility with the most widely accepted interpretations of quantum mechanics prevented them from receiving proper attention.

According to Randell L. Mills the free electron is "*is a spinning two-dimensional disk of charge. The mass and current density increase towards the center, but the angular velocity is constant. It produces an angular momentum vector perpendicular to the plane of the disk*" [29]. As in our proposed model the intrinsic angular momentum of free electron is $\hbar$ but there is an important difference in charge distribution shape and speed. A constant angular velocity for a flat charge distribution implies that the charge speed is not always equal to the speed of light as strictly demanded by our model [30]. Mills' theory [31,32] "*assumes physical laws apply on all scales including the atomic scale*" in agreement with Occam's razor principle, is based on simple fundamental physical laws and is highly predictive.

Using geometric algebra and starting from Dirac theory, David Hestenes has proposed a ZBW model according to which "*the electron is a massless point particle executing circular motion in the rest system*" and "*with an intrinsic orbital angular momentum or spin of fixed magnitude $s = \hbar/2$*" [1]. The phase of the probability amplitude wave function is related to the ZBW rotation phase, a concept usually hidden in the traditional mathematical formalism used in quantum mechanics based on complex numbers and matrices. However, we should remark that the concept of point-like charge in quantum mechanics should be considered unrealistic. It violates Occam's razor principle and may be used only as a first approximation. We remember also that in our model the value of intrinsic angular momentum for a free electron is $\hbar$ and that the "spin" is interpreted as the component of the angular momentum along an external magnetic field as in Stern–Gerlach experiment (see Section 3.2). Another interesting electron model has been proposed by David L. Bergman [15,16]: according to this model the electron is a very thin, torus shaped, rotating charge distribution with intrinsic angular momentum of the electron equal to its spin value $s = \hbar/2$. The torus radius has a length $R = \hbar/mc$ and half thickness $r = 8R\mathrm{e}^{-\pi/\alpha}$, where α is the fine structure constant.

7. Electromagnetic Composite At Compton Scale

If the electron is a current loop whose radius is equal to the reduced electron Compton wavelength, it is reasonable to assume the possibility of existence of "super chemical" structures of pico-metric (1 pm $= 10^{-12}$ m) dimensions. These dimensions are intermediate between nuclear (1 fm $= 10^{-15}$ m) and atomic scale (1 Å $= 10^{-10}$ m).

A simple ZBW model of the proton consists in a current loop generated by an elementary positive charge that rotates at the speed of light along a circumference with a length equal to the proton Compton wavelength ($\approx 1.32141 \times 10^{-15}$ m) [33]. According to this model the proton is much smaller than the electron ($r_\mathrm{e}/r_\mathrm{p} = m_\mathrm{p}/m_\mathrm{e} \approx 1836.153$). A hypothetically very simple structure formed by an electron with a proton at his center would have potential energy

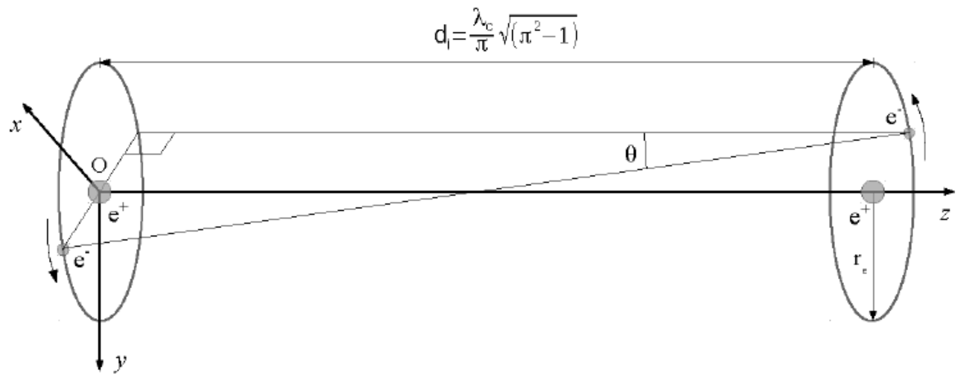

Figure 3. UDH protons distance.

of $-e^2/r_e \approx -3.728$ keV corresponding to a photon wavelength of $\lambda_\varphi \approx 3.325 \times 10^{-10}$ m. This structure may be created starting from atomic hydrogen or Rydberg State Hydrogen only in specific environments, as materials with high free electron density and with lattice constants and energy levels allowing a resonant absorption of 3.7 keV photons. A high electron density can be obtained in "swimming electron layers" formed when a metal is heated in contact with materials, such as SrO, with low work functions [34,35].

The hypothesis of existence of Compton-scale composites (CSC) has been experimentally confirmed by Holmlid [2,3,36]. The inter-nuclear distance in Ultra-Dense Deuterium (UDD) of ≈ 2.3 pm, found by Holmlid [2], seems compatible with deuteron–electron (or proton–electron in Ultra-Dense Hydrogen(UDH)) structures where the ZBW phases of adjacent electrons are correlated. Such distance may be obtained imposing, as a first step, the condition that the space–time distance $d_\square$ between adjacent electrons rotating charges is a light-like vector:

$$d_\square^2 = d_\vartriangle^2 - c^2\delta t^2 = 0, \tag{78}$$

$$\left[d_\square^2 = d_\vartriangle^2 - \delta t^2 = 0\right]_{\mathrm{NU}},$$

where $d_\vartriangle$ is the ordinary euclidean distance in space. This condition is satisfied if $d_\vartriangle$ is equal to electron Compton wavelength ($d_\vartriangle = \lambda_c$), $\delta t = T$ is the ZBW period and the phase difference between adjacent electrons is equal to π. In this case from a direct application of the Pythagorean theorem we can find the internuclear deuteron distance d_i as shown in Fig. 3.

$$d_i = \frac{\lambda_c}{\pi}\sqrt{\pi^2 - 1} \approx 2.3 \times 10^{-12} \text{ m}, \tag{79}$$

Figure 4. UDH model.

Figure 4 shows a hypothetical chain of these deuteron–electron pairs. We must remark that the hypothesis of existence of exotic forms of hydrogen is not new and has been proposed in different ways by many authors (Mills [31], Dufour [9], Mayer and Reitz [8,37], Krasnoholovets, Zabulonov and Zolkin [38] and many others). A very interesting result has been obtained by Jan Naudts starting from the Klein–Gordon equation for the hydrogen atom. Naudts found a low-lying eigenstate in witch "hydrogen" has a deep energy level $E_0 \approx -m_e c^2 \alpha \approx -3.728$ keV and a radius $r_e = \hbar/m_e c \approx 3.9 \times 10^{-13}$ m $(0.0039$ Å$)$ [39].

Indirect support for these hypotheses comes also from the numerous claims of observation of anomalous heat generation in metal–hydrogen systems. We must remark that these hypothetical "Compton Scale Composites" should be electrical neutral or negatively charged objects that cannot be stopped by the Coulomb barrier. For this reason they may generate unusual nuclear reactions and transmutations, considering the different sizes, time-scales and energies of this composites with respect to the particles (such as neutrons) normally used in nuclear experiments.

Mayer and Reitz, starting from a ZBW model of the electron, propose a three body system model at the Compton scale, composed by a proton and two electrons [8]. F. Piantelli, in patent application WO 2012147045 "Method and apparatus for generating energy by nuclear reactions of hydrogen adsorbed by orbital capture on a nanocrystalline structure of a metal", proposes an orbital capture of "H- ions" by nickel atoms in nano-clusters as a trigger for Low Energy Nuclear Reactions [40]. The orbital capture of the negatively charged structures at pico-metric scale described by Mayer and Reitz may be viewed as an alternative explanation to the capture of the much larger H- ions.

8. Conclusions

In this paper the authors want to underline that simplicity is an important and concrete value in scientific research. Connections between very different concepts in physics can be evidenced if we use the language of geometric algebra, recognizing also the fundamental role of the electromagnetic four-vector potential in physics.

The application of Occam's Razor principle to Maxwell's equations suggests, as a natural choice, a Zitterbewegung interpretation of quantum mechanics, similar but not identical to the one proposed by D. Hestenes. According to this framework, the electron structure consists of a massless charge distribution that rotates at the speed of light along a circumference with a length equal to electron Compton wavelength. Following this interpretation the electron mass–energy, expressed in natural units, is equal to the angular speed of the ZBW rotation and to the inverse of the orbit radius. Inertia has a pure electromagnetic origin related to the vector potential generated by the ZBW current. Moreover, in this framework the Heisenberg "uncertainty principle" derives from the relation between a particle ZBW radius and its angular momentum. The proposed model supports the ideas of some authors [3,8] that the ZBW may explain the existence of "super-chemical structures,“ such as ultra-dense deuterium, at pico-metric scale. A preliminary hypothesis on the structure of Holmlid's UDD, in which the ZBW phase of adjacent electrons are synchronized, has been presented demonstrating with good agreement Holmlid's experimental results. Pico-chemistry reactions and composites with intermediate energy values between nuclear and chemical ones can emerge as a key concept in understanding the origin of anomalous heat and the unusual nuclear reactions seen in many metal–hydrogen systems, as already suggested by some researchers in the field of condensed matter nuclear science.

"It is a delusion to think of electrons and fields as two physically different, independent entities. Since neither can exist without the other, there is only one reality to be described, which happens to have two different aspects; and the theory ought to recognize this from the outset instead of doing things twice!" – A. Einstein, cited in [41].

"In atomic theory, we have fields and we have particles. The fields and the particles are not two different things. They are two ways of describing the same thing, two different points of view" – P.A.M. Dirac, cited in [42].

Acknowledgements

Thanks to Salvatore Mercurio, former professor of physics at the North University of China (NUC), Taiyuan, Shanxi, for interesting discussions on the nature of electric charge and on hypothesis of existence of "super-chemical" reactions. Many thanks also to the reviewers for their interesting advice and beneficial suggestions and to Jed Rothwell for the English revision of the manuscript.

References

[1] D. Hestenes, Quantum mechanics from self-interaction, *Found. Phys.* **15**(1) (1985) 63–87.
[2] S. Badiei and P.U. Andersson and L. Holmlid. High-energy Coulomb explosions in ultra-dense deuterium: time-of-flight-mass spectrometry with variable energy and flight length, *Int. J. Mass Spectrometry* **282**(1–2) (2009) 70–76.
[3] L. Holmlid and S. Olafsson, Spontaneous ejection of high-energy particles from ultra-dense deuterium D(0), *Int. J. Hydrogen Energy* **40**(33)(2015) 10559–10567.
[4] K. Huang, On the zitterbewegung of the Dirac electron, *Amer. J. Phys*. **20**(8) (1952) 479–484.
[5] B.G. Sidharth, Revisiting Zitterbewegung, *Int. J. Theoret. Phys.* **48** (2009) 497–506.
[6] D. Hestenes, Mysteries and insights of Dirac theory, *Annales de la Fondation Louis de Broglie* **28** (2003) 3.
[7] D. Hestenes, *Hunting for Snarks in Quantum Mechanics*, In P.M. Goggans and C.-Y. Chan (Eds.), American Institute of Phys. Conf. Series, Vol. 1193, pp. 115–131, December 2009.
[8] F.J. Mayer and J.R. Reitz, Electromagnetic composites at the Compton scale, *Int. J. Theoret. Phys*. **51**(1) (2012) 322–330.
[9] J. Dufour, An introduction to the pico-chemistryworking hypothesis, *J. Condensed Matter Nucl. Sci.* **10** (2013) 40–45. www.iscmns.org/CMNS/JCMNS-Vol10.pdf.
[10] Y. Aharonov and D. Bohm, Significance of electromagnetic potentials in the quantum theory, *Phy. Rev.* **115** (1959) 485–491.
[11] D. Hestenes, *Reforming the Mathematical Language of Physics*, Oersted Medal Lecture 2002, 2002.
[12] D. Hestenes, Spacetime physics with geometric algebra, *Amer. J. Phys.* **71**(3) (2003) 691–714.
[13] G. Bettini, Clifford algebra, 3- and 4-dimensional analytic functions with applications, Manuscripts of the Last Century. viXra.org, Quantum Phys.:1–63, 2011. http://vixra.org/abs/1107.0060.
[14] D. Hestenes, Zitterbewegung modeling, *Found. Phys.* **23**(3) (1993) 365–387.
[15] D.L. Bergman and J.P. Wesley, Spinning charged ring model of electron yielding anomalous magnetic moment, *Galilean Electrodynamics* **1**(5) (1990) 63–67.
[16] D.L. Bergman and C.W. Lucas, Credibility of common sense science, *Found. Sci.*, Vol. 6, No 2 (2003) 1–17.
[17] Shau-Yu Lan, Pei-Chen Kuan, Brian Estey, Damon English, Justin M. Brown, Michael A. Hohensee, and Holger Müller. A clock directly linking time to a particle's mass. *Science*, January 2013. http://science.sciencemag.org/content/early/2013/01/09/science.1230767.full.pdf.
[18] F. Santandrea and P. Cirilli, Unificazione elettromagnetica, concezione elettronica dello spazio, dell'energia e della materia, 2006. Atlante di numeri e lettere. http://www.atlantedinumerielettere.it/energia2006/labor.htm.
[19] R.A. Treumann, W. Baumjohann and A. Balogh, The strongest magnetic fields in the universe: how strong can they become? *Frontiers Phys.* **2** (2014) 59.
[20] Q.-H. Peng and H. Tong, The Physics of strong magnetic fields in neutron stars, *Mon. Not. Roy. Astron. Soc.* **378** (2007) 159.
[21] D. Dew-Hughes, The critical current of superconductors: a historical review, *Low Temperature Phys*. **27**(9) (2001) 713–722.
[22] J. Dolbeault, M.J. Esteban and M. Loss, Relativistic hydrogenic atoms in strong magnetic fields, *Annales Henri Poincaré* **8** (2007) 749–779.
[23] J. Dolbeault, M. J. Esteban and M. Loss, Characterization of the critical magnetic field in the Dirac–Coulomb equation. ArXiv e-prints, December 2007.
[24] O. Consa, Helical model of the electron, *General Sci. J.* (2014) 1–14. http://gsjournal.net/Science-Journals/Research%20Papers/View/5523
[25] R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman Lectures on Physics: Mainly Electromagnetism and Matter*, Vol. 2, Basic Books, 2011.
[26] D. Hestenes, Real spinor fields, *J. Math. Phys.* **8**(4) (1967) 798–808.

[27] D. Hestenes, The Zitterbewegung interpretation of quantum mechanics, *Found. Phys.* **20**(10) (1990) 1213–1232.
[28] A.L. Parson and Smithsonian Institution, A magneton theory of the structure of the atom (with two plates), Number v. 65 in Publication (Smithsonian Institution), Smithsonian Institution, 1915.
[29] BlackLight Power, Periodic table of elements of the first twenty-electron-atoms solved with the grand unified theory of classical Physics. http://www.millsian.com/images/theory/Periodic-Table-Poster-light.pdf.
[30] A. Rathke, A critical analysis of the hydrino model, *New J. Phys.* **7** (2005) 127.
[31] R.L. Mills, J.J. Farrell and W.R. Good, *Unification of Spacetime, the Forces, Matter, and Energy*, Science Press, Ephrata, PA 17522, 1992.
[32] R.L. Mills, *The Grand Unified Theory of Classical Physics*, Blacklight Power, 2008.
[33] D.L. Bergman, The real proton, *Found. Sci.* **Vol. 3**, No 4, (2000). http://commonsensescience.org/pdf/articles/the_real_proton.pdf.
[34] F. Celani, E. Purchi, F. Santandrea, S. Fiorilla, A. Nuvoli, M. Nakamura, P. Cirilli, A. Spallone, B. Ortenzi, S. Pella, P. Boccanera and L. Notargiacomo, Observation of macroscopic current and thermal anomalies, at high temperature, by hetero-structures on thin and long Constantan wires under H_2 gas, *Int. Conf. on Condensed Matter Nucl. Sci., ICCF-19*, Padua, Italy, 13–17 April 2015.
[35] H. Hora, G.H. Miley, J.C. Kelly and F. Osman, Shrinking of hydrogen atoms in host metals by dielectric effects and inglis-teller depression of ionization potentials, *Proc. 9th Int. Conf. on Cold Fusion (ICCF9)*, pp. 1–6, 2002.
[36] L. Holmlid, Excitation levels in ultra-dense hydrogen p(-1) and d(-1) clusters: structure of spin-based rydberg matter, *Int. J. Mass Spectrometry* **352**(2013) 1–8.
[37] F.J. Mayer and J.R. Reitz, Thermal energy generation in the earth, *Nonlinear Processes GeoPhys*. **21**(2) (2014) 367–378. http://www.nonlin-processes-geophys.net/21/367/2014/.
[38] V. Krasnoholovets, Y. Zabulonov and I. Zolkin, On the nuclear coupling of proton and electron, *Universal J. Phys. Appl.* **10**(3) (2016) 90–103.
[39] J. Naudts, On the hydrino state of the relativistic hydrogen atom. ArXiv Phys. e-prints, July 2005.
[40] F. Piantelli, Method and apparatus for generating energy by nuclear reactions of hydrogen adsorbed by orbital capture on a nanocrystalline structure of a metal, Nov. 2012. WO Patent App. PCT/IB2012/052,100.
[41] C. Mead, *Collective Electrodynamics: Quantum Foundations of Electromagnetism*, MIT Press, Cambridge, 2000.
[42] A. Babin and A. Figotin, Neoclassical theory of electromagnetic interactions: a single theory for macroscopic and microscopic scales, *Theoretical and Mathematical Physics*, Springer, London, 2016.